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Study of Modulation Techniques for Multiple Access Satellite Communications

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Study of Modulation Techniques for Multiple Access Satellite Communications

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GODDARD SPACE FLIGHT CENTER

National Aeronautics and Space Administration

GREENBELT, MARYLAND

FEDERAL SYSTEMS DIVISION

International Business Machines Corporation

ROCKVILLE, MARYLAND

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SECTION 1

INTRODUCTION

1.1 Program Goals

This is the first of two technical reports concerning modulation techniques for communications via an active, stationary satellite. Multiple-access communication using small, relatively inexpensive ground stations is the principal interest in this study. Many of the terminals will probably be located in underdeveloped areas of the world. The goal is to specify a system configuration and to choose optimum modulation techniques for such a system. In this program there is no direct attempt to study multiple-access to and between "gateways" (large stations located in the vicinity of major cities), although some of the techniques developed here should be of interest to this problem as well.

The major goals of the first half of this program, the results of which are included in this report, are:

a. Specification of system configuration, system parameters, performance requirements, and a criterion for selecting candidate modulation techniques.

b. Selection of several candidate modulation techniques which appear to be promising and worthy of further, more detailed study during the final phase of the program, and preliminary specifications of a system design framed around these techniques.

c. Development of the background material necessary for the more detailed and final phase of the study.

To reach these goals, four major areas of study were pursued:

1. Specification of Design Point System Model (DPSM)
2. Conventional Multiplexing Techniques
3. Pseudo-Noise (PN) Multiplexing Techniques
4. Special Device Studies

The first task is completed; the results will, of course, be used in the final phase, just as they were used during this phase of the program.

Conventional multiplexing (which also received detailed attention during the first phase) will likewise continue with emphasis narrowed to two techniques. These will be:

SSB/FDM-up; composite FM-down

PCM/TDM-up; TCM/TDM down

PN multiplexing has been explored in detail during the first phase, and will continue to receive attention during the last phase with emphasis narrowed to two techniques, viz.,

PN modulation with matched filter reception

PN modulation with correlation-locked reception

Special device studies received some attention during the first half of the program. Principal emphasis was on the Travelling Wave

Tube (TWT) and its effect on transmission for various types of modulation. Detailed study of the TWT will continue through the remainder of the program. Particular emphasis will be on optimum use of the tube for power amplification of PN and conventional signals. The hard limiter and its effect on the selected modulation techniques will also be studied in more detail along with other important devices.

1.2 Introduction to Report

This report consists of an introduction and four major technical areas corresponding to the major areas of study. The last section contains conclusions and recommendations.

Section 2, "Evaluation Procedures and Criteria," specifies the Design Point System Model and includes system performance objectives, and a discussion of criteria for selecting a system's modulation technique.

Section 3, "Satellite Communications Using Conventional Multiplexing Techniques," classifies the conventional modulation techniques, and discusses time division and frequency division multiplex operations. Seven candidate combinations of conventional modulation and multiplexing are subjected to preliminary analysis. Functional block diagrams of ground transmitters and receivers, and satellites are presented for the two candidates that are found to qualify. Estimates are also presented for transmission bandwidth and ground transmitter power requirements for the two qualifying techniques.

Callup functions are analyzed for operation with the conventional modulation systems. Calculations are presented for two modes of operation, viz., common channel and narrow-band channel. Callup system functional block diagrams are also presented.

Section 4, "Satellite Communications Using Pseudo-Noise (PN Multiplexing Techniques," lays the background for understanding this class of techniques, develops the mathematical theory of PN multiplexing, and specifies the detailed logical configurations of the two techniques chosen for further study. Sections 4.1 - 4.4 develop the PN concepts and techniques for the engineer who is well-rounded in communications and modulation techniques but who is not familiar with this relatively new field.

Section 4.5 deals with the theory of asynchronous multiplexing. Here, the channel capacity, signal-to-noise ratio, and error probabilities for common channel PN systems (with various detection schemes) are the important results. It is not essential to go through the details of the mathematical analysis if one is satisfied to accept the final result.

Section 4.6 discusses the logical configurations of transmitters, receiver, and callup logic in detail for the two PN candidates chosen for further study.

Section 5, "Special Device Studies" summarizes the pertinent literature concerning TWT's and discusses the potential influence of the tube's characteristics on multiple access communications performance.

Section 6 discusses the conclusions drawn from this study, and recommends four modulation techniques and systems (two conventional and two pseudo-noise) for further study.

SECTION 2

EVALUATION PROCEDURES AND CRITERIA

This section presents the results of Phase I of the Modulation Techniques Study. The purpose of this phase was to study evaluation procedures and criteria. In Subsection 2.1 the system problem is defined in its generic form, prospective evaluation procedures are discussed, and the specific procedure selected for evaluating candidate modulation techniques is presented. In Subsection 2.2 the details of the Design Point System Model (DPSM) are presented and discussed. In Subsection 2.3 a discussion of evaluation procedures of a more generalized nature is undertaken, both to indicate some of the evaluation procedures that were rejected, and to outline the generalized analyses that will be continued throughout the remainder of the study.

2.1 Problem Statement

2.1.1 Generic Problem Formulation

The general problem of interest to the study can be described as that of providing access to a communication satellite from a large number of small and intermediate size ground stations. The specific type of communication satellite to be investigated is a stationary satellite employing an active repeater. Operation will be in the microwave band near 5 gc/s. Though these frequencies will be shared with other services, power flux densities will be controlled.

The ground stations to be employed may have different effective radiated powers or receiver sensitivities. In addition, the stations will present amounts of traffic which vary with time, indicating that fixed channel assignments will not be efficient. Some arrangement is, therefore, necessary to permit the actual number of channels available through the satellite to be much less than the number of potentially required channels. For these relatively small ground stations (compared with the large "gateways"), it is assumed that the principal purpose of the communication satellite system is to provide "satisfactory" voice channels, not necessarily channels which would be characterized as "excellent," or "toll quality."

Within the generic problem outlined above, an investigation is to be conducted to determine the optimum modulation technique for future satellite systems to provide an improved degree of multiple access. Where system model parameters indicate that different modulation techniques would be preferable for different system configurations, this factor is to be reflected in the study recommendations.

2.1.2 Prospective Evaluation Criteria

The many criteria that have been employed in past studies of multiple access communication satellite systems are discussed in Subsection 2.3. Ideally, all of these criteria might be combined in a single comprehensive equation which attempts to calculate the "goodness" of a system. Unfortunately, as indicated in 2.3.2, this approach has several important shortcomings:

1. The goodness function is usually difficult to formulate even in an approximate way.
2. The procedure for maximizing the goodness function is not clear.
3. Trade off procedures are difficult to obtain.

For these reasons, a less generalized approach was sought, with the principal problem being to choose the most appropriate evaluation procedure for combining and employing the multitude of available evaluation criteria. At the other end of the generality spectrum, all but one of the criteria and parameters could be fixed and various modulation techniques could be evaluated to determine which technique obtained the "highest" score for the variable criterion. A "middle ground" between this restricted approach and the completely generalized approach is desired.

The essence of the problem of selecting an evaluation procedure is, therefore, to reduce the generality of the analysis to the point where it is most compatible with the scope of the study and provides the most meaningful results. This reduction requires the selection of the most significant criteria and the restriction of the values of as many of the system parameters as possible.

The many parameters and criteria associated with a system can be conveniently subdivided into "cost-related" and "effectiveness-related" categories. Since a quantitative determination of cost is not the intention of this study, the term "equipment penalty" or "equipment complexity" will be used in the remainder of this report to refer to cost-related items.

2.1.3 Selection of Evaluation Procedure

Equipment complexities are extremely important in this study which is concerned with the small station problem. The very reason that small stations exist is because the user cannot afford or cannot justify the expense of providing a high performance terminal for his limited traffic requirements. For this reason, in this study, the equipment complexity required to implement systems of prescribed performance will be the standard for judging candidate modulation techniques.

The remaining task in the selection of an evaluation procedure is the restriction of an appropriate number of the system parameters. The approach of fixing <u>all</u> parameters is often preferred when a system is close to its final definition. At that time, many restrictions have been imposed; hence, there is not nearly as much flexibility for optimizing the modulation technique. Where possible, however, the establishment of rigid specifications on all portions of a system is to be avoided prior to the selection of some of the more important approaches to the system design. Flexibility in the selection of system parameters is needed if each modulation technique candidate is to be given a fair appraisal.

Since the particular stationary communication satellite system of concern to the study has not yet been fully defined, it is neither necessary, nor desirable to affix values to all system parameters. Several of the parameters, however, are fairly well established, either by international agreements (these will serve as guidelines) or by equipment capabilities and costs. In addition to these "primary" parameters, there are "secondary" parameters that have not as yet been specified. This flexibility should be reflected in the optimization of the system design for each candidate modulation technique.

Thus, the procedure that has been selected for evaluating candidate modulation techniques is to emphasize a particular (but not complete) set of "primary" system parameters. The parameters for this Design Point System Model are presented and discussed in Section 2.2. Candidate modulation techniques which meet the requirements inherent in this system model will be proposed and a recommendation based on system complexity will be made.

An inherent part of the design procedure for optimizing each candidate modulation system will be the consideration of some of the more important criteria discussed in Subsection 2.3. Sound engineering judgment must, of course, be employed in the design optimization process to reflect such factors as, e.g., satellite reliability, spectrum utilization, expandability, graceful degradation, etc.

2.2 Design Point System Model

This subsection presents the parameters of the Design Point System Model (DPSM) and the reasons for their selection. Variations to the DPSM that will be examined subsequent to the detailed analysis are also included, as well as a discussion of methods for extrapolating from the design point parameters.

2.2.1 Characteristics of the DPSM

2.2.1.1 Satellite Parameters

The most significant parameters in the satellite equipment are the antenna gain and the RF output power. These are assumed to be 18 db and an average of 10 watts (at a nominal frequency of 5,000 gc/s) respectively. For the purpose of intermodulation noise analysis, the transmitter output stage is assumed to be a travelling wave tube.

2.2.1.2 Ground Station Parameters

It is assumed that 50 ground stations are simultaneously using the communications satellite system. Forty of these stations are of the "small" variety (Type I), employing a 15 ft parabolic antenna and a 600°K receiver. Ten of the ground stations are of "intermediate" size (Type II), employing a 30 ft parabolic antenna and a 300°K receiver. Each of the small stations is to be capable of transmitting and receiving a minimum of two voice channels; each of the larger stations, a minimum of 12 voice channels. Thus, the satellite is required to transmit a total of at least 200 one-way voice channels, i.e., 100 two-way conversations through the system.

The callup and channel assignment system must be sufficiently flexible to permit re-allocation of unused channels to busy ground stations. The system must provide service for at least, say, 1000 separate addressees (i.e., sets of receiving equipment). (Actually, system capability is limited only by the number of active users, not by the number of subscribers.) Ten seconds will be acceptable for the callup time, but one second is desirable. A "busy signal" is required. Provisions for queueing and priority assignment should be considered in the system design.

With respect to the quality of the voice channels, the system objectives call for a 20 db signal-to-noise ratio (sinusoidal tone in a 4 kc/s bandwidth) at the output of the ground station receiver, if analog modulation techniques are to be used. For digital transmission of voice, a 20 kilo-bits per second data rate, with a bit error probability of 0.01 is needed.

The objective for the design margin for the satellite-to-ground link is 5 db. This figure will allow for miscellaneous losses, and for the degradation of component performances.

Tables 2-1, 2-2, and 2-3 summarize the parameters of the DPSM.

2.2.2 Justification for DPSM Parameters

2.2.2.1 Satellite Parameters

The gain achievable from the satellite antenna is dependent on the global coverage requirement, and is further restricted by the stabilization capabilities of the satellite equipment. The 18 db assumption is based upon the type of stabilization equipment which should be available in advanced synchronous satellites. The selection of an appropriate value for the transmitter RF power output is based upon projected equipment capabilities and international agreements. The TWT is generally accepted as the most likely output stage for the near future, because of its reliability and bandwidth capabilities. Projections

TABLE 2-1
SATELLITE CHARACTERISTICS

Antenna Gain*	18 db
RF Power	10 watts
Processing	Variable

* Assuming global coverage

TABLE 2-2
GROUND TERMINAL CHARACTERISTICS

	Antenna Diameter	Effective Receiving Effective Receiving	Equivalent Noise Band (with satellite specified)
Type I	15 feet	600° (Tunnel Diode)	11 mc/s
Type II	30 feet	300° (Parametric Amp.)	90 mc/s

TABLE 2-3
VOICE CHANNEL REQUIREMENTS

Analog	4 kc/s bandwidth with S/N=20db
Digital	20 kb/s data rate with P_e =0.01

of the state-of-the-art indicate that reliable, space-worthy TWT's capable of 10 watts of (undistorted) RF power output at 5 gc/s will be available in the next few years. Moreover, international agreements reached at the 1963 International Telecommunications Union meeting for the maximum power flux density for communication satellites establish approximately the same output power level. If the Range Equation is solved with the specified maximum received power density of -130 dbw per square meter*, an 18 db satellite antenna and a synchronous altitude satellite, the satellite power output must be less than 25.4 watts as shown below:

$$\frac{S}{A_R} = \frac{P_S G_S}{4 \pi R^2}$$

where

$\frac{S}{A_R}$ = received power density (=130 dbw/m^2)

P_S = transmitted power

G_S = transmitting antenna gain (= 18 db)

R = distance between satellite and receiver

As an example, further details regarding power density limits specify that if wide deviation frequency or phase modulation is used, the power in any 4 kc/s band shall not exceed -149 dbw per square meter. In the case of other types of modulation, the allowable figure

*The agreement does not specify whether this figure is an average, a peak, or a value not to be exceeded a certain percentage of the time.

is -152 dbw/m^2 in a 4 kc/s band.

To analyze the effects of signal distortion it will be necessary to select a particular output/input voltage characteristic as representative of the transfer characteristic for spacecraft traveling wave tubes. A fourth order polynomial characteristic[1] (superscript is reference) is currently being evaluated for the purpose of studying the effects of intermodulation distortion. For the operating region of interest, this polynomial reduces to the sum of a linear term and a cubic term.

With pseudo-noise techniques, phase distortion is more critical than non-linear amplitude distortion. TWT characteristics are being studied with the aim of finding a procedure for choosing an operating point which maximizes the signal-to-noise ratio at the output of a coherent receiver.

In the pseudo-noise case it is desirable to multiplex in the satellite by using a hard limiter preceding the TWT. This allows a more constant level signal to be applied to the TWT. This should improve performance. It is well known that for a signal-to-noise ratio input of less than unity, the output signal-to-noise ratio of a hard limiter is reduced approximately 1 db.

2.2.2.2 Ground Station Parameters

Various estimates have been made for the total number of ground stations in a multiple access communication satellite system.[2] In general, these range between 10 and 100. The number 50 was selected as representative of this range.

The equipment capabilities of the "small" (Type I) station were essentially determined by establishing low cost as a major objective. With this in mind, the decision was made to select a tunnel diode receiver, and to use a 15 foot diameter antenna. The characteristics of the "intermediate" (Type II) size ground station were determined by the necessity for reflecting a substantial difference in station capabilities. A 9 db difference in receiving capability was selected. This improvement can be obtained by using a ground station antenna of twice the diameter (+6 db), and a receiver with half the effective temperature (+3 db) of the Type I station. The better performing receiver might be an uncooled parametric. This differential suggests the possibility of different signal strengths depending upon the nature of the transmitting and receiving ground stations. The specific number of allowable active voice channels in the system is derived from receiver threshold considerations and ground station equipment penalty factors. Using the example of a single sideband up, phase modulation down system, the receivers in the small ground stations will be just at threshold with 200 channels, assuming the use of feedback receivers and the system margin objective of 5 db for the down link. To provide the flexibility that is desired in all multiple access systems, it is necessary to have additional equipment available at the ground stations for use with channels that have been released by the stations to which they are normally assigned. Many elements of the ground station receiving and transmitting equipment have costs that vary linearly with the number of channels.

The analog signal-to-noise ratio and digital bit rate and bit error probability level have been selected to provide the desired "satisfactory" voice channels. In determining the relationship between the analog signal-to-noise ratio specification and international standards, it is first necessary to convert the S/N to a test-tone-to-noise ratio. It has been shown that for 240 channels and CCIR-CCITT performance, using single sideband transmission, the conversion from signal-to-noise ratio of a sinusoid in a 4m kc/s band to a test tone-to-noise ratio in a 3.1 kc/s band (psophometrically weighted) results in the following approximate relationship:

$$(T.T./N)_{\substack{3.1\ kc/s\ ch.\\ \text{psoph. weighted}}} = [10 \log_{10} (S/N)_{4m\ kc/s} + 9]\ db$$

where $(S/N)_{4m\ kc/s}$ is the signal-to-noise ratio in a 4 m kc/s bandwidth.[1] The resultant test tone-to-noise ratio is therefore 29 db. An examination of the assumptions behind the above equation indicates that the error resulting when applied to a 200 channel system can be expected to be small. Equivalent equations are available for other types of modulators. The use of compandors has generally been assumed to be necessary to achieve toll quality channel performance. (See for example, references 1 and 2). The reduction in noise achievable through the use of compandors has been variously cited to range from 13 db to more than 30 db.[1 & 4] Thus, depending upon the investment in compandors and the capabilities of advanced compandors, the resulting voice signal may approach quite closely the standards for international communication. Additional uncertainties, such as the noise that will be introduced on the land lines in the areas served by the small ground stations--and the S/N requirements compatible therewith--preclude the possibility of finding an invariant requirement which will be guaranteed to satisfy the system requirements without undue expenditure. In this light, it is important to point out that variations in the system requirements can be readily extrapolated from the design point system by examining the effects of the Range Equation and adjusting some of the system parameters (e.g., ground station receiving capabilities) to meet the new requirements. Certain of the variations discussed in Subsection 2.2.3 will consider the possibility of providing improved voice channel performance.

The foregoing remarks apply equally well to digital voice communications. The bit rate and bit error probability selected have again been chosen to provide a "satisfactory" voice channel.[5]

2.2.3 Variations of the DPSM Parameters

Subsequent to the detailed investigation of modulation techniques for the DPSM parameters, variations of these parameters will be considered to provide recommendations for systems other than the design point problem. Some examples of the variations that may be examined are:

a. Data will be considered as a system transmission requirement in combination with voice; the system parameters will otherwise remain fixed.

b. A smaller number of stations with the same system parameters but more stringent voice channel requirements.

c. Larger stations, with the same total number of stations but with more stringent voice channel requirements.

d. A larger number of stations with the same parameters as the DPSM will be examined.

e. Smaller ground stations in the same numbers as for the DPSM.

f. A different TWT transfer characteristic with the system parameters otherwise unchanged.

g. A different type of transmitter output stage (e.g., electrostatic klystron, amplitron, varactor harmonic generator) with the same system requirements.

h. An increased number of ground station types.

i. A more restricted RF bandwidth (e.g., 10mc/s or less)

j. Teletype channels only, with smaller ground stations.

2.3 Generalized Evaluation Procedures

This section discusses efforts made during this study to bring together, in a single comprehensive equation, the quantities affecting transmission. This generalized analysis is a continuing effort which will be pursued for the remainder of the study in parallel with the principal evaluation procedure discussed in Subsections 2.1 and 2.2. Progress to date on the generalized evaluation procedures is included in this report to draw attention to: (1) the way parameters enter into system performance, (2) the improvement to be gained using different modulation processes, and (3) the effect of interfering signals.

2.3.1 Previous Studies

Previous communication satellite studies have been principally concerned with fixed load gateway-to-gateway transmission. Even the studies addressed to the multiple station access problem have not dealt with the use of small stations, and often the problems of varying traffic and call-up procedures are not analyzed in depth.

These studies have been directed to the problem of commercial telephonic transmission; they, therefore, usually have assumed CCIR-CCITT recommendations. The resulting performance would be quite high; particularly with regard to signal-to-noise ratio. The DPSM objectives are somewhat lower, but are also aimed at providing a high level of intelligibility; and with the addition of techniques such as companding, performance should come close to approximating CCIR-CCITT recommendations.

In the past, modulation studies have made evaluations in terms of system communication channel parameters, number of available channels, bit error rate, signal fidelity with increasing traffic load, access time and flexibility of access, system complexity and growth potential, compatibility with existing communication links, reliability and economy; but in the case of specific communication satellite programs, more than anything else, feasibility within a very short time period was required.

The last consideration has often led to pairs of recommendations; one for the near future and a second for the period when space components are more fully developed. The near-time recommendation usually consists of a higher power, narrow band, FDM up-link; aboard the satellite, a frequency shift, and possibly a compound frequency modulation of the composite signal or an increase of depth of modulation if only one signal is present; and a low power, wide band transmission on the down-link. The reasoning here is that the up-link arrangement conserves RF spectrum, while the down-link makes a necessary, but undesirable, trade of bandwidth to make up for the lack of RF power aboard the satellite. The assumption of minimal signal processing in orbit is made for the sake of reliability.

Future system recommendations usually assume major (sometimes orders of magnitude) increases in satellite RF power; and possibly component reliability that would allow significant on-board signal processing. The in-orbit processing could reduce ground transmitter requirements, and reject unintentional interfering signals.

Technical improvements may make higher power feasible; but the necessity of sharing frequencies with other services using the same part of the radio spectrum requires limiting power densities as noted in Subsection 2.2.2.1. Therefore, future com-satellite down-links must either use very sensitive ground receiving stations (with narrow beam antennas, to restrict signals from unwanted sources), or some form of modulation that provides an improvement factor. Because this study is concerned with small stations, the emphasis will be on modulation that offers signal-to-noise improvement.

Pseudo-noise multiplexing as a possible approach to multiple access satellite systems has received very little attention. The techniques are relatively new. They have been applied to military anti-jam communication systems, to interplanetary ranging, and to communications over interplanetary distances. These techniques have certain properties not shared by conventional systems, which appear attractive for the multiple access problem considered here.

2.3.2 Minimum Qualifying Performance

A simple preliminary calculation can be made that will identify techniques with significant deficiencies, thus avoiding the necessity of completely evaluating every conceivable type of modulation. Briefly, the procedure is this:

Since the down-link is the least substantial, solving the Range Equation (assuming a Type I DPSM ground station) for signal-to-noise ratio will identify the modulation and multiplexing combinations that qualify, come near, or are far from qualifying. This is particularly true for conventional techniques. In this qualifying S/N calculation, certain factors are intentionally ignored; these are, noise from the up-link, intermodulation, and interference from other sources. The equation then is

$$S/N = \frac{P_S G_S A_R K_1 K_2}{4\pi R^2 M k T_R B_R n D}$$

The majority of the symbols are those commonly used in Range Equation calculations; n is the number of channels, D is a derating factor applied to the TWT (to avoid saturation with high peak signals), and K_1 and K_2 are modulation improvement factors. All are explained in the Glossary. The equation assumes operation above threshold, a condition that must be determined in a separate calculation for every system.

In the case of pseudo-noise techniques the useful part of the Range Equation is simply,

$$W_N = \frac{P_R}{N_o} = \frac{P_S G_S A_R}{4\pi R^2 M\, k T_R}$$

where P_R is the received signal power, and W_N is the equivalent noise bandwidth (i.e., the band in which the ratio of signal-to-thermal noise is 1). With knowledge of W_N and the channel bandwidth, the performance of pseudo-noise multiplex systems can be evaluated.

2.3.3 Effectiveness of Use of the Transmission Medium

After qualifying in preliminary tests, modulation-multiplexing schemes should be examined to determine the effectiveness of their utilization of the transmission medium. Here, the word "utilization" has the special meaning of consumption, thus hindering or preventing the use by others. The medium can be thought of in terms of its degrees of freedom, viz., frequency, time, and spatial separation.

Since even "small" ground antennas form very narrow beams ($\theta < 1^{\circ}$), communication satellite systems can be thought of as using a form of space multiplexing with respect to each other. However, this is not necessarily so for other services sharing the same frequencies; hence, the need for restricting the power density to -130 dbw/m^2.

An interesting measure of performance is channels per unit of bandwidth on the down-link. This can be found by rearranging the equation used in the preliminary qualification procedure.

2.3.4 Practicality of Critical Components

In the analysis of effectiveness it will be necessary to make a number of estimates regarding component performance. Before final recommendations are made, it should be determined whether, indeed, the assumed operation can be obtained in the desired time period at a reasonable cost. The relative costs of the principal techniques should be determined to assure that technical comparisons are being made consistently.

2.3.5 Reliability

The chief consequence of unreliable components is increased system cost. There will be a natural interest in the effects of putting as much complexity as possible aboard the satellite; therefore, it is of interest to see how satellite repeater expected life affects costs.

The total system cost can be written as (see Glossary for definition of symbols):

$$C = C_o + T_s \cdot C_a$$

The two parts of the cost can be expanded as follows:

$$C_o = N_t \cdot C_t + \frac{N_r}{P} \cdot C_r + \frac{N_r}{P\ N_p} \cdot C_b$$

$$= N_t \cdot C_t + \frac{N_r}{P}\left(C_r + \frac{C_b}{N_p}\right)$$

$$C_a = N_t \cdot C_m + \frac{N_r}{P \cdot T_r} \cdot C_r + C_b\left(\frac{N_r}{P \cdot T_r \cdot N_p}\right)$$

$$= N_t \cdot C_m + \frac{N_r}{P \cdot T_r}\left(C_r + \frac{C_b}{N_p}\right)$$

The complete expressions can now be combined and rearranged to show terminal and orbital relay costs

$$C = \overbrace{N_t(C_t + T_s \cdot C_m)}^{\text{Terminal Costs}} + \overbrace{\frac{N_r}{P}\left(1 + \frac{T_s}{T_r}\right)\left(C_r + \frac{C_b}{N_p}\right)}^{\text{Relay Costs}}$$

Differentiating the above with respect to T_r gives the rate of change of cost with respect to the life relays.

$$\frac{dC}{dT_r} = -\frac{T_s N_r}{T_r^{\,2} P}\left(C_r + \frac{C_b}{N_p}\right)$$

For the system under analysis, N_r and $N_p = 1$, and, in the advanced time period, P will also be almost one. Further, C_b, the cost of the booster, may be expected to be many times C_r, the cost of the repeater. Therefore,

$$\frac{dC}{dT_r} \approx - \frac{C_b T_s}{T_r^2}$$

Because C represents the total communication satellite system cost, the cost per channel is C divided by the number of channels. Similarly, the rate of change of cost per channel, for varying satellite life, can be found by dividing the above equation by the number of channels.

2.3.6 Growth Potential

Any of the following could constitute growth:

a. Enlargement of the number of subscribers (without increasing the system capacity), perhaps, to take advantage of more efficient operational procedures.

b. Increased numbers of channels terminated at each ground station (without increasing the system capacity). With some of the candidate modulation techniques, even at the "small" (Type I) stations, all of the system channels are received with adequate carrier-to-noise ratio to be useful.

However, each channel must be provided with a demodulator, (and probably) a compandor, an echo suppressor, and other ancillary equipments.

c. Increased overall system capacity by "up-grading" ground receiver performance, say, by installing larger antennas, and/or more sensitive receivers, and in general "cleaning-up" front-end components to reduce their losses and noise contributions. To grow by "up-grading," it must be assumed that each of the Type I station channels can be traded for a larger number of channels if more sensitive receivers are used.

d. Increased satellite capacity. However, there is little room here for expansion without violating one or more of the "ground-rules." For example, the restriction on power density would allow about 3 db more radiated power, and global coverage with a single antenna limits gain to 18 db. (Very narrow beams directed to individual receivers is a conceptual possibility, but this would violate the ground rules and would be a markedly different system.)

Another way to increase system capacity is through more efficient signalling. This possibility applies principally to spread-spectrum modulation systems that are clutter-limited. If faster-acting, lower price components become available, larger time-frequency products may be used to reduce self interference.

GLOSSARY

A_R = Aperture of ground receiver antenna (effective)

Type I -- 100 ft.^{2} } DPSM values.
Type II -- 400 ft.^{2}

A_S = Aperture of satellite receiver antenna (effective).

= $G_S \lambda_1^2 / 4\pi$.

A_T = Aperture of ground transmitter antenna (effective).

= Same as A_R .

B = Channel bandwidth.

= 4000 cycles per second, DPSM value.

C = Cost, total system ($).

C_b = Cost of launching one booster ($/booster).

C_m = Cost of operating and maintaining one terminal for one year ($/terminal × year).

C_r = Cost per relay ($/relay).

C_t = Cost per terminal, original ($/terminal).

D = Derating factor for satellite RF power output to reduce intermodulation and limiting (dimensionless).

G_R = Gain of ground receiver antenna.

= $4\pi A_R / \lambda^2$.

DPSM = Design point system model.

G_S = Gain of satellite receiver and transmitter antennas.

= 63 = 18 db, DPSM value.

G_T = Gain of ground receiver antenna.

= $4\pi A_T / \lambda_1^2$.

k = Boltzmann's constant.

= 1.38×10^{-23} (Watts sec/Deg. K).

K_1 = Up-link modulation improvement factor, (dimensionless).

= A variable parameter.

K_2 = Down-link modulation improvement factor, (dimensionless).

= A variable parameter.

m = Number of multiplexed channels.

M = Margin, up-link, (dimensionless).

= 5 db, DPSM value.

N = Noise, (Watts).

N_o = Noise density, (Watts/cycle per second).

N_p = Number of relays launched per booster (relays/booster).

N_r = Number of relays, minimum, in the system at any time (relays).

N_t = Number of terminals (terminals).

P = Probability of successfully launching a booster (dimensionless)

P_R = Signal power (total) received from satellite, (Watts).

P_S = Undistorted satellite RF power output.

= 10 Watts, DPSM value.

R = Range from ground station to satellite.

= 1.3×10^8 ft. (representative), DPSM value.

S = Signal power, (Watts).

TT = Test tone power, (Watts)

T_r = Time, length of expected life of each relay (years).

T_R = Temperature (system effective noise) of ground receiver.

Type I $600^\circ K$ DPSM

Type II $300^\circ K$

T_s = Time, length of system life (years).

W_N = Equivalent noise band, (c/s).

λ_1 = Wave length used on up-link.

λ_2 = Wave length used on down-link.

SECTION 3

CONVENTIONAL MULTIPLEXING TECHNIQUES

3.1 Classical Modulation Techniques

The term classical modulation techniques, as used in this report, refers to non-pseudo-random methods. This includes the generic types, viz., amplitude modulation, angle modulation, pulse time modulation, and pulse code modulation, plus all their variations.

3.1.1 Classification by Baseband Treatment

One useful scheme of classifying the techniques is based on the treatment of message waves at baseband. The breakdown is:

a. Continuous-time, continuous-amplitude

b. Continuous-time, discrete-amplitude

c. Discrete-time, continuous-amplitude

d. Discrete-time, discrete-amplitude

Examples of modulation techniques fitting these four categories are shown in Figure 3-1. Continuous-time and continuous-amplitude means that all points in the original wave are represented in the transmitted signal. Discrete-amplitude refers to quantization, and discrete-time means that a single value of the message wave is used to represent the wave during some time period (most likely the Nyquist interval, which is the inverse of twice the highest frequency component). There seems to be no useful example of continuous-time, discrete-amplitude modulation. Combinations of modulation are often used, for example, PCM-PM.

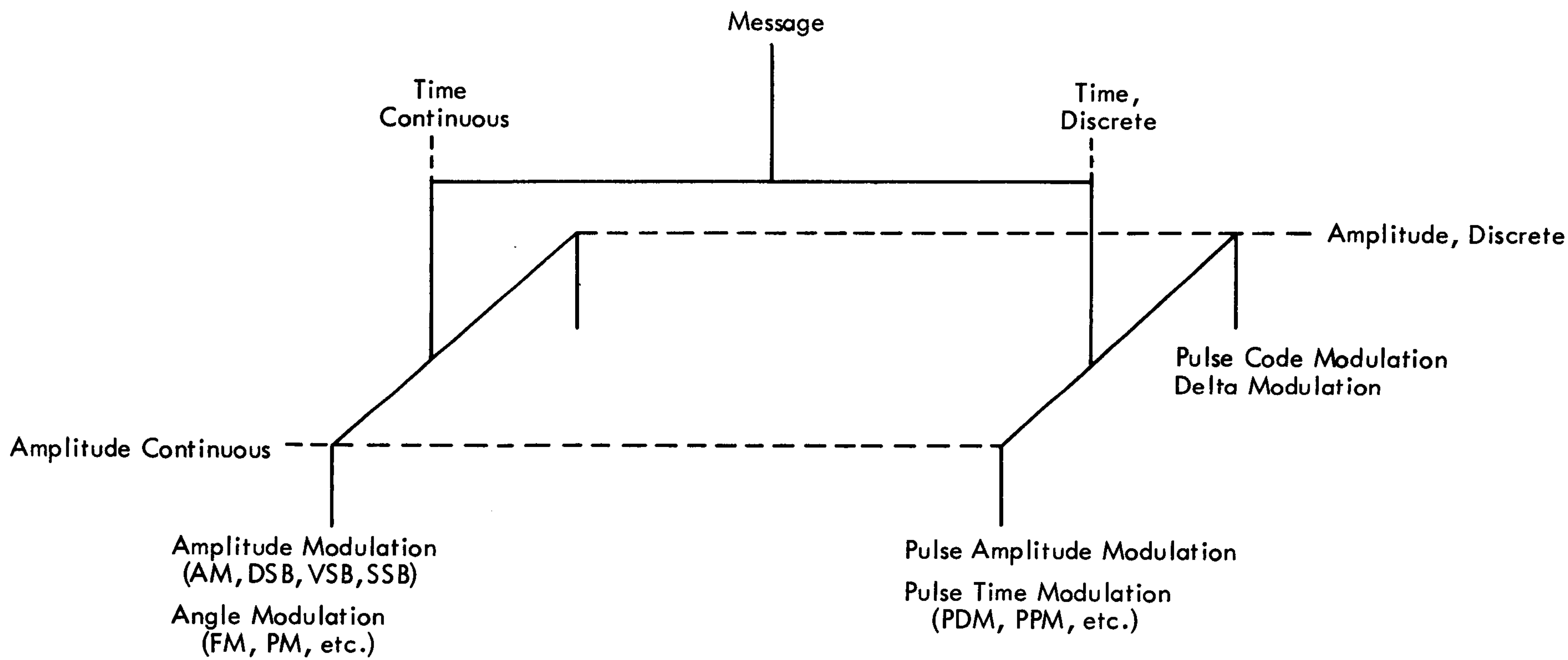


Figure 3-1. Classification of Conventional Modulation Techniques

3.1.2 Time Division Multiplexing

Any signal (continuous- or discrete-time) can be broken into sections, stored, and sent out with high data rate (or wide bandwidth) bursts. This allows time division multiplexing (TDM) of both continuous- and discrete-time signals. There are several possible benefits to be derived from burst transmission. First, only one signal is present at a time; thus, if each signal has a limited and constant-peak amplitude (as with FM), a peak power limited repeater can be operated at its maximum level; that is, no derating is necessary to allow for the peaks that will result when a number of unrelated signals are added. Also, because only one signal is present at a time, no channel cross-talk is produced (barring TDM difficulties).

If the bursts are long enough, there is a second advantage to burst transmission, viz., the time synchronization problem is reduced. Since the minimum delay with a stationary satellite is about 250 milliseconds, the added delay for burst transmission may be quite large without influencing the system's performance. Therefore, with the same fraction of time allowed for time guard slots, timing accuracy throughout the complex of stations need not be kept as well as if signals were interleaved bit by bit, or "word" by "word."

A third advantage of TDM transmission in burst comes from the possibility of sharing terminal ancillary equipments such as encoders, compandors, etc.

The chief disadvantage of sending in bursts stems from the need for storage at transmitters and receivers. Storage readout must be very fast, and any shared ancillaries will also have to be capable of high speed (wide band) performance.

3.1.3 Frequency Division Multiplexing

Although probably more commonly accepted, frequency division multiplexing (FDM) is a more sophisticated method of separating transmission than TDM. Time division is based on one user at a time, thus no cross effects. FDM, on the other hand, depends on the fact that when two sinusoids of different frequencies are multiplied together, the average value is zero. That is, sinusoids are overlapping "orthogonal" waves, while time division systems depend on non-overlapping waves. There are other types of overlapping orthogonal waves, and recently there has been considerable interest in quasi-orthogonal wave forms. In systems that use quasi-orthogonality, a specification is placed on the amount of interaction allowed between the different signals. The interaction (or crosscorrelation as it is called) may be made as small as desired. Quasi-orthogonality will be taken up in more detail in Subsection 3.2.

FDM demands knowledge of frequency by all parties involved. Motion of, say, the repeater will cause a doppler frequency shift. Fortunately, with a "stationary" satellite, relatively small shifts can be expected. Without careful frequency corrections, however, substantial frequency guard bands (analogous to time guards in TDM) will be required.

With FDM a large number of independent signals will be added together at the satellite receiver. The "central-limit" theorem states that the sum of a large number of independent random variables will approach the gaussian distribution no matter what the distribution of the components, provided that no one of the components is comparable with the sum. Thus, even if constant amplitude signals, such as FM, are used, the combined FDM signal through the satellite will have a gaussian amplitude distribution.

Since the gaussian distribution extends to plus and to minus infinity, any physically realizable repeater must clip parts of the composite signal. The clipping can be thought of as adding the negative of the amount of clipping at the instants of clipping. This "added" wave is, of course, distortion. The distortion will be made up of many frequencies, some of which will fall outside of the composite signal frequency band, and therefore may be rejected; but some will fall in the band, and be of consequence.

Because part of the clipping results from the combination of two or more signals, the resulting distortion is termed intermodulation. Intermodulation may be suppressed, in part, by the types of modulation that suppress thermal noise. The degree of suppression depends both upon the type of modulation and the nature of the intermodulation.

It is common practice to specify the percent of time clipping is allowed. The figure 0.003% was used in Reference 6. Recalling that the average power of a gaussian distributed signal is the square of its standard deviation (and working from a table for cumulative probability) it is possible to find the clipping level-to-average power ratio. See Figure 3-2. Because TWT ratings are expressed for a sine wave (which has a two-to-one peak-to-average power ratio), the clipping level-to-average ratio must be divided by two to obtain the derating factor D. For 0.003% clipping D must be 9 (i.e., 9.5 db).

Because of the fortunate constant geometry of the stationary satellite, it can be expected that all channels will arrive at the repeater with approximately the same signal strength. Therefore, there should be no difficulty with a strong signal capturing the repeater, as might occur with some types of modulation. If all n signals arrive with equal strength at a repeater, each retransmitted signal can be assumed

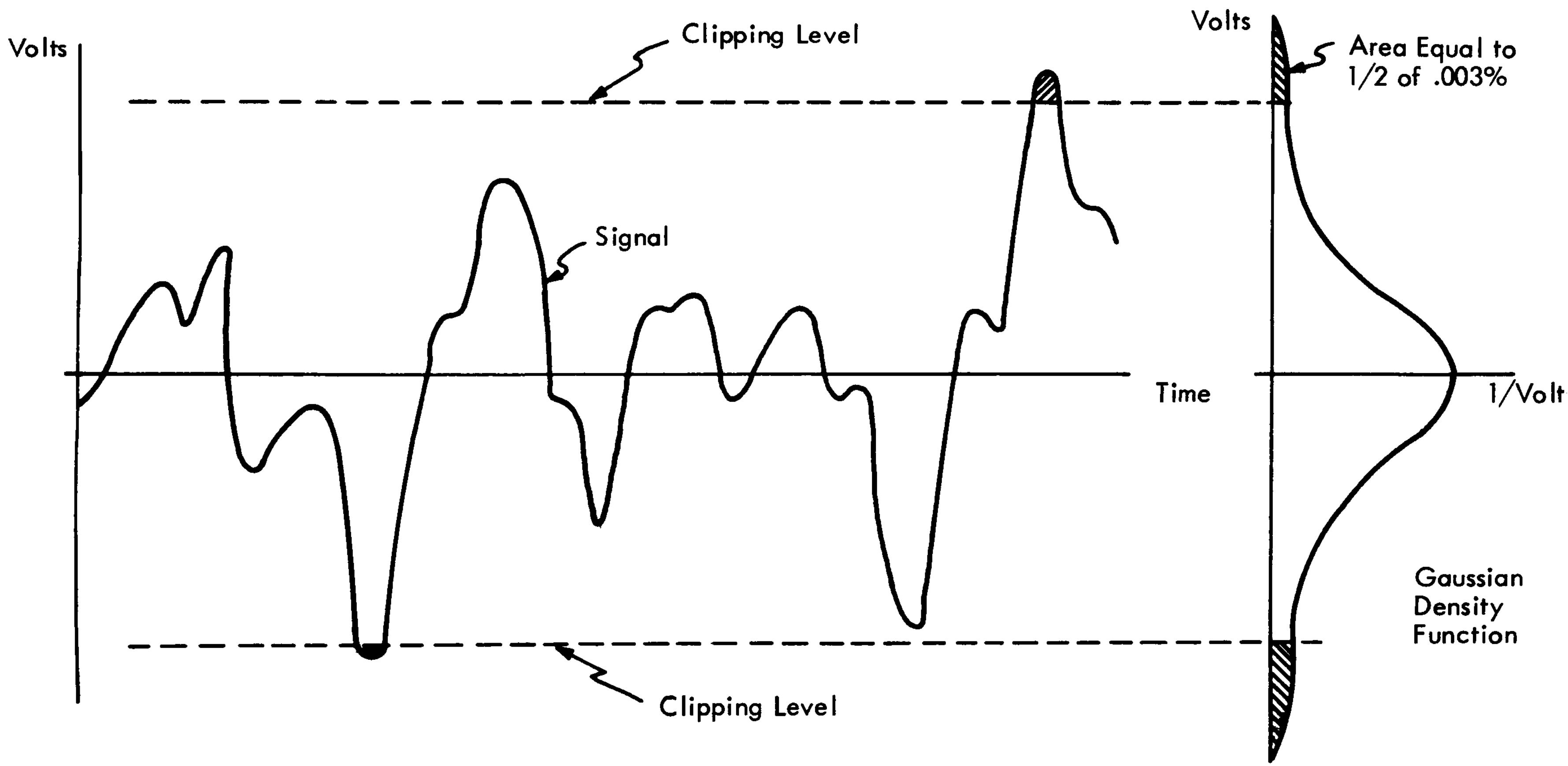


Figure 3-2. Clipping

to have one n^{th} the output radiated power. But even when assigned, a channel is not necessarily active. In telephony, one party normally listens while the other talks, and even the talker pauses between words and between sounds within words, so that an assigned channel may, indeed, be active only about one fourth of the time. Therefore, with a large number of channels, the transmitted power per channel may be made four times one n^{th} the total power. This is a 6 db improvement. Signals such as SSB are automatically off when there is no message input. A way would have to be found, however, of turning off, for example, an FM signal when there is no input. There are doubtlessly problems such as keeping an FMFB receiver operating, and intra-word pause might not be usable; but the principle probably can be made to work if needed.

3.1.4 Satellite Repeater Functions

Aboard an active repeater three basic things must happen to the input signal:

a. The signal is received, and in so doing noise is added to the system.

b. The signal received at the repeater is processed in preparation for retransmission. If the processing includes detection of, for example, PCM, a fraction of the noise introduced at the relay's receiver may be suppressed.

c. The power of the processed signal is raised to an appropriate level, and is transmitted. In the power amplification stage particularly there is usually a significant amount of (inter-modulation) distortion introduced.

To proceed with a preliminary analysis it will be assumed that the noise and the distortion introduced in the satellite is negligibly small compared with the noise introduced at the ground station receiver. Later, satellite and ground transmitter parameters will be determined by choosing acceptable relationships between the various disturbances. Temporarily, it will be assumed that the system is operating above its threshold. That this is indeed true must be verified by a secondary preliminary calculation. With antennas, RF output, and the receiver specified in the DPSM, signal processing is the chief interest.

Four fundamental functions can occur in the processing portion of the repeater:

a. Frequency translation

b. Change of degree of modulation

c. Detection

d. Modulation

These may be used in combination or alone (with the exception of detection, which must be followed by modulation). Figure 3-3 shows block diagram arrangements. Amplifiers have been omitted.

It is expected that all satellite designs will employ a frequency translation to provide isolation between transmitters and receivers at ground stations and at the satellite itself.

Because the channels are not multiplexed at a common point in a multiple access system, it is not easy to increase the degree of modulation by a simple process such as frequency multiplication, as is done in Relay.

Each ground station could send up its channels by narrow-band frequency modulation with one sideband suppressed (SSBFM). The signal envelope for SSBFM is not constant, as with conventional FM. Also, with the same average power, some of the otherwise not significant higher-order sidebands become significant, so that (in general) the amount of spectrum occupied with SSBFM is more than one-half that of conventional (double sideband) FM, unless the deviation ratio is much less than 1. The other set of sideband frequencies can be recovered by reinserting a carrier and limiting.[7] With both sidebands present, frequency multiplication can be used to obtain the desired deviation ratio.

With SSBFM, the up transmission could have any desired deviation

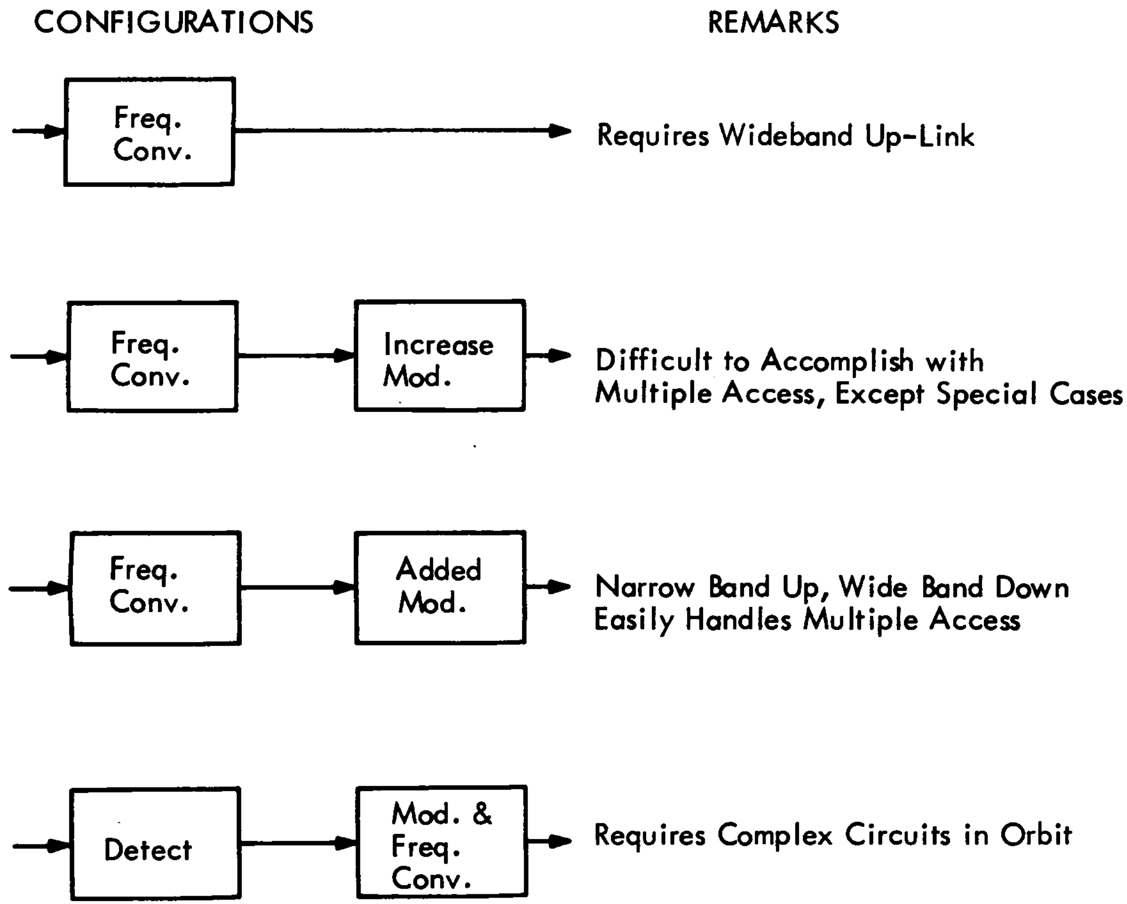


Figure 3-3. Satellite Signal Processing Arrangements

ratio, but probably in the interest of spectrum conservation the choice would be narrow-band FM; that is, first sideband only with the carrier suppressed. In this case there is no difference between SSBFM and SSBAM. The system described above is, indeed, the one to be used with the Advanced Syncom, except that the up transmission is described as SSBAM, and the generation of the other set of sidebands is referred to as modulation.

Other types of compound modulation of the composite of the signals arriving at the satellite are possible, but, in general, it will be desirable to use a form that yields a constant average power so that the RF output device (assumed to be a TWT) may be used to the best of its abilities. This means a double sideband angle modulation.

Detection and remodulation aboard the satellite would improve the signal-to-noise ratio before retransmission, thus allowing the use of less ground transmitter power. It would also benefit a system working in a poorly controlled electromagnetic environment. In general, however, it is believed that the reduced reliability brought with the added complexity outweighs the advantages of detection in orbit.

3.1.5 Preliminary Comparisons of Modulations

Calculations were performed to determine the signal-to-noise ratios achievable with seven combinations of classical modulations together with frequency and time division multiplexing. The formula used was the one given in Subsection 2.3.2

$$S/N = \frac{P_s G_s A_R K_1 K_2}{4\pi R^2 M k T_R B_R n D}$$

To establish minimum qualifying performance, DPSM values for the Type I (small) ground terminal and for the satellite can be substituted into this equation.

$$S/N = \frac{10 \times 63 \times 10^2 K_1 K_2}{4\pi \times 1.69 \times 10^{16} \times 3.16 \times 1.38 \times 10^{-23} \times 600 \times 200 \times B_R D}$$

$$= 5.63 \times 10^4 \times \frac{K_1 K_2}{B_R D}$$

$$= 47.5 + 10 \log_{10} K_1 + 10 \log_{10} K_2 - 10 \log_{10} B_R - 10 \log_{10} D \text{ db}$$

The performance objectives are:

Analog - 4 kc/s bandwidth with $S/N > 20$ db

Digital - 20 db/s data rate with $P_e < 0.01$.

The computations placed each of the candidates in one of three categories, viz., "qualifies," "near," or "far." "Qualifies" means that the technique meets the S/N (or P_e) objectives. The term "near" is used to characterize approaches that fall short of the objectives by less than five (i.e., 7db), and which, therefore, deserve further attention to see what could be done to bring them up to an acceptable level of performance. The combinations of modulation and multiplexing that result in "far" performance are probably beyond being recovered.

Table 3-1 presents the signal arrangements employed on the "up" and "down" links, and the type of onboard satellite processing used. The word "compound" between the "up" and "down" links means that a further step of modulation is applied to the composite signal arriving at the satellite. That is, each of the separate signals coming from the ground station transmitters becomes a subcarrier transmission of the composite down link. A straight line indicates an essentially "transparent" repeater. In practice, as noted before, a frequency shift would be used to provide transmit-receiver isolation at the satellite and at all ground terminals.

3.1.5.1 Case I. SSB/FDM - Compound - Composite FM

It is assumed that enough pre-emphasis is used on the high channels to make the composite multiplexed and the demultiplexed individual channel signal-to-noise ratios the same.

The values needed to complete the performance calculations are:

$D = 1 \quad (0 \text{ db})$

$K_1 = 1 \quad (0 \text{ db})$

$K_2 = \frac{3}{2} m^2 \times \frac{B_R}{B}$

TABLE 3-1

NON-PSEUDO RANDOM MODULATION

	UP		DOWN
I	SSB/FDM	(Compound)	Composite FM
II	FM/FDM	(Compound)	Composite FM
III	FM/FDM		FM/FDM
IV	SSB/FDM		SSB/FDM
V	PCM/TDM		PCM/TDM
VI	PCM/FDM		PCM/FDM
VII	PPM/TDM		PPM/TDM

B is equal to 4000, and it was found that a modulation index (m) of 2.18 was needed. Therefore,

$$K_2 = \frac{3}{2}(2.18)^2 \times \frac{B_R}{4000}$$

$$= 1.78 \times 10^{-3} \ B_R$$

$$= -27.5 + 10 \log_{10} B_R \ \text{db}$$

The B_R in the expression for K_2 cancels the B_R in the performance equation, therefore, the performance is

$$S/N = 47.5 - 27.5$$

$$= 20.0 \ \text{db}$$

If the activity factor is taken into account, the signal-to-noise ratio will be 26 db.

By convention, most authors present their carrier-to-noise (C/N) threshold figures in terms of noise in a band twice the width of the base band. For the example at hand, that would be 2nB, since the discriminator must process all n channels simultaneously. The S/N formula previously used can be modified for the C/N calculation by dividing by 2B and letting K_1 and K_2 be unity.

$$C/N = 47.5 - 10 \log_{10} 2 \times 4000$$

$$= 47.5 - 39$$

$$= 8.5 \ \text{db}$$

This ratio (with an m equal to 2.18) is essentially at the FM improvement threshold, provided an FMFB receiver is used.

Clearly, SSB/FDM----composite FM "qualifies" (i.e., meets the signal-to-noise power ratio objectives.

3.1.5.2 Case II. FM/FDM - Compound - Composite FM

The output signal-to-noise power ratio for this combination of processes will be the carrier-to-noise power ratio multiplied by the product of the two FM improvement factors.

There are two cases of interest: (a) where a significant improvement factor is realized from both stages of FM and (b) where the satellite FM process is employed to give a constant amplitude multiplexed signal, but m is kept small, so that there is no significant processing gain.

In the first case the up-link FM signal will have a bandwidth (using Carson's rule) of about $2(m_1 + 1)B$. Therefore, on the down-link the C/N into the discriminator must be measured in a band of $4(m_1 + 1)nB$. Even if m_1 is negligibly small, the noise band in which the carrier-to-noise ratio is measured is twice that in Case I. Case II(a) does not meet the threshold requirement, therefore, but is sufficiently close to be categorized as "near." If assigned, but non-active, channels could have their carriers turned off, systems with small m might work, but with no great advantage.

In the second case, m_2 is kept small and individual subcarrier signals are selected out by reference to the carrier. A discriminator is not used for the first stage of detection. This second FM step is kept essentially linear. Ordinarily, FM is a non-linear process and superposition does not hold.[8] That is, each channel puts crosstalk into all others, unless m is small. With a small m_2 the presence of a significant amount of carrier is assured since the first zero of the J_o function (zero order Bessel function of the first kind) occurs when its argument (the deviation ratio) is equal to 2.4. The carrier-to-noise power ratios can now be made to approach within about 3 db of those in Case I for each of the n channels, since both P_S and the predetector noise band are divided by n. Beyond m equal to 1.1 the FM process does not approximate linearity. At that point one half of the available power is in the carrier (hence, the 3 db difference from Case I), and there is about 2% distortion due to higher order sidebands. The principal advantages are the simplicity of the satellite multiplexing equipment, and the constant power output. Case IIb is also classified as "near." With non-active-channel carriers turned off, Case IIb would "qualify."

3.1.5.3 Case III. FM/FDM - FM-FDM

In this case a large number of uncoordinated FM signals (in disjoint frequency bands) are sent to the satellite where they are amplified (shifted in frequency) and retransmitted. Although all of the incoming signals are of constant (peak) amplitudes, by the Central Limit Theorem the instantaneous output will have a gaussian amplitude distribution. Since the satellite RF device (probably a TWT) will be saturated by some of the extreme peaks of the gaussian signal, it is necessary to determine what fraction of the time saturation is permissible, and from that figure to select a derating factor, D, relating peak power to average power. A value of 9.5 db is reasonable for D, corresponding to about 0.003% of the time saturation.

Because of the power derating, the carrier-to-noise ratio will be reduced by the factor D. This will put the system to the left (the wrong side) of the threshold. Therefore, Case III is an example of "far" from qualification. With carriers turned off for the non-active channels, the carrier-to-noise ratio would still be at least 3 db below threshold, therefore only coming "near" to qualifying.

3.1.5.4 Case IV. SSB/FDM - SSB/FDM

Again, as in Case III, the multiple uncoordinated signals give rise to a gaussian-distributed output signal. Therefore, the power output from the satellite must be reduced by a factor D. Further, with single sideband there is no modulation process improvement factor. Using the parameters already mentioned in other sections, a calculation was made to find the signal-to-noise power ratio. The result was a S/N of about 2.5 db. Case IV must be considered "far" from qualifying. Taking activity factor into account, the signal-to-noise ratio is 8.5 db and the system is still "far" from qualifying.

3.1.5.5 Case V. PCM/TDM - PCM/TDM

In a binary pulse system the significant quantity analogous to C/N is the pulse energy-to-noise density ratio (E/N_o). From this figure a number of authors have worked out the probability of making an error (P_e) in deciding whether a "1" or a "0" has been sent.[9] For speech, a P_e of about 0.01 has been assumed to be satisfactory.

Using the same parameters used in previous examples and solving for E/N_o for a data rate of 20,000 bits per second, the ratio is 4.5 db. The values of E/N_o needed for a P_e of 0.01, for various types of pulse transmission, are shown in Table 3-2.

At 20 kb/s, Case V "qualifies" for coherent PSK, and nearly qualifies for the remaining modulations on the list.

Maintaining RF coherence for PSK and PCM word interlacing from each ground transmitter might be difficult. However, there is no reason why each transmitter should not transmit word bursts of (say) 100 (or more) bits; that is, several words at a time. This, of

TABLE 3-2

COMPARISON OF BINARY TRANSMISSION SYSTEMS

Type of Transmission	E/N_o for $P_e = 0.01$
Coherent PSK	4.2 db = 2.62
Differential-Coherent PSK	5.9 db = 3.88
Coherent Keyed Carrier Coherent FSK (ρ =0)	7.0 db = 5.00
Non-Coherent Keyed Carrier	8.2 db = 6.60
Non-Coherent FSK	8.9 db = 7.76

course, would require buffer memory devices at the transmitters and receivers.

Because only one signal is present at a time, the TWT can be operated at its peak output; that is, no derating is necessary, and (as noted before) terminal ancillaries can be shared.

3.1.5.6 Case VI. PCM/FDM - PCM/FDM

Again, as in Cases II and IV, the simultaneous presence of many uncoordinated signals in the satellite output requires TWT output power derating. If D is taken as 9.5 db and bit rate as 20 kb/s, E/N_o will be -0.5 db. Therefore (again referring to Table 3-2), Case VI "nearly" qualifies for coherent and differential coherent PSK.

3.1.5.7 Case VII. PPM/TDM - PPM/TDM

Provided the ratio of peak signal power ($\hat{S}$) to mean noise (N) is high, the output signal-to-noise power ratio for an analog PPM system is given by the following equations:

$$S/N = \frac{1}{2}\left(\frac{B_p}{2nB} - 1\right)\left(\frac{\hat{S}}{N}\right) \tag{3-1}$$

The new symbol B_p represents the bandwidth of the pulses. For two hundred 4 kc/s channels and a S/N of 20 db, B_p must be almost 50 mc/s. If the pulse duration (τ) is the inverse of B_p, then the ratio S/N can be calculated by adapting the general formula used for the other cases.

$$\hat{S}/N = 47.5 + 10 \log_{10} 200 \quad 10 \log_{10} 5 \times 10^7$$

$$= 47.5 + 23.0 - 77.0$$

$$= -6.5 \text{ db}$$

This is far below the pulse signal-to-noise ratio believed necessary. The probable reason is that the time occupied by the pulse is only 1/30 of the available time slot. Some other equally-wideband, high duty factor, pulse time system would probably give much better results.

3.1.5.8 Summary

The results of the above sections are:

SUMMARY OF RESULTS

Case	Combination	Result
I	SSB/FDM - COMP. FM	Qualifies
II	FM/FDM - COMP. FM	Near*
III	FM/FDM - FM/FDM	Near
IV	SSB/FDM - SSB/FDM	Far
V	PCM/TDM - PCM/TDM	Qualifies at 20 kb/s
VI	PCM/FDM - PCM/FDM	Near at 20 kb/s
VII	PPM/TDM - PPM/TDM	Far**

* If non-active-channel carriers are turned off, II(b) qualifies.
**Some other high duty factor, pulse-time system of modulation might give better performance.

3.1.5.9 Conclusions

There are two principal conclusions from the above analysis:

a. The use of multiple carriers on the down-link requires derating the TWT. Therefore, the composite signal at the satellite should be angle-modulated to form a single carrier signal, or TDM should be used.

b. FM threshold requirements, even with a feedback receiver, demand that the noise band not exceed 2nB when Design Point System Model parameters and objectives are used.

3.2 Functional Block Diagrams and Further Preliminary Analysis

In Paragraph 3.1.5, two modulation combinations were found to "qualify," viz., "SSB/FDM - composite FM" and "PCM/TDM-PCM/TDM." Functional block diagrams are presented below to illustrate the operation of ground stations and satellites with these two types of systems.

3.2.1 Ground Station Diagrams

3.2.1.1 SSB/Composite FM Ground Station

The combination of modulation techniques considered in this section are SSB/FDM on the up link and composite FM on the down link. Generic forms of ground station transmitting and receiving equipment are shown in Figures 3-4 and 3-5. Intermediate stages of amplification have been omitted for clarity. Although only two channels

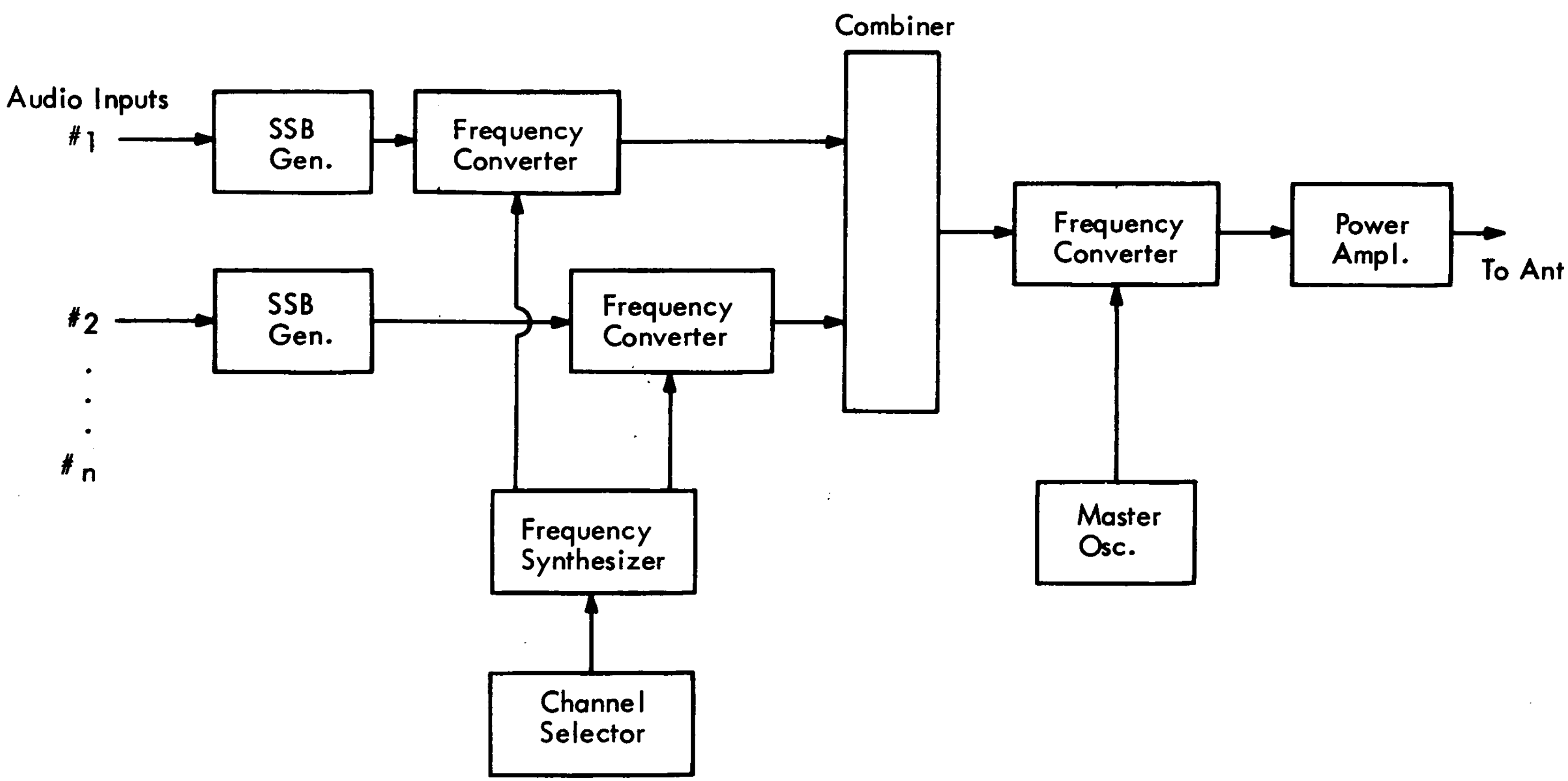


Figure 3-4. SSB/FDM Transmitter

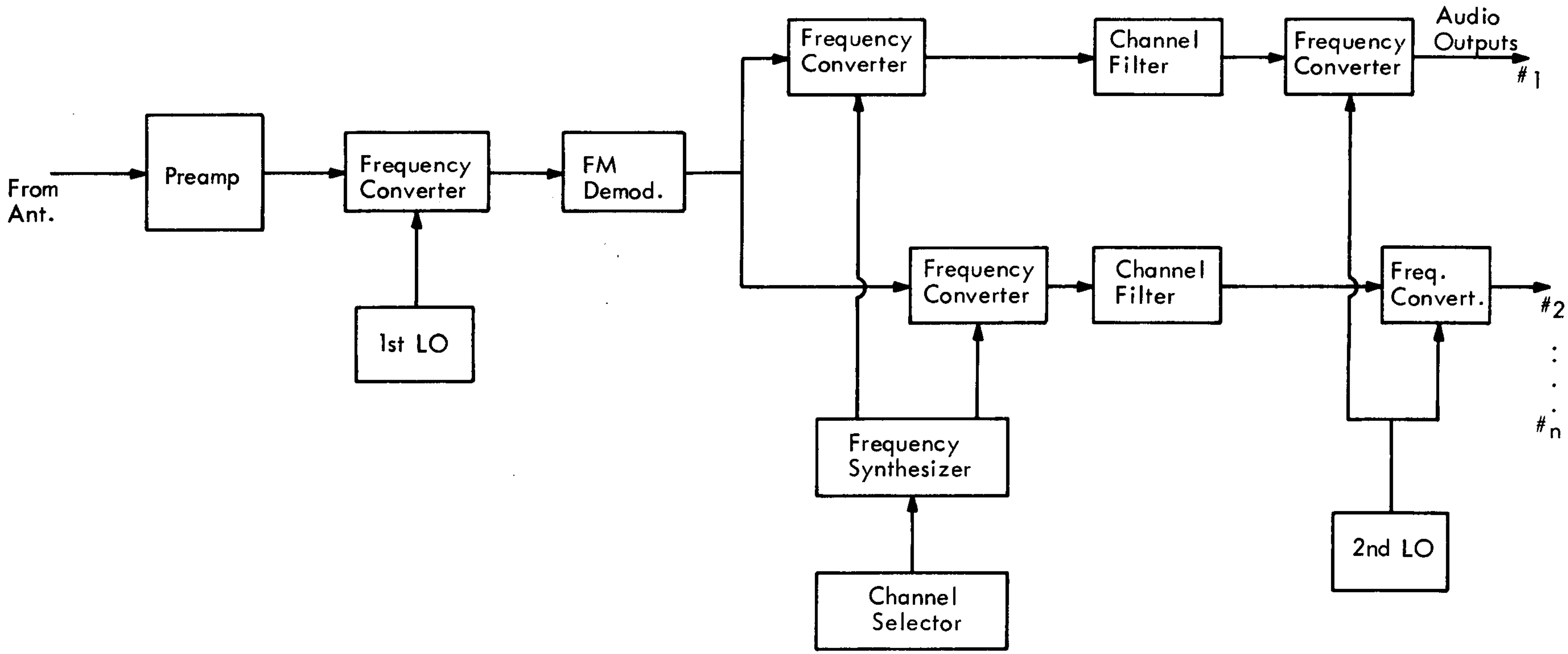


Figure 3-5. Composite FM Receiver

are shown, the actual number would, of course, be determined by the size of the station.

Transmitter. Each input signal feeds a single-sideband generator consisting of oscillators, mixers, and filters. To facilitate the SSB modulation, processing is done at a comparatively low frequency. All channel outputs are initially at the same frequency, thus simplifying equipment requirements.

Channels are multiplexed by shifting them in frequency to selected locations in the IF spectrum. Channel position is controlled by a Channel Selector which picks up the proper output from a frequency synthesizer in accordance with instruction received from the system's central station.

The channels are combined to form an FDM signal, and then shifted up to the transmission frequency. This requires the use of a Combiner, a Frequency Converter, and a Master Oscillator. The power level of the signal is then raised by the Power Amplifier and transmitted to the satellite.

Receiver. The composite FM signal from the satellite is raised in level by the preamplifier and converted to IF by mixing with the first local oscillator. The original FDM stack is then recovered by the FM Demodulator. Demultiplexing of the SSB signals is accomplished in an operation which is the inverse of that employed in the transmitter. Channel allocation information is fed into the Channel Selector which determines the proper frequency synthesizer output to shift the desired signal into the passband of the Channel Filter. Each filter passes one channel and rejects all others. (This filtering is best done at a 2^{nd} intermediate frequency.) It is then necessary to shift the selected channel down to base band, thus recovering the origin audio signal. The shift is accomplished by the 2^{nd} LO and the second set of Frequency Converters.

3.2.1.2 PCM/TDM System

The system considered here employs PCM with TDM on both the up and down links. The ground transmitting and receiving equipment is shown in generic form in Figures 3-6 and 3-7. The time synchronization system has been omitted as have intermediate stages of amplification.

Transmitter. Individual audio inputs are passed through a PCM Encoder, and output bits from each channel are stored in preselected locations in the Memory. The data rate up to this point is 20 kb/s. To decrease the time synchronization problems, information is read out in blocks of (say) 100 bits. Since there may be 200 active channels, it is necessary to read out each block at a rate of 4 megabits per

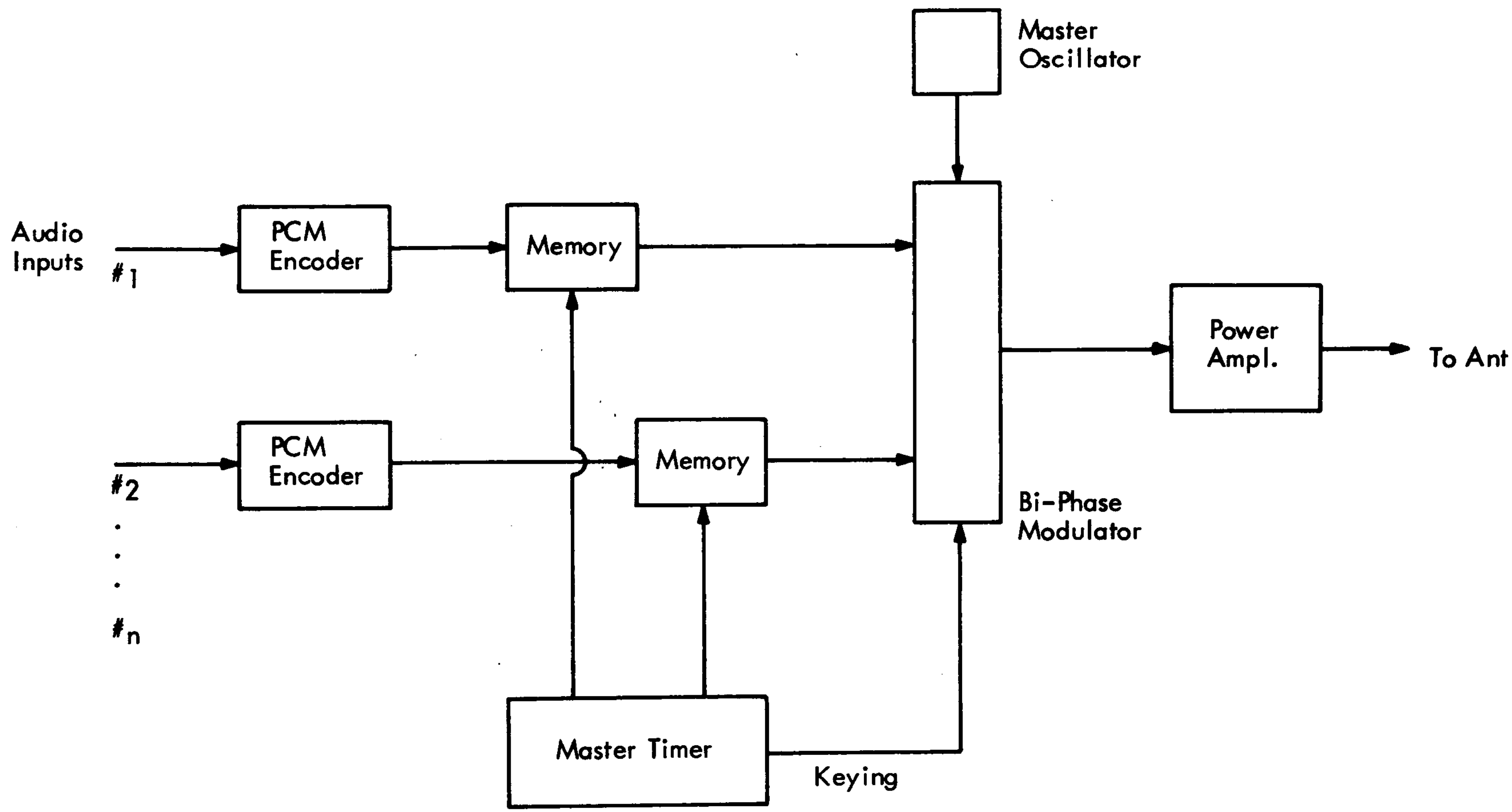


Figure 3-6. PCM/TDM Transmitter

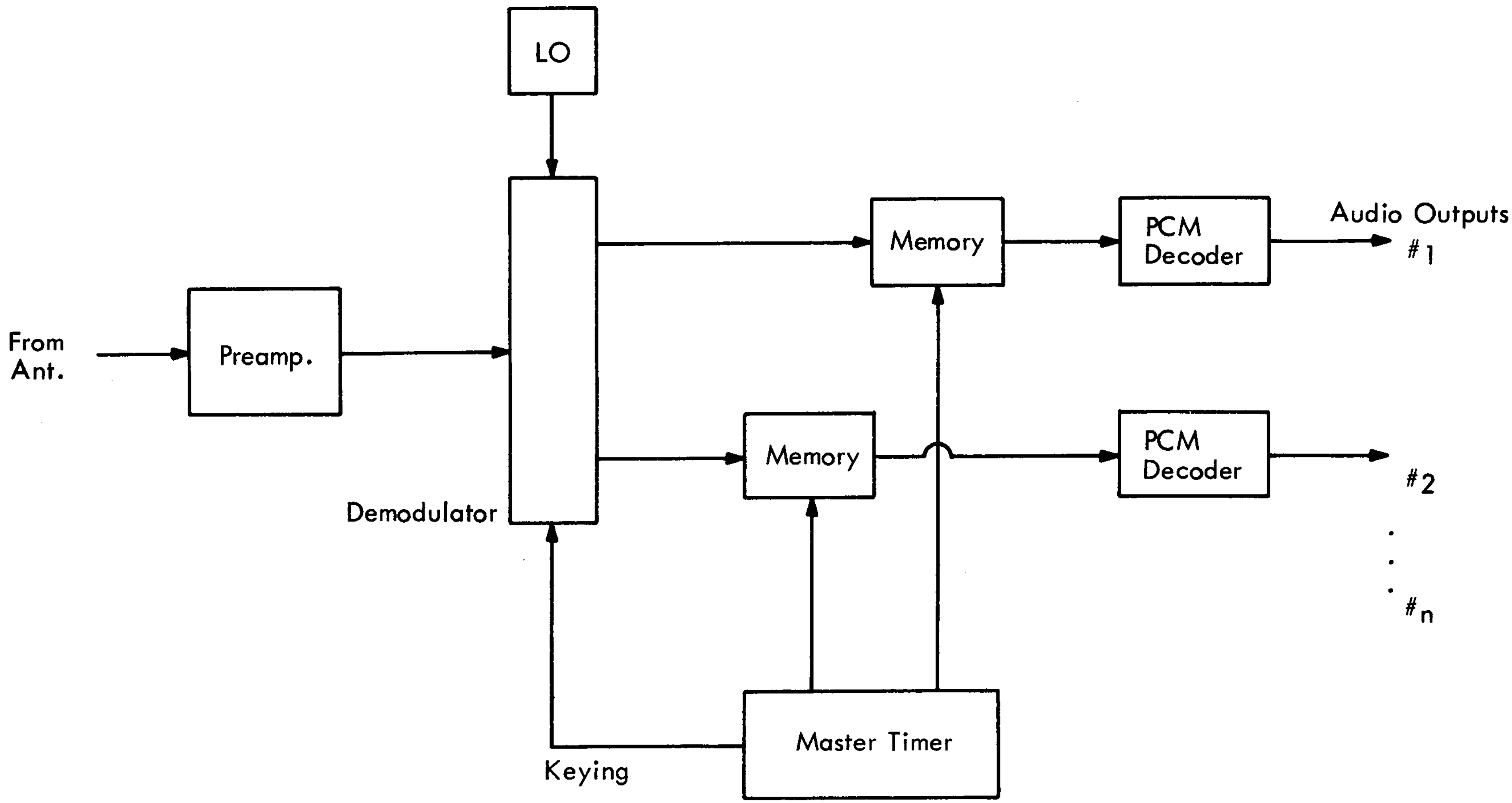


Figure 3-7. PCM/TDM Receiver

second (200 x 20 kb/s). The time slot, during which a particular memory location is read out, is determined by the Master Timer. The timer receives information on synchronization and station allocations from the system's central station. Each block of 100 bits is applied to the Biphase Modulator where the phase of the Master Oscillator signal is varied in accordance with the incoming binary digits. The Master Timer also applies a keying signal to the Modulator to gate out the RF signal when no information is being transmitted. A Power Amplifier then raises the signal to the desired level for transmission to the satellite.

Receiver. The PCM/TDM receiver is, in principle, just the inverse of the transmitter. The signal received from the satellite is first raised in level by the preamplifier. Blocks of data are recovered by means of the LO and the Demodulator, the latter is keyed-on only for periods when information is transmitted to the subject station. These data blocks are read into the Memory at 4 mb/s. The stored signal is read out at 20 kb/s, as controlled by the Master Timer. A separate PCM Decoder is provided for each active channel to reproduce the original audio signal.

3.2.2 Satellite Diagrams

3.2.2.1 SSB/FDM - Composite FM

Figure 3-8 shows a functional block diagram of a satellite for operation with this system of transmission. The signal, s(t), into the pre-amplifier is composed of n messages, $m_i(t)$, transposed in frequency to a position near the nominal up-link transmission frequency, f_u. Symbolically and ideally, this can be stated as

$$s(t) = \sum_{i=1}^{200} m_i(t) \cdot e^{j(f_u + 4000i + \varepsilon_i \ \phi_i/t)\, 2\,\pi t} \quad \text{For 4 kc/s channels.} \tag{3-2}$$

The symbols ε_i and ϕ_i represent uncontrolable frequency and phase errors associated with each message as it is transposed in the frequency domain. It is these errors that make it difficult to multiplex at the satellite.

Noise, n(t), is added at the satellite receiver. From the point of view of components that follow, viz., the FM modulator and later the FM demodulator, the satellite receiver noise is indistinguishable from signal. Therefore, the two may be lumped together to form the signal, s'(t), as seen by the FM components. Note that n(t) does not adversely affect the FM-demodulator threshold.

A frequency down conversion is performed for operational convenience, and then s'(t) is added to a sinusoidal carrier frequency

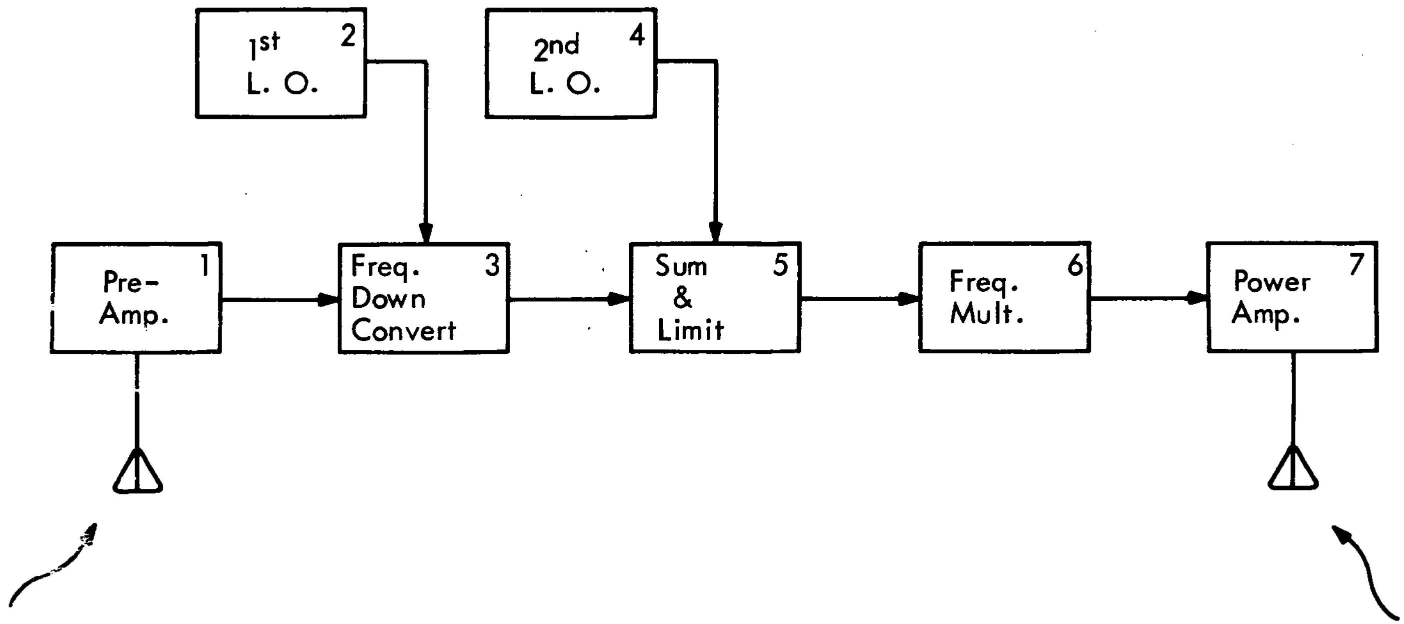


Figure 3-8. Satellite Functional Block Diagram
(SSB/FDM - Composite FM)

signal that is much larger in amplitude and slightly below (or above) in frequency, and when the combination is limited in amplitude, the result is that the carrier's zero crossings are altered in accordance with $s'(t)$. This may be interpreted as small deviation FM. (Or more properly, phase modulation; but the PM and FM are related by a linear operation, viz., differentiation.) Several stages of frequency multiplication can be used to increase the deviation ratio. Fluctuation noise arising within the frequency multiplier circuits is expected to be much smaller than that coming from the receiver.

If the frequency down conversion and multiplication are properly related, the signal out of the multiplier will be at the down-link transmission frequency. The signal, however, must first be passed through a power amplifier.

3.2.2.2 PCM/TDM - PCM/TDM

The functional block diagram for this satellite is shown in Figure 3-9. The satellite signal processing functions for this system consist of reception, translation of the signal to the down-link frequency, and the power amplification.

3.2.3 Frequency Requirements

3.2.3.1 SSB/FDM - Compound FM

The up-link frequency band, B_u, for $s(t)$ is

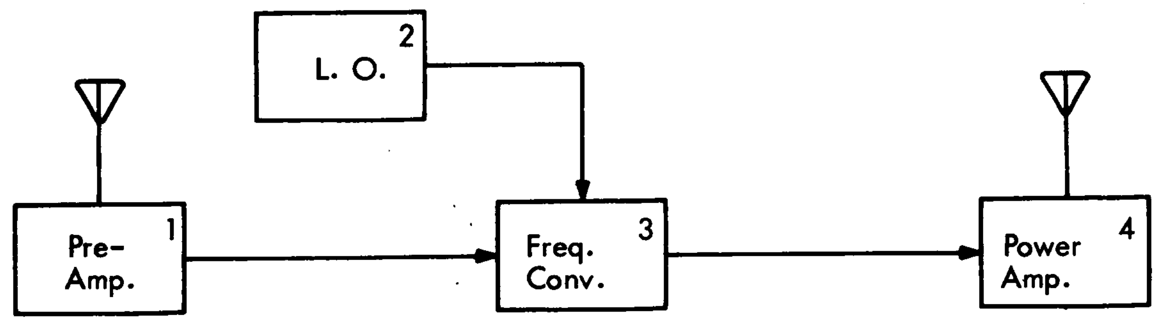


Figure 3-9. Satellite Functional Block Diagram
(PCM/TDM—PCM/TDM)

$$B_u = nB \tag{3-3}$$

For 200 channels, as in the DPSM case,

$$B_u = 200 \times 4000$$

$$= 800 \text{ kc/s}$$

The down-link band, B_d, can be calculated using Carson's rule. For the DPSM

$$B_d = 2(2.18 + 1)200 \times 4000$$

$$= 5.1 \text{ mc/s}$$

3.2.3.2 PCM/FDM - PCM/FDM

The up-link and down-link frequency bands are the same for this case. If the baseband pulse width is taken as the same as the bit rate, and if double sideband transmission is assumed, then,

$$B_{u,d} = 2nR \tag{3-4}$$

For the DPSM example

$$B_{u,d} = 2 \times 200 \times 20{,}000$$

$$= 8 \text{ mc/s}$$

3.2.4 Ground Transmitter Power Requirements

Good estimates of ground transmitter power requirements can be made by assuming:

a. The noise, N, in a received (base band) channel can be divided into two parts, viz., ground receiver derived, N_R, and satellite receiver derived, N_S; and setting a ratio between N_R and N_S.

b. Interference from other systems, and intermodulation from within the system are negligible.

c. An effective noise temperature, T_S, for the satellite receiver.

To obtain specific numbers, it will be assumed that (1) N_S should not be greater than one-tenth N_R, (2) T_S is equal to 800°K, and (3) transmission is between two DPSM Type I ("small") ground stations.

It should be noted that margins must be carefully handled. If the down-link signal is decreased, the satellite derived noise will be decreased by a similar amount. In the examples to follow, up-link margin, M_1, and down-link margin, M_2, are both taken to be equal to $\sqrt{10}$.

3.2.4.1 SSB/FDM - Compound FM

Using the 10 to one noise ratio selected above, and noting that N_S may be decreased by M_2, the following relationships can be written:

$$N_R = 10\ N_S/M_2 \qquad (3\text{-}5)$$

$$= \sqrt{10}\ N_S$$

$$\frac{2kT_R nB(m+1)}{\frac{3}{2} m^2 \left[\frac{nB(m+1)}{nB}\right]} = \sqrt{10}\ K_1 k T_S B \left(\frac{G_S A_R}{4 \pi R^2}\right) \qquad (3\text{-}6)$$

In Paragraph 3.2.2.1 it was pointed out that the FM modulator and demodulator cannot distinguish satellite derived noise from signal. Therefore, only the noise generated at the ground receiver (the left hand side of the equation) is affected by the FM improvement factor.

By cancelling wherever possible, this equation can be considerably simplified. The satellite power gain, K_1, may be written as the ratio of the power out of the satellite, P_S, to the power into the satellite. In turn, the power into the satellite is $(P_T G_S A_T) / (4\pi R^2)$.

$$\frac{nT_R}{m^2} = \frac{3\sqrt{10}}{4} \frac{P_S}{P_T}\left(\frac{4\pi R^2}{G_S A_T}\right)\left(\frac{G_S A_R}{4\pi R^2}\right) T_S \quad (3\text{-}7)$$

Because transmission is between similar ground stations, $A_T = A_R$. More cancellation is now possible, and the equation can be rearranged to give P_T:

$$P_T = \frac{3\sqrt{10}}{4} \times \frac{T_S\, m^2\, P_S}{n\, T_R} \quad (3\text{-}8)$$

Substituting the selected value of T_S and numbers from the DPSM,

$$P_T = \frac{3\sqrt{10}}{4} \times \frac{800 \times (2.18)^2 \times 10}{200 \times 600}$$

$$= 0.75 \text{ watts (without } M_1) \quad (3\text{-}9)$$

To allow for the up-link margin, M_1, and to insure linearity, the value of P_T just calculated should be multiplied by $\sqrt{10}$ and by 9. Thus

$$P_T = 21.3 \text{ watts per channel} \quad (3\text{-}10)$$

3.2.4.2 PCM/TDM - PCM/TDM

Using a line of reasoning similar to the one in Paragraph 3.2.4.1, the satellite-derived noise density in the energy-to-noise density ratio is set equal to one tenth the ground receiver noise, taking into account the down-link margin.

$$\frac{E}{N_o} = \frac{\dfrac{P_S G_S A_R T}{4\pi R^2} \Big/ M_2}{N_{o_R} + N_{o_S} \Big/ M_2} \tag{3-11}$$

$$N_{o_R} = \frac{10 N_{o_S}}{M_2} \tag{3-12}$$

$$= \sqrt{10}\; N_{o_S}$$

Again, as above, this equation can be expanded, terms cancelled, and rearranged to give the needed ground transmitter power.

$$P_T = \sqrt{10}\; \frac{P_S T_S}{T_R} \tag{3-13}$$

With the assumed satellite receiver temperature and DPSM values,

$$P_T = \sqrt{10} \times \frac{10 \times 800}{600}$$

$$= 42.1 \text{ watts peak (without } M_1\text{)}$$

To allow for the up-link margin, M_1, P_T should be multiplied by 10. Thus,

$$P_T = 133 \text{ watts peak}$$

The peak power is the same regardless of the number of channels. The average power is

$$P_{T_{av}} = \frac{133}{n}$$

$$= \frac{133}{200}$$

$$= .677 \text{ watts/channel}$$

3.3 Callup Systems

This section discusses important considerations related to the callup procedures for a system using conventional (i.e., non-pseudo-noise) modulation techniques. A possible implementation for a callup system is proposed here to indicate that the principal callup function can be accomplished with an equipment penalty that is small relative to that required for transmission of the voice channels.

3.3.1 Introduction

The functions that are to be accomplished by the callup system are similar to those in a commercial land-based telephone system. The particular callup problem of concern here is associated with systems employing conventional modulation techniques.

The first question to be raised concerning the callup system design is whether a coordinating (ground) station should be employed. One common argument against such a station is the reluctance of the participating countries to allow any one country the preferred position of having the coordinating station within its territorial boundaries. However, if the coordinating station executes only functions which are required to establish a link between station A and station B based on predetermined procedures initially agreed to by all parties concerned, this objection is eliminated. This "sealed box" type operation, perhaps in conjunction with the selection of a neutral site such as Geneva, should dispel all reservations to the use of a coordinating station. Additionally, in conventional systems there appear to be several disadvantages associated with the complete elimination of a coordinating station. Much of the callup logic would have to be duplicated at each ground station, thereby imposing an important additional equipment penalty upon the small stations. Billing and the monitoring of transmissions for illegal operation would become more complicated. Finally, the emergency functions which must inevitably require human intervention are best performed at a coordinating station.

For these reasons, the system which is proposed in Paragraph 3.3.4.2 as a possible means of performing the callup functions includes a coordinating station.

The second important design consideration is the intentional misuse of the callup system that might occur. It is assumed in this that all users will "play by the rules," and that jamming will not exist.

A further assumption is that the callup system is only required to establish a connection between ground stations. All further connections to complete a call are assumed to be beyond the responsibility of the callup system. In other words, the callup process is only concerned with the availability of stations, not with such matters as which receiver in the station should be used. Once a link has been established, the system will perform in essence like a wired system, using the assigned channel to complete the call between the two subscribers.

The callup system design will depend strongly on the degree of random access allowed. A completely random access system is one in which all channels are assigned on a first-come-first-served basis. Although this approach is appealing, the users may prefer some type of priority assignment on the channels. Such an assignment could be either on a permanent or a temporary basis. This would have the advantage of guaranteeing the individual stations at least limited access to the system even during maximum load times. Such priority or leasing arrangements could take many forms all of which involve additional callup equipment complexity. As an example of a very limited priority setup, one might assign to each station a high priority for one channel during specified hours to insure the rapid transmission of important information. If there are 50 stations and 200 channels, this still leaves 150 channels to operate in a random manner.

In addition to priority, a queueing function may also be desirable. It is important here to make clear that the queueing function does not refer to subscriber callup requests entering the individual ground stations. Queueing is meant to refer to callup requests made by stations for access to other stations. Such queueing would increase the callup equipment complexity but would reduce the traffic flow in the callup system.

3.3.2 Callup Functions

A callup subsystem must, at a very minimum, assign available channels (i.e., time or frequency allocations) to calling stations, release channels when calls are terminated, and provide busy signals.

In addition to these basic functions, there are numerous auxiliary functions that may or may not be needed. For instance, the coordinating station might also tally the number of active users at each voice station. It may also be desired to provide confirmation that a channel hook-up was successfully made. Therefore, if station A desires to call station B, which is known by the coordinating station to have no available receivers, it may return a busy signal to station A without ever having assigned a channel. Provisions may also be made for periodic polling of the ground stations to correct for any possible errors in the channel availability records.

The channel selection problem could be further complicated by allowing forms of priority or leasing. This would require additional logic to allow channel selections based on the constraints set by the priority or leasing arrangements.

There are several passive or monitoring functions that might be desired. For example, many systems may require accurate billing and accounting. System usage statistics could be compiled and used to adjust channel priorities or leasing arrangements.

There are several passive or monitoring functions that might be desired. For example, many systems may require accurate billing and accounting. System usage statistics could be compiled and used to adjust channel priorities and allocations.

It may also be necessary to monitor the system to insure compliance with the operating standards set for the system.

The equipment complexity at the coordinating station will depend strongly upon the number of ancillary or monitoring functions that are included. It may be that a small general purpose computer will be the most practical means of automating the callup functions. It should be stated, however, that any central callup subsystem will not be totally automatic. Some form of manual control will be necessary since unforseen emergencies are bound to occur, thereby requiring the diagnostic and corrective help of human intelligence.

3.3.3 Implementation Considerations

For the purpose of estimating equipment penalties, it can be assumed that an additional frequency band (or time slot) disjoint from the voice channels is used for the callup functions. It is conceivable that the callup band could be split and tucked into the guard bands of several or all of the voice channels to conserve bandwidth. Since the required bandwidth will be relatively small, however, such sophisticated approaches need not be examined at this time.

Inherent in any callup system is the random nature of the requests for channels and the release of channels. There are two potential methods of handling these asynchronous functions. One is to assign a narrow band channel to each station for requesting and/or releasing channels. The second method involves the use of a common channel with some method of prohibiting, or correcting, for overlapping messages. The former system relies on there being only a moderate number of stations while the latter system depends on a very low calling rate for success. Both of these conditions will exist in the systems under consideration; therefore both approaches have merit.

3.3.3.1 Common Channel

The common channel or "party line" approach has several potential solutions. All of these solutions depend for success on the fact that each station uses the common callup channel briefly and infrequently. This minimizes the probability that messages from two or more stations will overlap at the coordinating station.

One approach to this problem of overlapping messages is to reduce the probability of overlap until it is negligible. The most obvious way is to shorten the duration of the messages transmitted. Unfortunately this approach increases bandwidth and peak power requirements. Lutz[10] suggests two alternative solutions that slow promise. In the first of these, one transmits a short "priority pulse" simultaneously starting a delay-time gate on the receiving circuit. Receipt of one's priority pulse ahead of a pulse from some other station permits initiation of the call. Prior receipt of another priority pulse inhibits the call and requires another try. Coincidence of two or more pulses is still possible but quite improbable. The second solution suggested by Lutz involves prefixing calls with "holds" symbols longer than the delay time. To initiate a call, station A transmits a time assigned to it. If another tone is received in less than the delay time, it will inhibit the call. Also, if another station's tone arrives in partial or complete overlap with station A's tone, the resultant beat tine inhibits both calls.

In addition to approaches that minimize or prohibit overlap, there is another important approach. The philosophy here is to detect an overlap when it occurs and to "try again." A random number generator might be used to determine the delay before retransmission to prevent the overlapping stations from staying "in step." The probability of a successful transmission will be quite high after several attempts.

One can compute the probability of a successful transmission (no overlap) for one, two, three, etc. tries by making assumptions on the statistics of call channel transmissions. Defining

N_n = event that no overlap occurs on the n^{th} try

O_n = event that overlap occurs on the n^{th} try

E_n = event that a successful transmission occurred on the n^{th} try after n-1 unsuccessful tries.

The value of the index, $n = 1$, will be assumed to be the first try and $n = 2, 3, \ldots,$ the subsequent retransmissions. The probability that the first try will be a success will be

$$P(E_1) = P(N_1). \tag{3-14}$$

The probability that a successful transmission will occur on either the first or second try will be

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2), \tag{3-15}$$

since the events E_1 and E_2 are mutually exclusive. The probability, $P(E_2)$, that a successful transmission occurs on the second try will be the joint probability of failure on the first try and success on the second try.

$$P(E_2) = P(O_1,\ N_2) \tag{3-16}$$

This joint probability can be usefully expressed in terms of a conditional probability,

$$P(O_1, N_2) = P(O_1)P(N_2/O_1) = \left[1 - P(N_1)\right] \ P(N_2/O_1) \tag{3-17}$$

Therefore,

$$P(E_1 \text{ or } E_2) = P(N_1) + \left[1 - P(N_1)\right] \ P(N_2/O_1) \tag{3-18}$$

The probability of no overlap on the first try can be computed once a distribution of calls has been assumed. The joint probability, $P(N_2/O_1)$ will depend on both the initial distribution as well as on the distribution of the retransmitted calls that overlapped on the first try. In fact,

$$P(N_2/O_1) = P(R, N'_1) = P(R)P(N'_1) \tag{3-19}$$

where $P(R)$ is the probability of no overlap with messages that overlapped your own message on the first try and $P(N'_1)$ is the probability of no overlap with messages transmitted from the rest of the station. $P(N'_1)$ will be computed from the same distribution as $P(N_1)$ using the total number of stations minus the number with overlapping messages on the first try. Therefore, the probability of success on the first or second try will be

$$P(E_1 \text{ or } E_2) = P(N_1) + \left[1 - P(N_1)\right] \left[P(R)P(N'_1)\right] \tag{3-20}$$

This process, though laborious, can be extended to any number of tries.

To illustrate this consider a system with 200 channels. Assuming duplex calls (2 channels) lasting on an average of 5 min., the average call rate will be 0.33 call per second. If one has a common channel used only for channel release messages, the message rate on that channel will also be 0.33 message per second.

Assuming the calls are statistically independent, the probability that exactly k channel release messages are started in a time period of length t is equal to

$$P_k = \frac{(\mu t)^k}{k!} e^{-\mu t} \tag{3-21}$$

where μ is the mean call rate.

Assume that a station starts transmitting on the channel release band at time t_o and that all channel release messages are of duration t. Overlap will occur if another station starts a transmission with $\pm$ t seconds from t_o*. The probability of no overlap will be given by Equation (3-21) with k = 0; therefore,

$$P(N_1) = e^{-2\mu t} \tag{3-22}$$

where $\mu = 99/300 \approx 0.33$. For this example a message duration of 0.2 sec will be assumed. From Equation (3-14) the probability of

*Variations in transmission delays due to differences in station-to-satellite distance will cause a negligible error in the $\pm$ estimate of the overlap interval.

no overlap on the first try will be

$$P(N_1) = e^{-.13} = 0.88 \qquad (3\text{-}23)$$

Since the probability of more than two messages overlapping is negligible, there will be only one retransmitted message to copy with on the second try assuming overlap on the first try. If the delay till retransmission is a random number between zero and one second, the probability of no overlap on either the first or second try from Equation (3-20) will be

$$P(E_1 \text{ or } E_2) = 0.95 \qquad (3\text{-}24)$$

Probably the best way to detect the overlap is to use an error detecting code. Likely prospects are the "Cyclic Codes" discussed by W.W. Peterson and D.T. Brown.[11] These codes are easy to generate and are good at detecting burst type errors which occur from overlapping. A code requiring 6 redundant bits will be generated by a 6 bit shift register with feedback paths. Such a code will detect all burst errors of length 6 bits or less and will detect all but 2^{-b} of the rest, where b is the number of code bits added.

3.3.3.2 Narrow Band Channels

The obvious alternative to the common channel approach for satisfying the callup requirements is the assignment of one or more narrow band channels (or time slots) to each station. The elimination of the overlap problem by using separate bands is especially attractive when the number of stations and therefore the number of narrow band channels is not too large.

Since doppler must be considered with narrow band systems, it will be necessary to estimate the doppler shifts to be expected and (perhaps) compensate for them. It should be emphasized that this doppler effect occurs in a quite orderly fashion in a synchronous satellite and is therefore amenable to simple "standard" corrections based on precomputed doppler correction tables. The use of such corrections is attractive since they would not significantly increase the complexity of the callup subsystem.

In addition to the use of a standard correction, there are a number of other approaches to handling the doppler problem. If the shift is small enough, one can simply "open up" the bandwidth of the narrow band channels sufficiently to provide the necessary guard bands. This will, of course, increase the power requirements. If sufficient power is not available, "tracking filters" (i.e., phase locked loops) can be used. In the worst case, each narrow band channel would require its own tracking filter although it is likely that fewer tracking filters will suffice. Other alternatives are the use of more sophisticated doppler prediction with compensation at either the individual voice stations or the coordinating station, active satellite tracking and doppler measurement, and use of VHF for the narrow bands.

3.3.4 Conceptual Callup System

To facilitate the estimation of the equipment penalty associated with the callup subsystem, and to determine its impact on total system requirements, a conceptual callup system design will now be presented. No attempt has been made to optimize this design, since even this rather rudimentary approach has an unimportant effect on total system requirements.

The 50 ground stations of the Design Point System Model are assumed, in conjunction with a coordinating station with the characteristics of the intermediate size station of the DPSM (See Section 3.3.3.1). It is assumed that sufficient corrective measures have been applied (if required) to restrict the doppler uncertainty to 300 c/s.

The central callup station communicates with the other ground stations over a common callup band monitored by all stations. If station A desires to call station B, it first transmits a "tone" on its assigned narrow band channel. The arrival of station A's tone at the coordinating station sets a storage element to indicate the request. The callup station continuously polls the 50 storage elements until it finds a channel request. It then checks its record of channel availability, selects and transmits this information to station A on the common callup channel.

The coordinating station delays response to the next call request long enough to allow station A to contact station B on the callup channel. This arrangement has two advantages. First, it simplifies the logic at the coordinating station since it does not concern itself with the identification of the called party. Second, the possibility of overlap on the callup channel is eliminated by the synchronizing control of the callup station.

Assuming a nominal capacity of 1000 b/s for the callup channel, the coordinating callup station sends a synchronous code (every 0.1 second) with a 6-bit station address, 8-bit frequency assignment, and 16 bits of housekeeping and error control information. These 30 bits require 0.03 second. When they reach the ground station they are decoded and a new word is formatted (and immediately sent out on the callup channel), consisting of a 6-bit receiving station address, the 8-bit frequency assignment, and 16 bits of housekeeping and error control information. These 30 bits require 0.03 second again. After the 0.04 second "dead space" (which might be used for other control functions) the callup channel can be reassigned. Thus, the "service time" for the coordinating station queue is 0.1 second (although the entire time required for completing the connection will be about 1 second). In the unlikely case that 100 stations required connections, the coordinating station could assign them in approximately 10 seconds.

The station for which the call is intended tunes its transmitter to the assigned frequency and its receiver to the adjacent frequency. (Alternatively, two assignments could be made, or one of the frequencies could be "fixed" on a long-term basis.) This station then transmits essentially a new "dial tone" (or "busy signal" if busy) via the newly assigned transmitter frequency. The caller hears this tone and starts "dialing."

In evaluating this approach to the callup problem, it is most important to compute the power required in the spacecraft. The power required from the spacecraft transmitter is:

$$(E/N_o)_c = \frac{P_C G_S A_R \tau_c}{4\pi R^2 M_2 k T_R N_c}$$

where P_c is the satellite power required for the callup tones.

$(E/N_o)_c$ is the signal-energy-to-noise-power density ratio required in each callup band.

N_c is the number of callup bands.

τ_c is the integration time for each callup tone.

Assuming that system requirements dictate a probability of detection of 99% and a false alarm probability of 10^{-6}, the required $(E/N_o)_c$ is 15 db.[12] Let τ_c be 0.02 sec (-17 db with respect to one second), and N_c be 50 (17 db with respect to one)i.e., the worst case, all 50 stations requesting a channel simultaneously. For the Type II callup station and the parameters of the DPSM,

$$(E/N_o)_c = 69.5 + P_c + \tau_c - N_c \text{ dbw}$$

$$P_c = (E/N_o)_c + N_c - \tau_c - 69.5$$

$$= -20.5 \text{ dbw}$$

Therefore, with the total satellite power assumed to be 10 watts, the worst case callup tone power is much less than 1% of the total system power requirement. The actual ratio of callup tone power to total power will usually be even less since under normal conditions only a few of the stations will be transmitting tones simultaneously.

The additional satellite power required for the 1000 b/s channel used by all ground stations for callup purposes can be calculated in a similar manner. For this digital channel, however, the significant parameter is $(E/N_o)_{cc}$, the ratio of the signal energy per bit to the noise power density in the common callup channel, at the ground receiver. If it is assumed that a 10^{-6} bit error rate is required, Lawton[9] has shown that the necessary $(E/N_o)_{cc}$ is 11 to 14 db, depending upon the type of modulation selected. Assuming the higher requirement, the satellite power required in the 1000 b/s common callup channel is:

$$P_{cc} = (E/N_o)_{cc} + R_{cc} + 60.5 \quad \text{dbw}$$
$$= 16.5 \text{ dbw}$$

where Type I ground station parameters are assumed, and R_{cc} (the bit rate in the common callup channel) is 30 db (relative to one bit per second).

Figure 3-10 is a functional block diagram of the conceptual callup system described in Section 3.3.2. The transmitting station initiates a callup request by an input to the callup-code decoder that causes it to start the "request callup channel" tone generator. When the central coordinating station services the narrow band channel assigned to this transmitting station and detects the presence of a tone, it sends a message to the transmitting station receiver. This message will contain either a busy signal, indicating that all the available channels are in use, or a channel assignment. If the busy signal is sent, the "requesting callup channel" tone is turned off and a busy signal is sent to the output interface. A channel assignment return causes the channel number to be stored in the 30 bit storage. In addition, the "use callup channel" decoder turns off the tone generator and causes readout of the 30 bit storage.

This 30 bit message is addressed to the receiving station. The return from the addressed station will be either a busy signal or a confirmation. The busy signal will be detected by the "receiver busy signal" block which sends a busy signal to the output interface. A confirmation signal is detected by the receiver address decoder, the output of which selects the proper transmitter and receiver frequencies in preparation for voice communication.

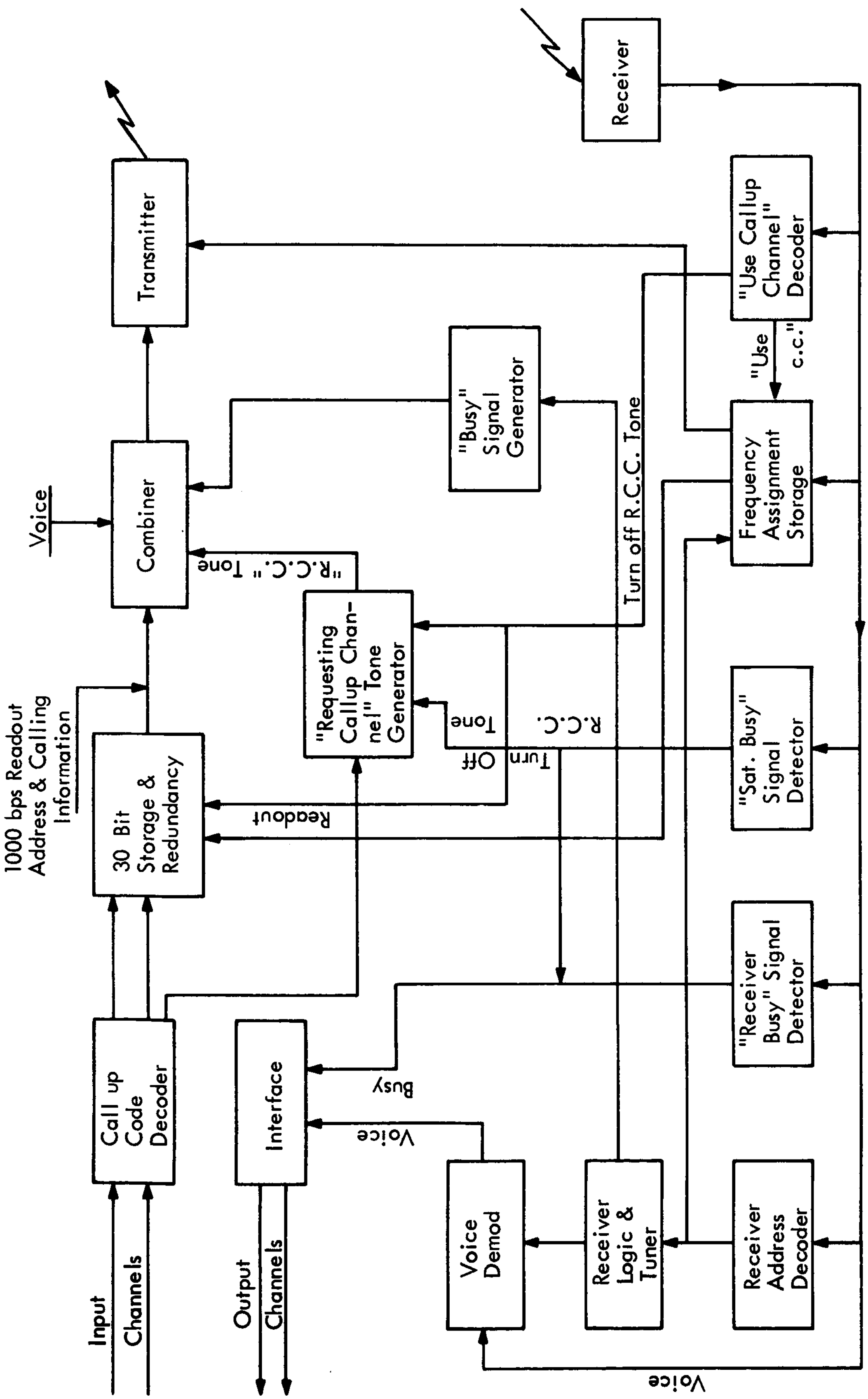


Figure 3-10. Functional Block Diagram for a Callup System

GLOSSARY

TDM	=	Time Division Multiplexing
FDM	=	Frequency Division Multiplexing
SSB	=	Single Sideband
FMFB	=	Frequency modulation with feedback
DPSM	=	Design Point System Model
SSBFM	=	Single Sideband FM
SSBAM	=	Single Sideband AM
TWT	=	Traveling Wave Tube
S/N	=	Signal-to-Noise
C/N	=	Carrier-to-Noise
D	=	Derating Factor
E/N_o	=	(Pulse) Energy-to-Noise Density Ratio
P_e	=	Probability of making an error
PSK	=	Phase Shift Keying
PCM	=	Pulse Coded Modulation
$\hat{S}$	=	Peak Signal Power
N	=	Mean Noise, Noise
PPM	=	Pulse Position Modulation
τ	=	Pulse Duration
B_p	=	Pulse Bandwidth

LO	=	Local Oscillator
T_S	=	Effective Noise Temperature
N_R	=	Ground Receiver Derived Noise
N_S	=	Satellite Receiver Derived Noise
K_1	=	Satellite Power Gain
P_S	=	Power out of Satellite
P_T	=	Power into Satellite

SECTION 4

SATELLITE COMMUNICATIONS USING PSEUDO-NOISE (PN) MULTIPLEXING TECHNIQUES

4.1 Introduction

The term "pseudo-noise (PN) modulation" refers to a type of modulation in which an information-bearing signal with a relatively narrow bandwidth is converted--prior to transmission--to a new signal with a much greater bandwidth. PN-signals have properties which make them useful as subcarriers for multiplexing asynchronously many messages into a common frequency band. A satellite is a common multiplexing point for signals received from world-wide ground stations. Since it is impractical to place a switching central in the satellite, techniques must be devised which perform the equivalent function.

PN-signaling techniques are attractive candidates for this application since the number of "good" signals, i.e., pair-wise correlation functions have relatively low sidelobes, is large; as a result, many channels can be accommodated by one satellite system although the number of active channels (those that are in fact transmitting carriers) is limited by the capacity of the system.

From the communications reliability point of view it is immaterial which channels are active provided the number is limited. This bound on the number of active channels is not critical; generally, a relatively substantial number of channels can exceed it without overloading the system. If properly designed, a PN random access system will degrade relatively slowly with overload, but it must eventually reach a point where communications break down.

Under suitable conditions PN communications techniques can undergo hard limiting with little loss in performance.[13] This capability makes such techniques attractive for a TWT repeater satellite system. It is therefore possible to present to the TWT a signal of relatively constant amplitude at a point in its operating characteristic where optimum communications efficiency can be achieved.

Finally, PN communications techniques, in the presence of white gaussian noise combined with correlation reception, are optimum from the communications theoretical point of view. Since the mutual clutter has properties similar to thermal noise, near optimum communication can be approached practically.

The techniques employed in PN modulation can be classified as

- PN signal transmission with matched filter reception
- PN signal transmission with correlation-locked reception
- Frequency-time hopping communications systems

Bandpass matched filter systems use a correlation filter followed by an envelope detector. The output of the filter, in response to the PN-signal to which it is matched, is a large peak whose height is equal to twice the energy in the waveform. Off the major peak there are subsidiary peaks called sidelobes or clutter. The duration of the major peak is the reciprocal of the spread spectrum bandwidth. The fact that there is a sharp peak which is large compared to the sidelobe level makes the matched filter output an excellent synchronization signal. The delay resolution property inherent in this technique can be exploited simply, for the purpose of achieving efficient communications.

Correlation-locked and matched filter systems are similar from the theoretical point of view, but differ markedly in the implementation. Correlation-locked systems use a PN subcarrier which is modulated by the message. The message is extracted at the receiver by multiplying an exact replica of the PN subcarrier into the received signal and then narrow-band filtering this output. Once locked, the thermal noise and mutual interference can be reduced by the processing gain. (The numerical measure of processing gain is equal to the bandwidth-time (WT) product of the signal where W is the PN signal bandwidth and T is its duration.)

Using these techniques many transmissions can be multiplexed simultaneously and asynchronously into the same frequency band permitting more efficient utilization of the satellite channel. Hard limiting in the satellite can also be used to apply a constant amplitude level to the TWT.

A major difficulty with correlation lock techniques is that precise synchronization is required at all times. The problem of establishing precise synchronization in the call-up mode is more difficult if active correlation is used at the receiver rather than a matched filter. The implementation, in terms of a hardware component count, is, however, much less complex than for matched filters.

The correlation-locked technique is much like a MODEM which establishes a connection between terminals. Once a connection is established it behaves like a wire line connecting the terminals.

Frequency-time hopping techniques are pulse communication techniques where a pulse is transmitted (1) in a different frequency slot, (2) in a different time slot, or (3) in a different frequency and different time slot. It is also possible to transmit groups of pulses, each in a different frequency-time slot, for each message symbol and to change the pattern of the group for each message bit. In some cases multiple receivers are used for detecting each pulse of short duration while in others frequency stepping is used at transmitter and receiver. Synchronization is required since the time and frequency gates are opened in synchronism with the received signals. Simple frequency-time hopping is attractive since it permits spectrum spreading over extremely broad frequency bands, more so than by the correlation or matched filter techniques. This is effective in reducing mutual interference. If pulses of extremely short duration are used, the peak power must be extremely large in order to realize a large signal energy thus limiting the range of such systems. (These characteristics apply particularly to time-hopping.) The frequency-time hopping techniques do permit weak synchronization--for example, gating in the particular region of frequency-time in which a pulse is expected. Frequency and/or time hopping techniques permit common use of the frequency band by many subscribers; that is, random access communications techniques are possible by assigning each user a different frequency-time code. The law governing the performance of frequency-time hopping systems in the presence of mutual interference is different and less efficient as compared to correlation techniques. However, frequency hopping techniques are simpler to instrument than correlation or matched filter methods.

Finally, frequency hopping techniques are compatible with matched filter techniques and also with correlation techniques. In the latter case, the narrow-band filter output would generally be followed by an envelope detector, since it would be difficult to maintain RF phase coherence over such an extremely broad band.

Pseudo-noise communication is a relatively new area which has its foundations in modern communication theory. The mathematical disciplines of coding theory, statistical detection theory, and modulation theory play a significant part in the analysis and design of such systems. Sophisticated as the techniques may appear, a thorough knowledge and understanding of conventional modulation techniques and associated physical principles is sufficient to the understanding of the PN techniques from an engineering standpoint. Although the signals used are more complex than, for example, sinusoidal signals, the basic principle of modulation remains unchanged.

The number of available PN communications techniques is more numerous than the more popular modulation methods. In fact, the conventional modulation techniques can be superimposed on a PN subcarrier, though the message recovery operations are necessarily different. Considering the above, an attempt was made to develop a limited number of PN multiplexing techniques which are considered important and which are representative configurations of a larger class. The techniques are demonstrated with simple block diagrams along with a description of the basic operations. With this as a background the uninitiated in this new field will be able to better evaluate the PN random access techniques which are proposed for further study. A thorough reading of Sections 4.2 and 4.3 will permit the reader to study and understand the detailed configurations in Section 4.6 which are proposed for the Phase III study, and to assess their engineering and operational implications.

However, pseudo-noise techniques, much like conventional modulation, is intimately tied to mathematical theory. The mathematical theory of asynchronous multiplexing using PN techniques is therefore developed in detail. The theory attempts to describe the physical operations used in PN reception and aims at two major goals:

(a) Development of physically realizable performance measures which are functions of the channel parameters (i.e., bandwidth, time, signal power, noise power, number of users).

(b) Development of performance measures which can only be attained in theory (i.e., channel capacity).

The first goal gives insight to the interrelationship among the channel parameters, the possible trade-offs, and specifies the expected performance.

The second goal gives an absolute reference for comparing different modulation techniques on a theoretical basis.

The success of the mathematical theory in reaching the goals depends on how well it describes the real satellite communications channel. It is believed that the theory is sufficiently accurate for practical purposes.

In Phase III computer simulation will be used as a tool for studying and verifying the assumptions made in developing the theory. The communications simulator, although still an approximation to the real satellite channel, can take into account some of the effects which had to be neglected in the theory, and which will yield performance results which will serve as a valuable comparison with those predicted by theory.

A major part of the work reported on in this section was performed under IBM's internally sponsored Independent Research and Development Program (IRAD).

4.2 Matched Filter Techniques

4.2.1 Transmission Using PN Signal Alphabets

The output of a filter which is matched to the input signal is the autocorrelation function of the signal. At the instant of match, the output of the filter is a maximum proportional to the energy in the input signal. All other input signals having the same energy will necessarily have a smaller maximum output. Physically, the instant of match can be considered as the instant at which each elementary component of the spectrum adds in phase.

In communications we are particularly interested in bandpass signals. Spread-spectrum techniques make use of extremely complicated signal alphabets. The complexity of a signal is defined here as one which has large dimensionality. A pseudo-noise binary signal of finite duration having many 1's and 0's is an example of a signal having a large dimensionality. The signal output of a random noise generator consisting of many independent noise voltage samples is another example of large dimensionality. It is a mathematical convenience to use the analytic signal representation for studying the behavior of noise-like signal alphabets and communications systems.

Briefly, a complex bandpass signal of time duration T can be expressed mathematically as

$$Z_o(t) = Z(t) \exp[j\omega_o t] \qquad 0 \leq t \leq T \tag{4-1}$$

where

$\exp[j\omega_o t]$ = complex sinusoidal carrier of frequency

$Z(t)$ = complex envelope function

T = time duration of signal

The complex envelope function is defined as

$$Z(t) = a(t) \exp[j\phi(t)] \qquad 0 \leq t \leq T \tag{4-2}$$

where

$a(t)$ = envelope of modulated carrier

$\phi(t)$ = phase of modulated carrier

Another expression for Equation (4-2) is

$$x(t) = a(t) \cos \phi(t)$$

$$y(t) = a(t) \sin \phi(t) \tag{4-3}$$

where

$$a(t) = [x^2(t) + y^2(t)]^{1/2}$$

$$\phi(t) = \tan^{-1} y(t)/x(t) \tag{4-4}$$

Combining Equations (4-1) and (4-2) gives, for the general expression of a bandpass signal,

$$Z_o(t) = a(t) \exp(j[\omega_o t + \phi(t)]) \qquad 0 \leq t \leq T \tag{4-5}$$

The real part of Equation (4-5) is the physical bandpass signal.

Bandpass noise signals are represented mathematically as in Equation (4-5) where a(t) and ϕ (t) are now random variables. A linear bandpass filter can also be represented by Equation (4-5). In fact, the signal given by Equation (4-5) can be represented by an impulse generator of duration $T_{\Delta} << T$ which drives a linear filter whose impulse response, Equation (4-5), is shown in Figure 4-1.

If the output of Figure 4-1 is fed into a linear filter whose impulse response is the mirror image of the real part of $Z_o(t)$, the output of this second filter is the autocorrelation function of the real part of $Z_o(t)$, as shown in Figure 4-2.

Conceptually, the same result can be obtained by taking the output of Figure 4-1, recording it on tape, perhaps, and playing the recorded signal backwards into the filter Z(t).

The filter concepts demonstrated graphically in Figures 4-1 and 4-2 represent the basic building blocks of matched filter spread-spectrum communication techniques.

The dimensionality of a pseudo-noise signal waveform of the type shown in Figure 4-1 is given by its bandwidth-time product. This product, often referred to as the "processing gain," is the most important signal parameter of PN signals.

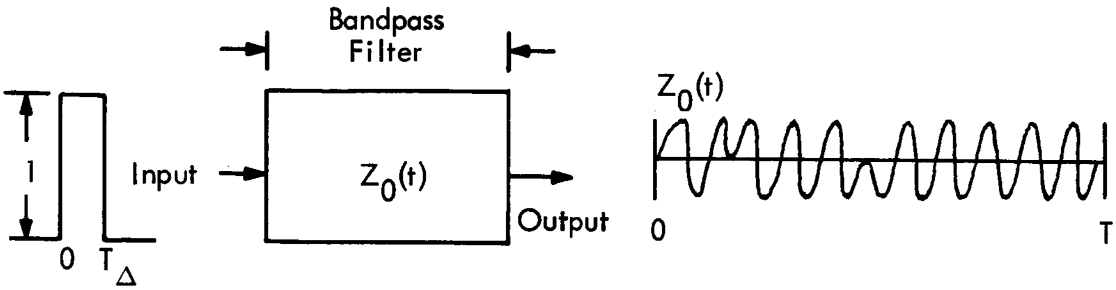


Figure 4-1. Impulse Response of Bandpass Filter

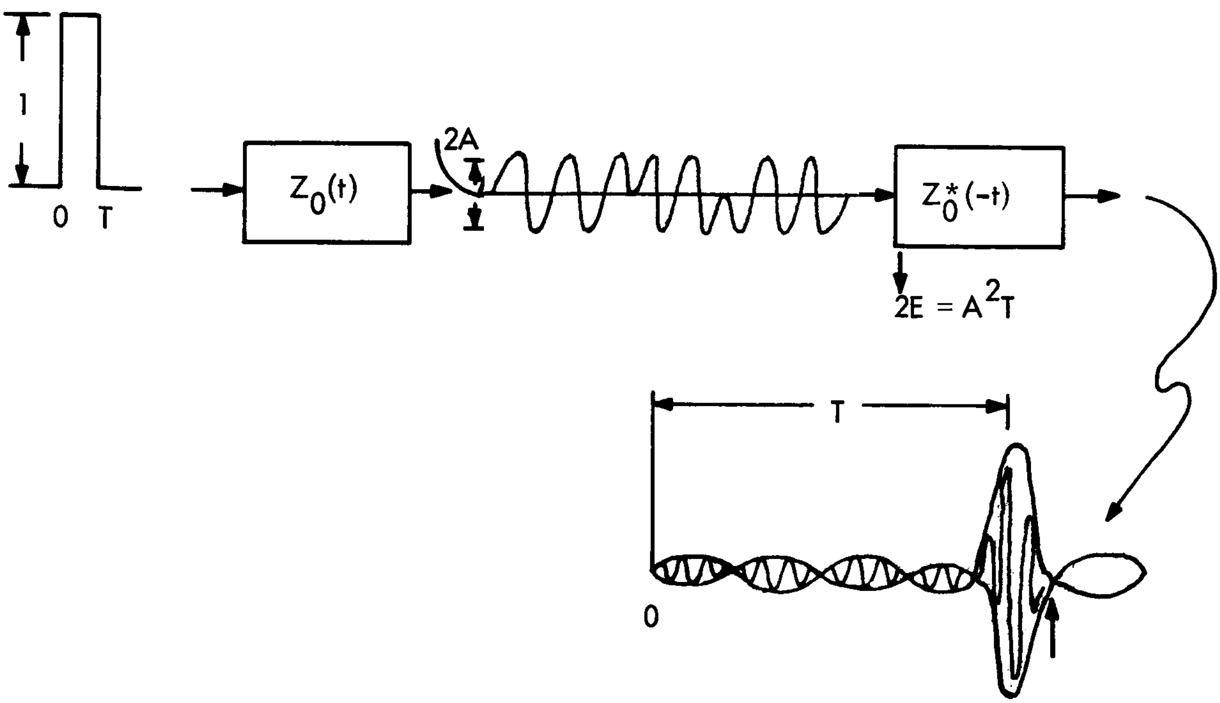


Figure 4-2. Bandpass Matched Filter Output

4.2.1.1 Discrete PN Signal Alphabets

A discrete pseudo-noise signal alphabet of "M" signals assigns a particular noise-like waveform to each of M possible results of measurement. We can call each of the M discrete values a message. In particular, if the message is binary (M=2), a PN wave is assigned to binary number 1 and another wave is assigned to binary 0. In certain applications, the absence of a waveform can represent one of the binary message symbols.

Binary Signal Alphabets. An extremely important pseudo-noise binary signal alphabet is obtained from Equation (4-5) by letting

$$a(t) = a_o = \text{constant} \qquad 0 \le t \le T$$

$$\exp[j\phi(t)] = \{1, 1, -1, -1, \ldots 1, -1, 1, 1, -1\}$$

$$= \text{pseudo-noise sequence of } N \text{ bits.} \qquad (4\text{-}6)$$

For this special case, Equation (4-5) is simply

$$Z_o(t) = \begin{cases} \exp\{j[\omega_o t + \phi_1(t)]\} \rightarrow \text{waveform for binary zero} \\ \exp\{j[\omega_o t + \phi_2(t)]\} \rightarrow \text{waveform for binary one} \end{cases} \qquad (4\text{-}7)$$

Each binary message symbol has the same carrier frequency ω_o but a different phase-reversal code $\phi(t)$. The phase reversal code performs the spectrum spreading.

In this type of system each subscriber is assigned a pair of PN signals (i.e., to represent its "one" and "zero") that are distinct from all other pairs. A receiver can only respond to its assigned signals; all other mutually interfering signals are suppressed.

The spectrum-spreading factor of such a system can easily be calculated. Let T be the duration of a message symbol and hence the duration of the PN signal alphabet and let ΔT be the duration of a phase reversal bit of the PN waveform $\exp[j\phi(t)]$. Such a waveform is illustrated in Figure 4-3. The spectrum spreading factor is

$$N = T/\Delta T = 2WT = W/W_o \tag{4-8}$$

where

$W = 1/2\Delta T$ = effective (low-pass) signal bandwidth,

W_o = effective low-pass bandwidth of message symbol.

Equation (4-8) very simply shows that the effective bandwidth of the binary message is increased by a factor N.

The average energy E in the waveform of Figure 4-3 when modulating a carrier is

$$E = \overline{A^2}\, T \tag{4-9}$$

where A is the amplitude of the carrier and $\overline{A^2}$ is the mean-square value. Prior to spreading, this energy occupies the band $2W_o$. Hence, the power density (the power per cycle of bandwidth) in the narrow-band case is

$$\frac{D(W_o)}{T} = \frac{E}{2W_o T} = \frac{\overline{A^2}}{2W_o} \tag{4-10}$$

After spectrum-spreading the power density is

$$\frac{D(W)}{T} = \frac{E}{2WT} = \frac{\overline{A^2}}{2W} = \frac{\overline{A^2}}{2NW_o}$$

$$= \frac{1}{N} \left(\frac{D(W_o)}{T} \right) \qquad (4\text{-}11)$$

where

$D(W_o)$ = energy density in narrow-band case,

$D(W)$ = energy density in broadband case.

Equation (4-10) shows that in the narrow-band case the power density after spreading is 1/N times the power density before spreading. This is shown in Figure 4-4.

In addition to using two different sequences per subscriber for encoding the binary message symbols into PN waveforms, we can use other techniques. A standard FSK (frequency-shift-keying) system has an alphabet given by

$$Z(t) = \begin{cases} \exp\,[j\Delta\omega t] & \text{"one"} \\ \exp\,[j\Delta\omega t] & \text{"zero"} \end{cases} \qquad 0 \leq t \leq T \qquad (4\text{-}12)$$

where

$$\Delta\omega = 2\pi/T$$

Equation (4-12) represents an orthogonal pair of waveforms in the time interval (0, T). These can easily be encoded into PN waveforms by multiplication with a noise-like carrier. The result of such an operation is

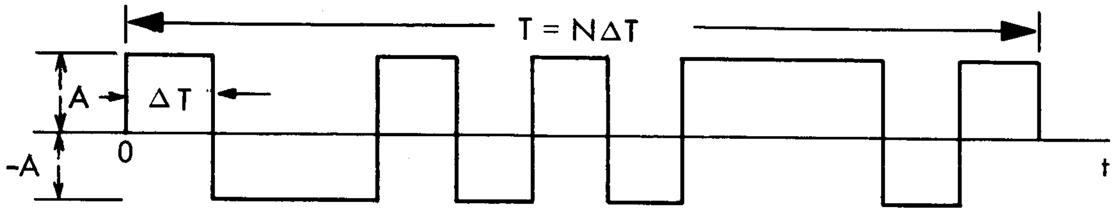


Figure 4-3. Phase Reversal Signal Address

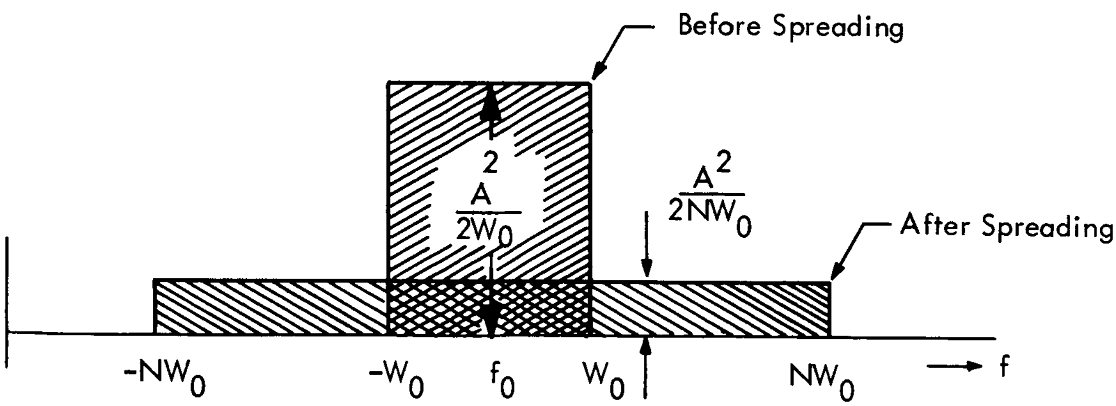


Figure 4-4. Power Density Spectrum Before and After Spreading

$$Z_0(t) = \begin{cases} \exp\{j\,([\omega_o + \frac{\Delta\omega}{2}]\; t + \phi(t)\,)\} \\ \exp\{j\,([\omega_o - \frac{\Delta\omega}{2}]\; t + \phi(t)\,)\} \end{cases} \qquad 0 \leq t \leq T \qquad (4\text{-}13)$$

In Equation (4-13) the phase modulation $\phi(t)$ can be phase-reversal modulation or $\phi(t)$ can be continuous analog PN angle modulation. In a system using FSK, each subscriber is assigned a different $\Delta\omega$. All subscribers use the same PN modulation ϕ (t).

The PN signal alphabet encoding procedure can be either passive or active. A passive PN signal generator uses a linear filter whose basic structure is modified by a PN sequence generator. The PN generator selects the signal address. An impulse generator combined with this filter is the PN signal address generator as shown in Figure 4-1. The active alphabet generator performs the operations indicated in Equations (4-7) or (4-12) using conventional mixing techniques.

Figure 4-5 shows a passive signal alphabet selector and generator in more detail. The binary message symbol 1 selects the impulse generator which, in turn, excites the corresponding bandpass filter. The output of the filter is a noise-like wave. The filter effectively "smears-out" the impulse in time. Similarly, each time the message source generates a 0 the impulse generator excites the corresponding filter. The PN sequence generator, once set, is

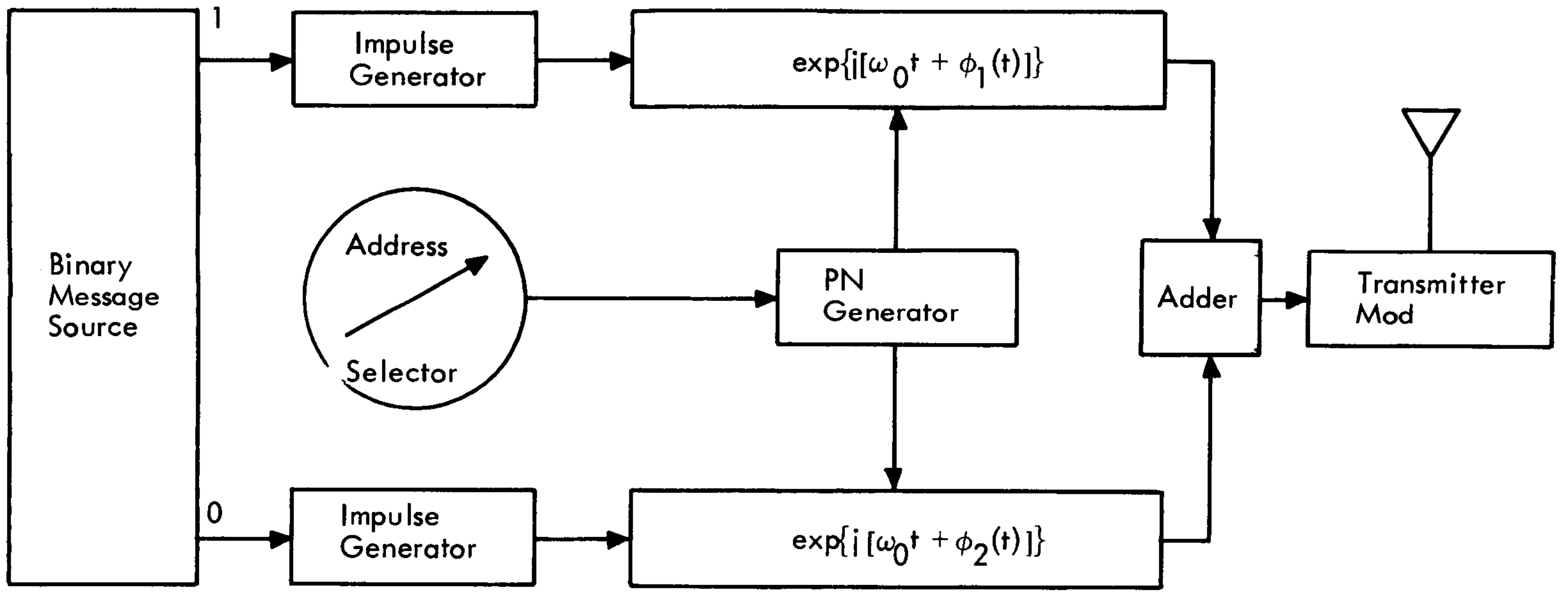
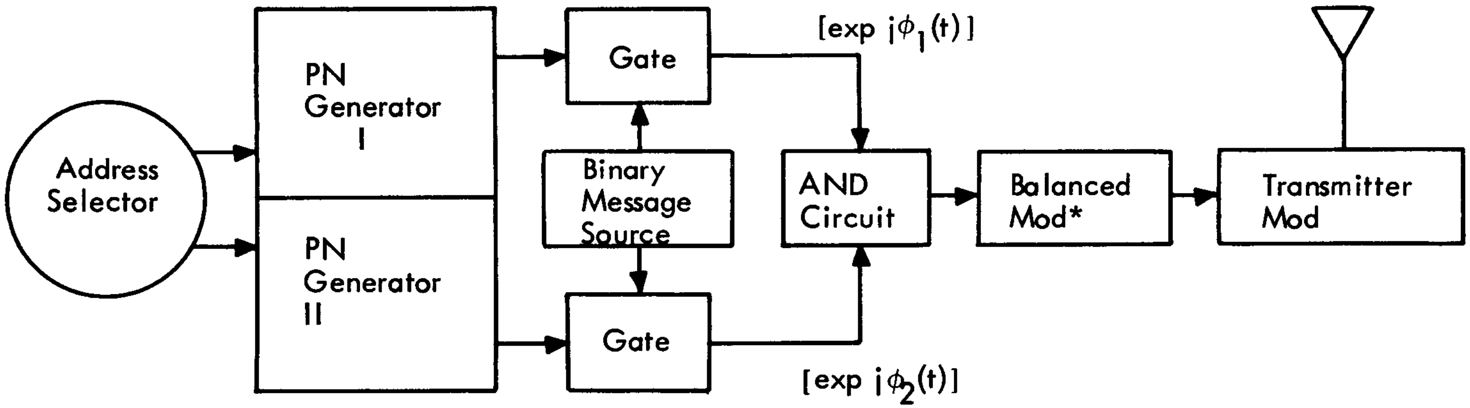


Figure 4-5. Passive PN Signal Selector and Generator (Message Encoding)

static; it modifies certain properties of the filter which generates the noise wave. The structure shown is very general in that $\phi(t)$ can be phase reversal or analog or FSK.

Figure 4-6 is an active technique for implementing Equation (4-7). Two PN sequence generators are shown driven in synchronism; Generator I corresponds to binary message symbol 1 and Generator II corresponds to message symbol 0. (In practice, a single PN generator can be used for generating the noise-like alphabets.) The binary message source, which is synchronized with the PN generator, selects the PN wave corresponding to 1 and 0. The encoded message symbol is fed into a balanced modulator where it is converted to the bandpass signal shown in Equation (4-7). This signal, in turn, is applied to the transmitter modulator.

Figure 4-7 is a block diagram of an FSK PN signal alphabet generator. The output of the balanced modulator is a PN subcarrier. The carrier is then tone-modulated by the frequency shift keying circuit. In this system the PN signal is the same for all subcarriers' addresses. Each subscriber is, however, assigned a different pair of frequency shifts. Thus, the address selector presets FSK_1 and FSK_2 to the correct address.



*Includes carrier generator. This convention will be followed throughout this report.

Figure 4-6. Active PN Signal Selector and Generator (Message Encoding)

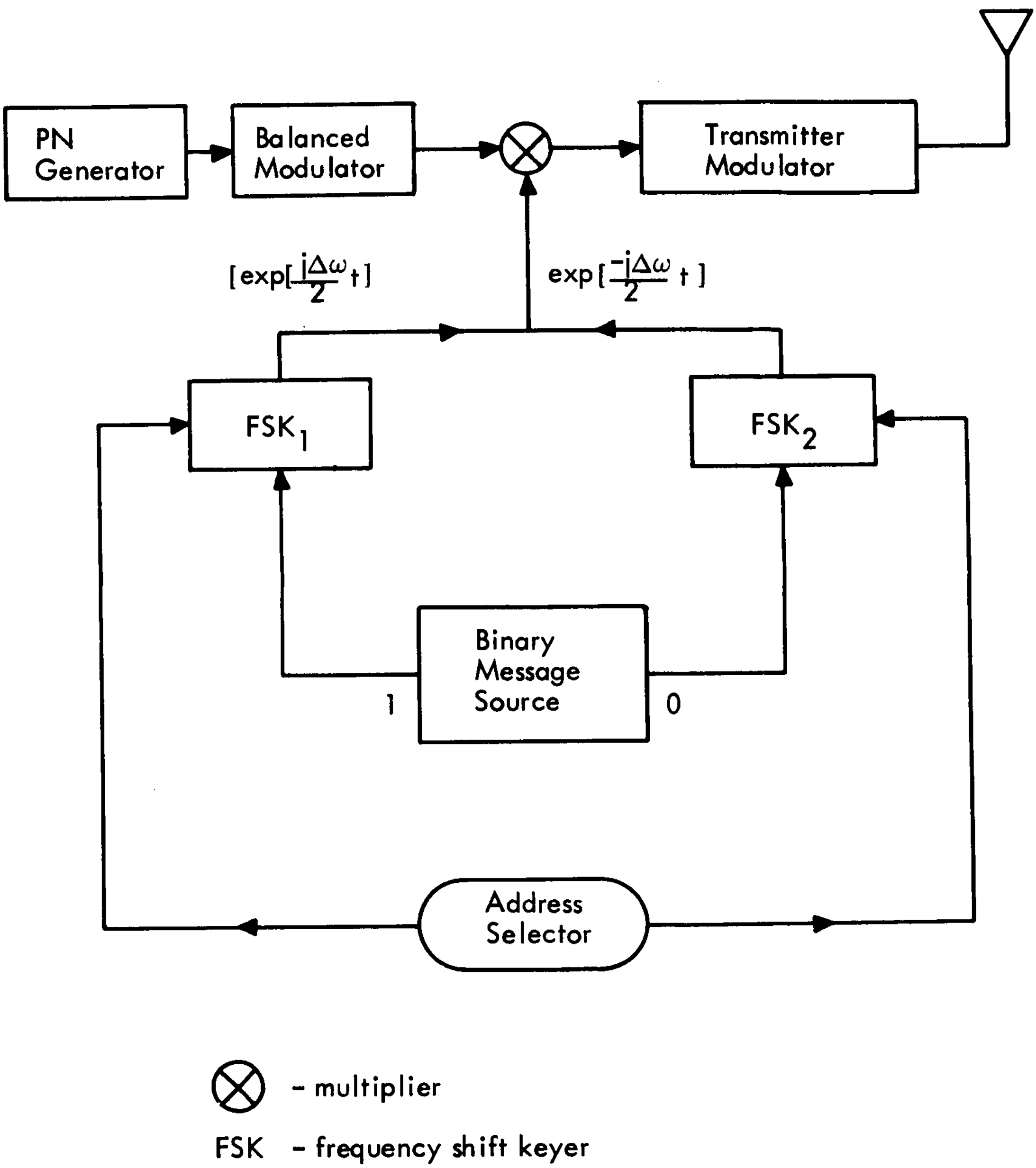


Figure 4-7. FSK PN Signal Generator and Selector (Message Encoding)

If an analog PN subcarrier is to be used, it is necessary first to filter the output of the PN generator. The balanced modulator is then replaced by a phase modulator. The rest of the circuitry remains unchanged.

The block diagrams discussed thus far are representative of the type of signal address generator and encoding techniques to be analyzed in greater detail in subsequent sections of this report.

Higher-Order Signal Alphabets. A higher-order alphabet assigns a signal waveform to each binary message sequence. If binary message sequences of length K bits are to be encoded, then 2^K PN signal waveforms are required for each subscriber. An advantage of using higher-order signal alphabets is that it is also a way of approaching the theoretical channel capacity limit. The attainment of this efficiency often comes about at the expense of extremely large bandwidths and extremely complex apparatus. One type of higher-order alphabet that is of some interest uses almost-orthogonal signals. An example would be the use of the delay resolution properties of PN signals. Here, the implementation is no more complex than the techniques discussed for binary alphabets.

In general, the higher-order PN signal alphabet can be expressed mathematically as,

$$Z_{oi}(t) = \exp\{ j[\omega_o t + \phi_i(t)]\} \qquad 0 \leq t \leq T \qquad (4\text{-}14)$$
$$i = 1, 2, \ldots, M$$

where M is the order of the alphabet. The number of message bits which can be encoded into the M signals is

$$K = \log_2 M \text{ bits.} \qquad (4\text{-}15)$$

Each PN signal has the spectrum-spreading code contained in the term $\exp[j\phi_i(t)]$. The spectrum-spreading signals can be generated by an ensemble of PN generators or derived from one PN generator. It is also possible to have an ensemble of linear filters all of which are modified simultaneously by a single PN generator. One such signal alphabet is the extension of the binary FSK noise-like alphabet.

A higher-order FSK alphabet can be expressed mathematically as

$$Z_{on}(t) = \exp\ j[(\omega_o + n\Delta\omega)\,t + \phi(t)]\} \qquad (4\text{-}16)$$
$$n = 0, \pm 1, \pm 2, \pm M/2 \quad 0 \leq t \leq T$$

Equation (4-16) represents a conventional multiple FSK signal alphabet with the spectrum-spreading angle modulation $\phi(t)$ which can be phase reversal modulated or analog PN modulated. The PN modulation takes each orthogonal sinusoidal waveform and spreads it over the broadband W. In practice each subscriber in a communications system may be assigned an alphabet of M frequency shifts. When a party is called the signal address generator is preset to the M frequency shifts. All signals have the same spreading code. Matched filters at the receiver can recognize each waveform. Figure 4-5 shows the power density distribution of such an alphabet before and after spectrum spreading. For illustrative purposes a rectangular representation of the spectra is again used, recognizing that this is not the true situation.

Figure 4-8(a) is a pictorial representation of the narrow-band signal alphabet while Figure 4-8(b) shows the spread-spectrum alphabet. From Figure 4-8(a) note that bandpass filters can separate a mixture of the narrow-band signals; broadband filters will not separate a mixture of the spread-spectrum signals. However, matched filters will separate the latter. This fact will be demonstrated in Section 4.2.2 where matched filter reception will be considered.

The block diagram of higher-order PN signal alphabet generators are the logical extensions of those shown for the binary alphabets. Figures 4-5, 4-6 and 4-7, and hence will not be shown here.

Previously it was shown that a higher-order alphabet can be generated by employing multiple FSK. A corresponding technique

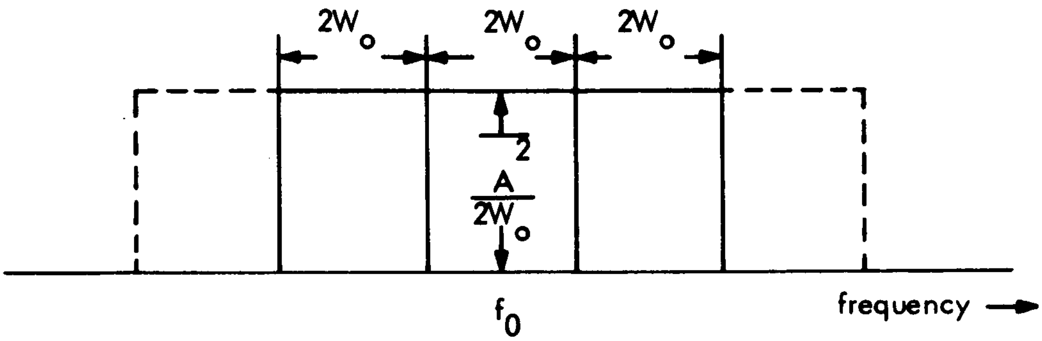


Figure 4-8(a). Narrow-Band Multiple FSK Signal Spectrum

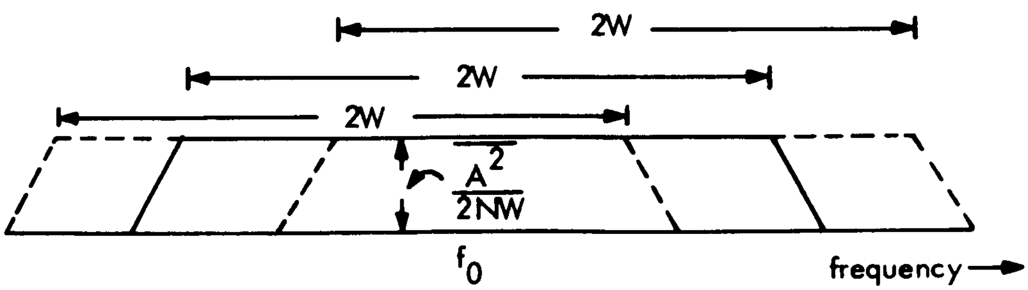


Figure 4-8(b). Spread Spectrum PN Multiple FSK Signal for generating a higher-order alphabet in the time domain is to shift a pulse with respect to a reference into one of the M time slots. This is a form of digital pulse-position modulation and is particularly applicable to signal alphabets that have the delay resolution property such as PN alphabets combined with matched filter reception. The PN encoding is achieved by feeding the pulse train into a matched filter, which smears it in time, or by using an active encoding procedure. At the receiver, each pulse is reconstructed by a matched filter. This technique is attractive because a single matched filter is required at the receiver. Figure 4-9 is a block diagram which demonstrates this modulation technique.

The PPM pulse train is smeared out by the matched filter. If the maximum deviation is one-half the time between samples and if the PN signal duration is equal to this time, synchronization at the receiver can be maintained. Of course, in a system of this type there are gaps in time. This can be extremely desirable since it increases the effective WT product without increasing the complexity of the matched filter. As long as there are gaps in the transmitted signals the duty factor is less than unity and hence more active users of the channel can be accommodated. The time gaps in the signals also reduce the effects of strong signal interference. If the duty factor is unity, a strong signal can capture the channel. If there are long time intervals of zero signals, as may be the case here, there is a reasonable probability that the strong signal will not overlap the desired one. If the matched filter receiver is preceded by a hard limiter, such a situation has no significant effect on the performance.

4.2.1.2 PN Alphabets as Sounding Signals for Time-Varying Media

A communication channel is said to be quasi-stationary if its characteristics change slowly with time relative to the message rate. If the time-varying channel can be represented by a linear filter, then the impulse response describes it over the period of time for which it is stationary. Physical channels characterized by multi-paths, doppler, and also meteor burst channels, fall into this category. In this case, the impulse response of the channel (or medium) can be measured and proper compensation can be made at the receiver in order to optimize reception. A signal used for such a measurement is called a "sounding" signal. The output of a matched filter when a PN signal passes through a linear filter of unknown impulse response is the impulse response of the unknown filter. This follows from one of Norbert Wiener's famous theorems and is illustrated in Figure 4-10.

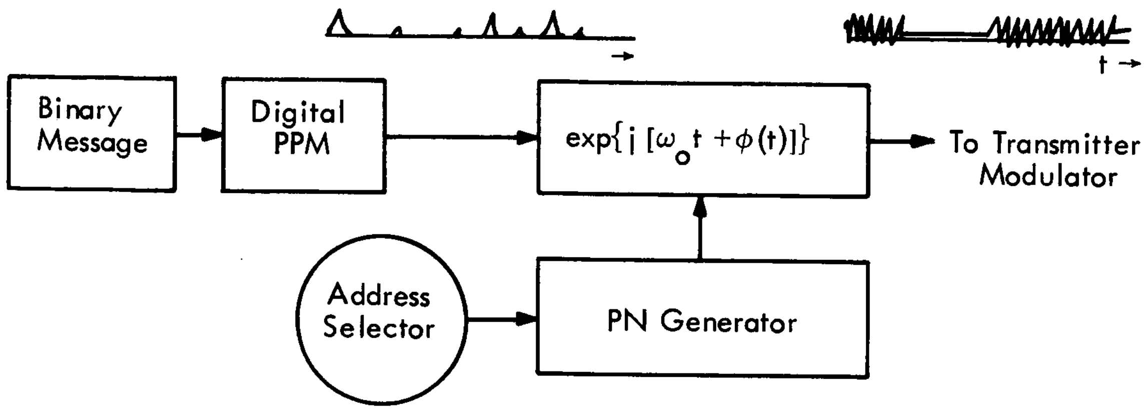


Figure 4-9. Digital PPM and PN Signal Encoder

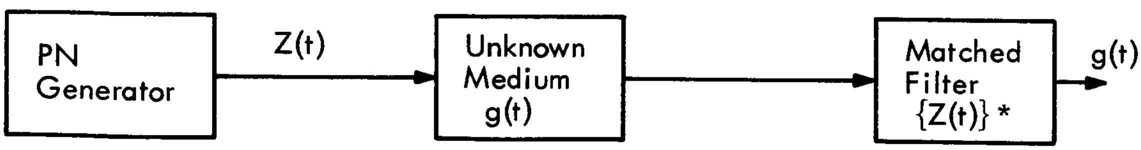


Figure 4-10. Measurement of Impulse Response of Medium

The PN waveforms combined with matched filters are also useful for establishing and maintaining message synchronization if required. This also comes about because of the delay resolution capability, the same property which is used in multipath resolution and in spread-spectrum radar for that matter.

In many applications, signals have a doppler shift. In this case the received alphabet can be represented mathematically as in Equation (4-16). It should be clear, however, that PN signals of the form $\exp\{j[\omega_o t + \phi(t)]\}$ will be mismatched at the receiver if the doppler shift is, say, $\Delta\omega > 2\pi/T$. This results in attenuation of the correlation peak at the receiver. One way of countering such a problem is to track doppler by using a frequency discriminator at the output of the matched filter. Acquisition can be achieved by the transmitter varying its carrier frequency until the receiver is locked-in. A two-way communication link is required here since the receiver must notify the transmitter when it is acquired. Acquisition in the presence of doppler can be difficult because of strong mutual interference. It is possible, however, to assign a separate channel for the purpose of establishing doppler lock.

Another approach to doppler acquisition is to use a bank of filters matched to

$$Z_{on}^{(i)}(t) = \exp\{j[(\omega_o + n\,\Delta\omega)t + \phi_i(t)]\}$$

$$n = 0 \pm 1, \pm 2, \ldots$$

A doppler shift will therefore cause one of the filters to respond, which supplies a doppler estimate and, if desired, time synchronization. Thus, each subscriber is assigned a PN signal address which is received as $Z_{on}^{(i)}(t)$. The receiver must have a filter matched to each possible doppler shift. Once a signal is received, the doppler estimate can be obtained and subsequently removed by doppler correction and tracking. If no doppler correction is used, frequency shifted signal addresses, Equation (4-16), are undesirable here since doppler introduces signal address ambiguity; that is, doppler can cause the unintended receiver to respond to call.

4.2.1.3 Synchronization by Using Matched Filters

The matched filter output in response to a PN signal is inherently a "good" synchronizing signal since the output is a pause of short duration. It is therefore possible to establish precise synchronization and to maintain it.

If desired, it is also possible to gate the receiver in the neighborhood of an expected signal arrival. Here, precise synchronization is not required provided the sidelobes of the autocorrelation function are substantially below the maximum.

Errors in detection can occur if the mutual interference produces an error during the time the gate is open.

Finally, if the signals have desirable correlation properties, the matched filter receiver can be wide open in time. If the receiver is preceded by a hard limiter the false threshold crossing error rate can be held constant, independent of the output clutter, while an erasure can occur only when the mutual clutter and noise during the presence of a signal pulse modifies the desired signal sufficiently so as to cause the matched filter not to respond.

4.2.2 Matched Filter Reception and Demodulation

The most complex part of a spread-spectrum modulation system is the receiver. This is particularly true for the case of matched filter systems. As shown in Section 4.2.1 either passive or active PN signal alphabet generation can be employed. At the receiver, there must be a matched filter which performs the crosscorrelation operation between the signal-interference combination and the locally "stored" PN signal alphabets. The stored PN signal alphabet is represented by the impulse response of the filter. In the absence of noise and interference, the matched filter output is the autocorrelation function of the input and hence has a definite maximum at one instant.

A linear filter can be considered a crosscorrelation; the output of the filter is the crosscorrelation function between the input signal and the mirror image of the impulse response, or equivalently, the impulse response and the mirror image of the input signal. Thus, if $Z(t)$ is a complex input signal, then $Z^*(-t)$ is the impulse response of a filter matched to $Z(t)$ (where Z^* represents complex conjugate of Z). If the filter has a finite duration response time, its impulse response is of the form

$$\begin{aligned} Z_1(t) &= Z^*(T-t) \\ &= \{Z(t)\}^* \qquad 0 \leq t \leq T \end{aligned} \tag{4-17}$$

where $\{Z(t)\}^*$ is the symbolic representation for impulse response of a filter matched to $Z(t)$. All receiver filters in this section will be represented by this symbolism. In general, the received signal can be represented by an additive complex noise process,

$$Z(t) = Z_{oR}(t) + Z_n(t) + Z_c(t) \tag{4-18}$$

where

$Z_{oR}(t)$ = received PN signal,

$Z_n(t)$ = thermal noise receiver,

$Z_c(t)$ = clutter introduced by other users of the channel.

In case of multiplicative noise, such as fading channels, we assume that the time-varying channel changes slowly compared to the message rate so that appropriate corrections can be made at the receiver. Time-varying media, doppler, etc., are perturbations on the transmitted signal $Z_o(t)$. The other interference is assumed additive and independent. Here we will assume that the internal receiver noise $Z_n(t)$ is white gaussian.

Figure 4-11 is a block diagram demonstrating a matched filter communication system. The transmitted carrier, modified by the medium, is received by a broadband RF front end at which point thermal noise is added.

Hard Limiting. The broadband output is fed into a hard limiter followed by a zonal filter which selects the broadband signal centered about the first harmonic of the IF. The combination is refered to as a band pass limiter (BPL). In the presence of severe broadband thermal noise the signal-to-noise ratio at the output of the hard limiter is for all practical purposes the same as the input. The effect of the hard limiter is equivalent to an AGC acting off the total signal plus interference. The degradation in output signal-to-noise with limiting when compared to the IF output without limiting varies from 1 - 3 db depending on the type of random noise.

A relatively higher degradation of signals occurs at high input signal-to-noise ratios. However, since the input signal-to-noise is

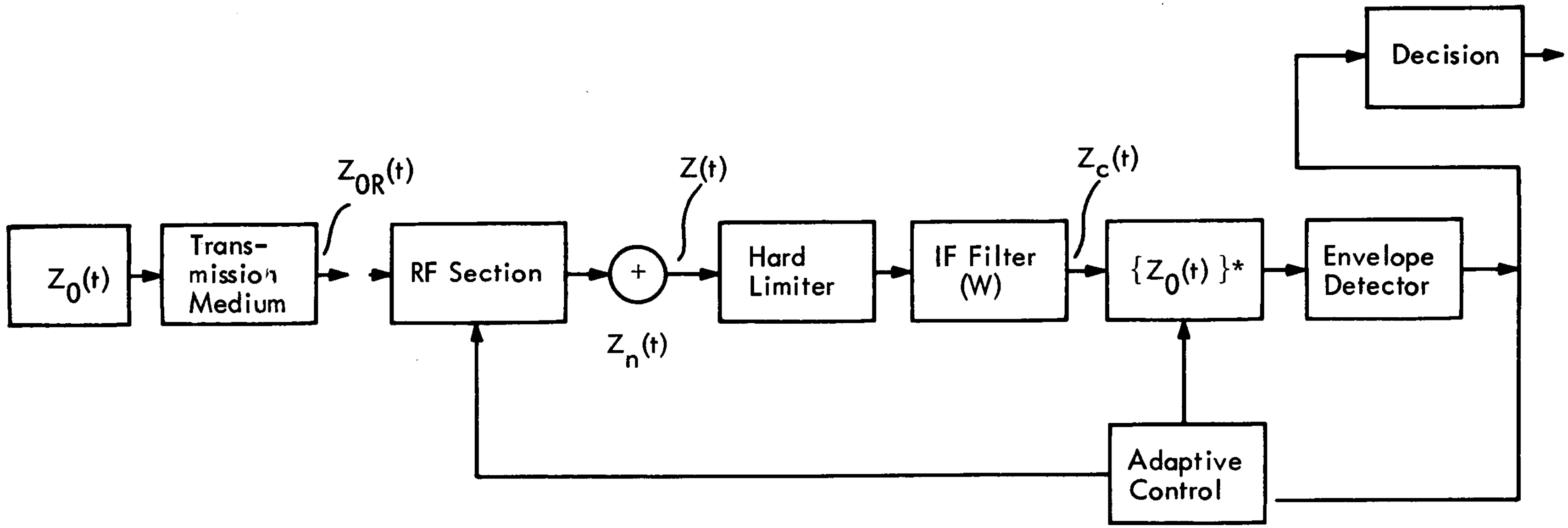


Figure 4-11. PN Communication Channel With Adaptive Control

high, the degradation is insignificant. The important point is that when the input signal-to-noise is small, for example when degradation caused by limiting cannot be tolerated, performance is hardly affected. The matched filter, of course, will build the signal up so that it will have adequate signal-to-noise ratio prior to envelope detection. In random access systems the PN signals are, almost always, immersed in noise and are therefore negligibly affected by hard limiting.

The hard limiter is a very simple yet extremely important non-linear operation in spread-spectrum matched filter techniques. In the absence of signal, the output power of a PN signal matched filter preceded by a hard limiter is independent of the input interference. In systems which have a fixed threshold of detection this prevents strong signal interference from capturing the receiver. Of course, when signal is present, the effect of the mutual interference is to push the desired matched filter output peak towards and eventually below the threshold as the interference increases. The hard limiter also eliminates certain design problems because it normalizes the dynamic range of the input.

In the case of two sinusoidal carriers lying in the same band, both having large signal-to-noise ratios but one signal dominating the other, the relative signal-to-noise ratio at the output is at most 6db down, the weaker signal being suppressed relative to the strone one. However, if the weak sinusoidal signal and strong one are deeply immersed in the noise, both will emerge from the bandpass limiter in the same ratio as they would in the absence of limiting; there is no suppression for all practical purposes.

In a random access system used for repeater satellite communications, it is desirable to have a hard limiter in the satellite preceding the TWT. The hard limiter is an asynchronous multiplexer which can supply a constant level signal to the TWT. The signals at the output of the limiter emerge with the same relative powers with which they enter.

Adaptive Control. The signal $Z_c(t)$ is fed into the filter $\{Z_o(t)\}^*$ which is matched to the transmitter signal. The adaptive control feeds back doppler information to the RF section voltage-controlled oscillator correcting for doppler shift. In some cases the matched filter can also be controlled to compensate for doppler compression and expansion on the envelope. The adaptive control also contains means for multipath combining. The output of the envelope detector is fed into a decision mechanism.

4.2.2.1 Reception of Discrete Signal Alphabets

In Section 4.2.1.1 we discussed techniques for generating PN alphabets. Here we will discuss PN receiver structures that are matched to those alphabets.

<u>Binary Matched Filter Receivers for PN Alphabets.</u> Assume that the receiver clock is synchronized to the binary message source at the transmitter. The receiver matched to the binary signal alphabet, in Equation (4-7), has an impulse response (using the symbolic representation for matched filters previously defined) of

$$Z_o(t) \quad * \quad \begin{array}{l} \nearrow \quad \exp\{ j[\omega_o t + \phi_1(t)]\} \; * \\ \\ \searrow \quad \exp\{ j[\omega_o t + \phi_2(t)]\} \; * \end{array} \qquad (4\text{-}19)$$

The block diagram of the binary matched filter receiver, which corresponds to the transmitter in Figures 4-5 and 4-6, is shown in Figure 4-12.

The mixture of signal and noise is fed into the bandpass limiter (BPL) and then into the matched filter. The matched filter will select the desired signal and suppress the undesired ones. Each matched filter output is fed into an envelope detector (ED) and then into a decision device. The optimum decision device chooses a message signal that is the largest at the instant of sampling.

There are four modes of operation possible at the receiver

(a) PN bit synchronous operation

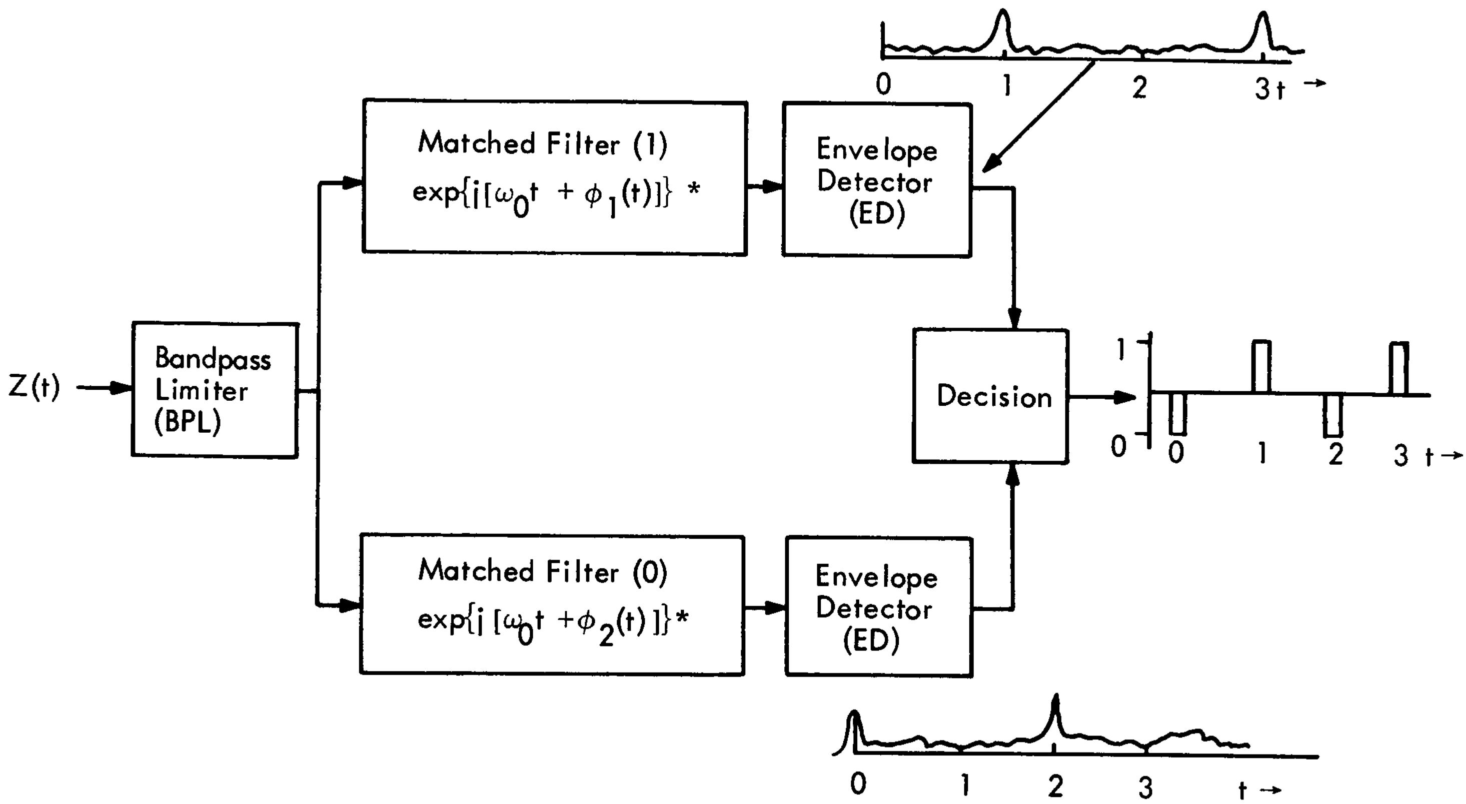


Figure 4-12. PN Matched Filter Binary Receiver

(b) PN quasi-synchronous operation

(c) Message bit synchronous operation

(d) Asynchronous operation

In the PN bit synchronous operation the synchronized receiver samples the decision circuit precisely at the expected peak of the matched filter.

In the PN quasi-synchronous operation a gate is generated which samples the decision device in the neighborhood of the peak (i.e., similar to range gating a radar receiver). The largest value during the gate interval is chosen from each matched filter and comparison is made between these outputs.

Message bit synchronization is a form of asynchronous operation where each detector is fed independently into a threshold decision device. The threshold is set sufficiently high so that the probability of a noise pulse exceeding the threshold is small, but where a signal-plus-noise pulse has a high probability of exceeding the threshold. (This is equivalent to the radar terms, false, alarm, and probability of detection.) As long as the actual transmitted PN signal is short in duration compared to the message bit duration, this operation can be termed asynchronous over an interval. This type of operation is shown in Figure 4-13.

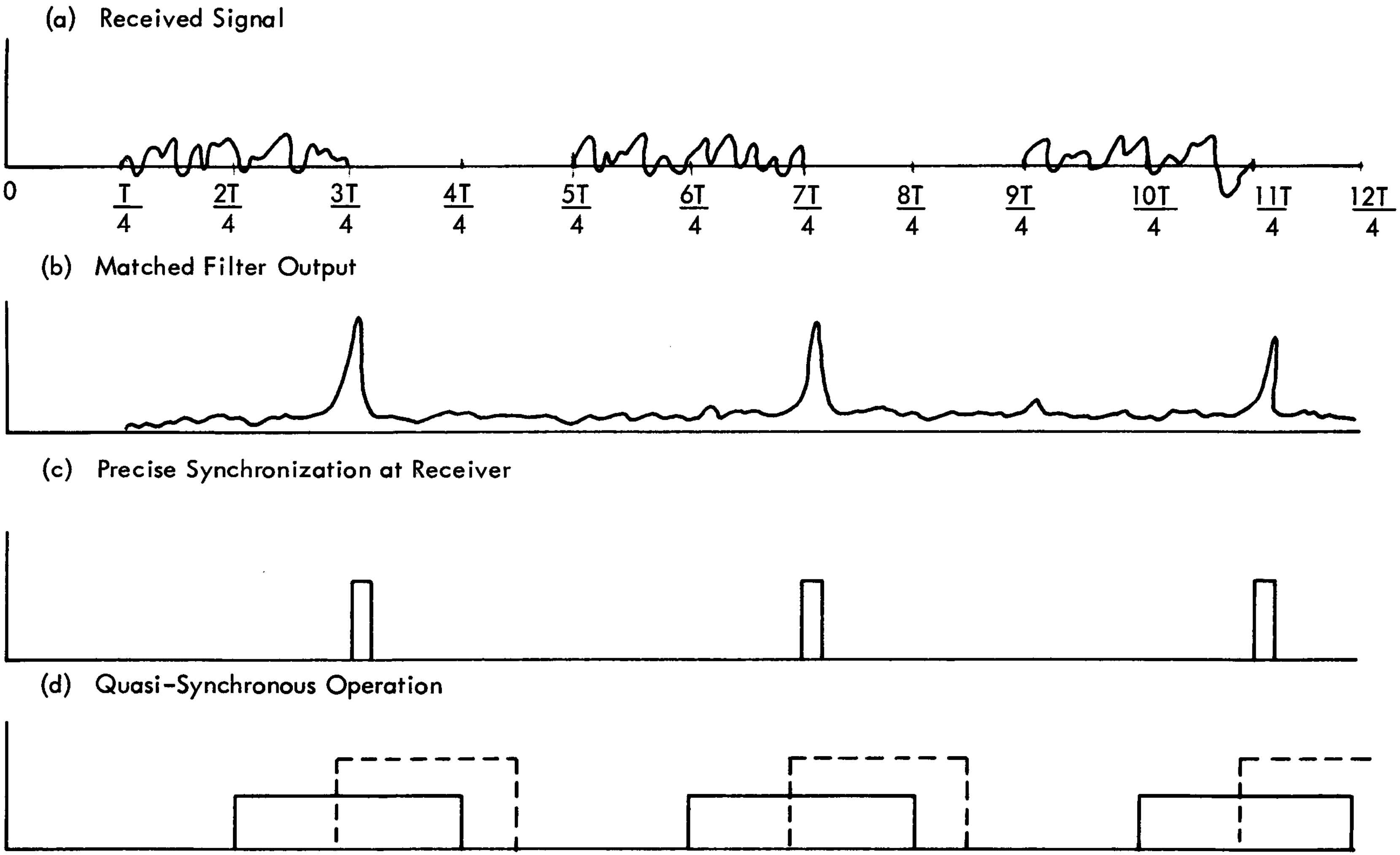


Figure 4-13. Timing Diagram Showing Synchronous and Quasi-Synchronous Operation

Figure 4-13 is an exaggerated timing diagram showing quasi-synchronous operation. The PN signals are purposely transmitted with gaps to permit quasi-synchronous operation. In random access systems this mode of operation is very desirable. The matched filter output is shown in Figure 4-13(b). Figure 4-13(c) shows the meaning of precise synchronization while Figure 4-13(d) shows quasi-synchronous reception. In the case of precise synchronization the local reference is in time coincidence with the received matched filter output peak. In quasi-synchronous operation, a long gate is opened in the expected region of the peak. This permits the receiver clock to be out of phase with respect to the transmitter clock. Time synchronization can easily be established with PN signal alphabets and matched filters because of the high delay resolution inherent in these techniques.

Asynchronous operation is possible if the information modulation used does not require synchronization. Delta modulation is an example of such binary modulation. In this case only 1's are transmitted in turn requiring a single matched filter in Figure 4-12. The decision device here is a fixed threshold at the output of the envelope detector. The output of the threshold device is fed into a delta demodulator (or integrator) where the analog message is recovered. In this application the limiter maintains a constant number of false threshold crossings independent of the noise and interference. Of course, strong interference reduces the desired peak, causing an increased number of errors. Similarly, thermal noise also causes false decisions.

The FSK pseudo-noise alphabet generator in Figure 4-7 has essentially the same matched filter as shown in Figure 4-12. In this case we simply let

$$\begin{aligned} \phi_1(t) &= \frac{\Delta\omega t}{2} + \phi(t) \\ \phi_2(t) &= \frac{-\Delta\omega t}{2} + \phi(t) \end{aligned} \tag{4-20}$$

All previous comments on matched filter reception apply equally as well here.

Higher Order PN Signal Alphabet Matched Filter Receivers. The reception of higher-order alphabets is the logical extension of the receiver shown in Figure 4-12 for the binary case. Instead of two matched filters for an M-order alphabet there are M-matched filters each of which feeds an envelope detector. At a given instant of time all of the envelope detectors are sampled; the channel containing the largest value is the one which contains the message.

As previously described, spread-spectrum matched filter communication techniques have the delay resolution property which can be used as a higher-order alphabet. The matched filter alphabet generator is shown in Figure 4-9 and the corresponding receiver in Figure 4-14. We will show here a receiver which uses a predetermined threshold as a decision criterion rather than one which makes a decision based on the largest value.

In Figure 4-14 the receiver signal is fed into a bandpass limiter and then into a matched filter. The matched filter is wired so that it will respond only to one signal address. The output of the matched filter is fed into an envelope detector and then to a threshold device. Whenever a sample exceeds the threshold a pulse is generated. The PPM demodulator produces a pulse train which contains the analog message. An appropriate low-pass filter will extract the analog message. Operation is asynchronous in this mode.

If synchronous operation is desired, the output of the threshold detector can be used to start a clock. A PN signal can be transmitted at intervals to keep the clock synchronized, or it is also possible to maintain synchronism by adding a tone to the message which can be extracted from the PPM demodulator and used for synchronization.

4.2.2.2 Reception of PN Signals Through Time Varying Media

Time varying media that have slowly varying multiple paths require a medium sounding signal that can resolve the paths and measure the attenuation of each one. A PN signal followed by a

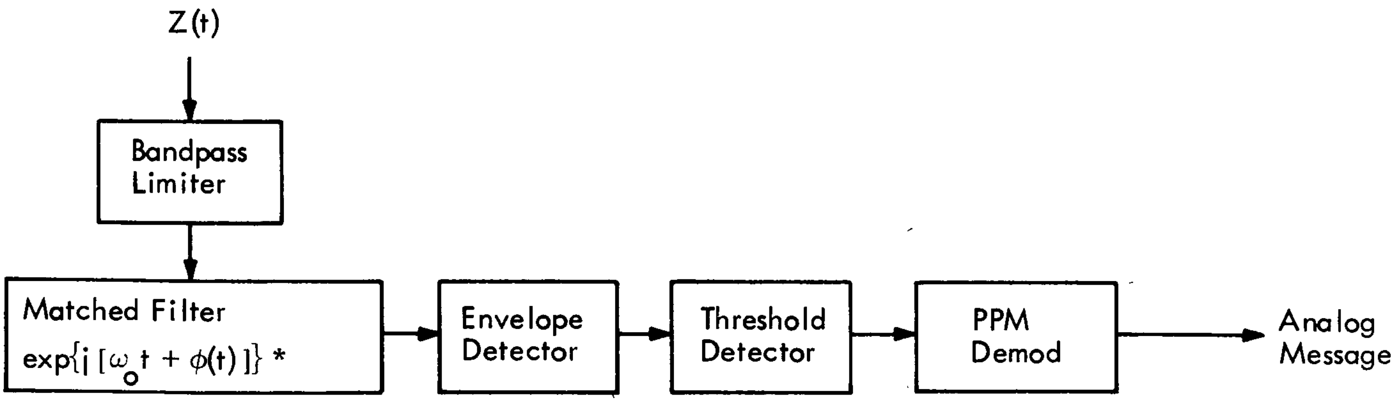


Figure 4-14. PPM Matched Filter Receiver

matched filter can measure the path structure so that appropriate multipath combining can take place at the receiver. A simple technique for achieving this is shown in Figure 4-15.

The output of the matched filter is a series of IF pulses whose envelope represents the path structure. This output is fed into a coherent integrator represented by a recirculating delay line having a delay equal to the duration of a message bit. The coherent integrator has a gain control in the feedback loop. Each time a path is recirculated it is multiplied by a factor $K < 1$; after r recirculations the attenuation is K^r. In this way a new path contributes more to the present coherent output than an old one. The coherent integration tracks the path structure as long as it changes slowly.

The integrated output is used as a "coherent" local oscillator for the present path. (Thus, an envelope detector is not required.) The outputs of the multipliers are integrated so as to add up all of the signal energy.

The signal-to-noise ratio at the output of the multipath combiners is

$$\eta^2_{MP} = \frac{2\dfrac{E}{N_o}}{\dfrac{8\,WT_o}{2\dfrac{E}{N_o}}\,\dfrac{1-K}{1+K} + \dfrac{4}{1+K}} \qquad (4\text{-}21)$$

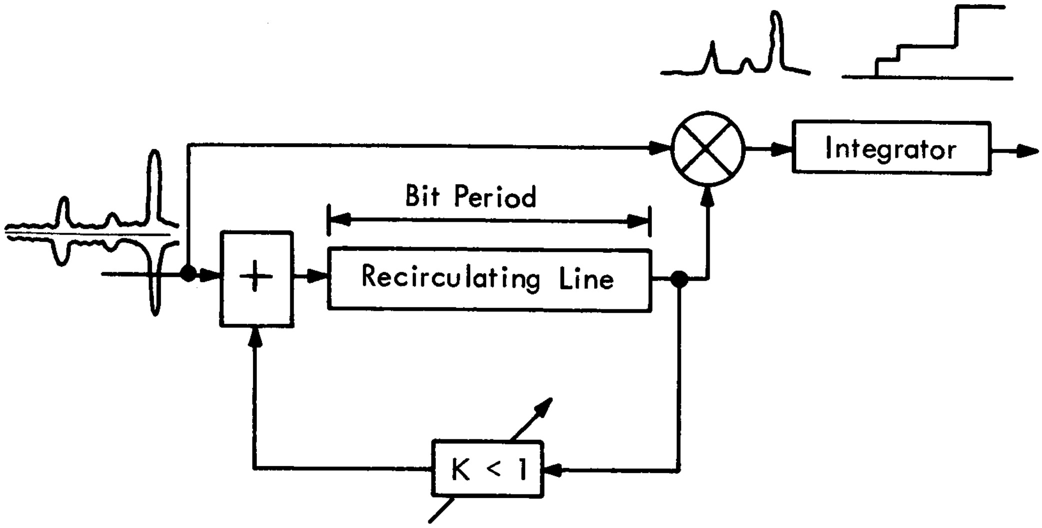


Figure 4-15. Adaptive Multipath Combiner

where,

T_o = multipath duration

N_o = total interference energy

K = gain of feedback loop ($K < 1$)

E = signal energy

4.2.2.3 Doppler Considerations in Matched Filter Systems

It should be recognized that a matched filter of the type described will respond only when the received waveform matches the impulse response quite precisely. For certain applications where a large doppler shift occurs, this can cause a loss of reception. In the case where frequency shifted noise waves are used, doppler shift can cause a signal to be accepted in an undesired receiver.

Assume that each address is a distinct PN sequence without any intentional frequency shift. Doppler acquisition can be achieved by having the transmitter search frequency until the intended receiver is acquired. The acquired receiver is now in a frequency tracking mode. A return signal from the receiver stops the transmitter. The received signals at the output of the matched filter can be fed into a frequency discriminator whose input is gated by the envelope detector output. The discriminator output feeds an integrator which in turn is fed back to the local oscillator which tracks the doppler. This is shown in

Figure 4-16. As long as the signal-to-noise ratio at the input to the discriminator (i.e., at output of matched filter) is, say, 3 db, the discriminator output is gaussian having a mean value of Δf_o which is the doppler shift and a standard deviation

$$\sigma_{\Delta f} = \frac{k\,W}{\sqrt{\frac{S}{\sigma_N}\;WT}}$$

$$= k\sqrt{\frac{W}{T}\;\frac{\sigma_N}{S}} \tag{4-22}$$

where k is a constant and S is RMS signal power.

Thus, after N integrations

$$\sigma_{\Delta f} = k\sqrt{\frac{WW_o\,\sigma_N}{S}} \tag{4-23}$$

where $W_o = 1/NT$ is the post discriminator low-pass filter bandwidth. As long as the doppler does not change too rapidly, it is possible to integrate out the frequency shift in a straightforward manner.

4.2.3 Logical Block Diagrams of Matched Filters

A matched filter for receiving PN signals is a complex device generally requiring delay-lines having the bandwidth-time product of the signals. There are a number of different techniques which perform the matched filter function. The technique which is chosen for

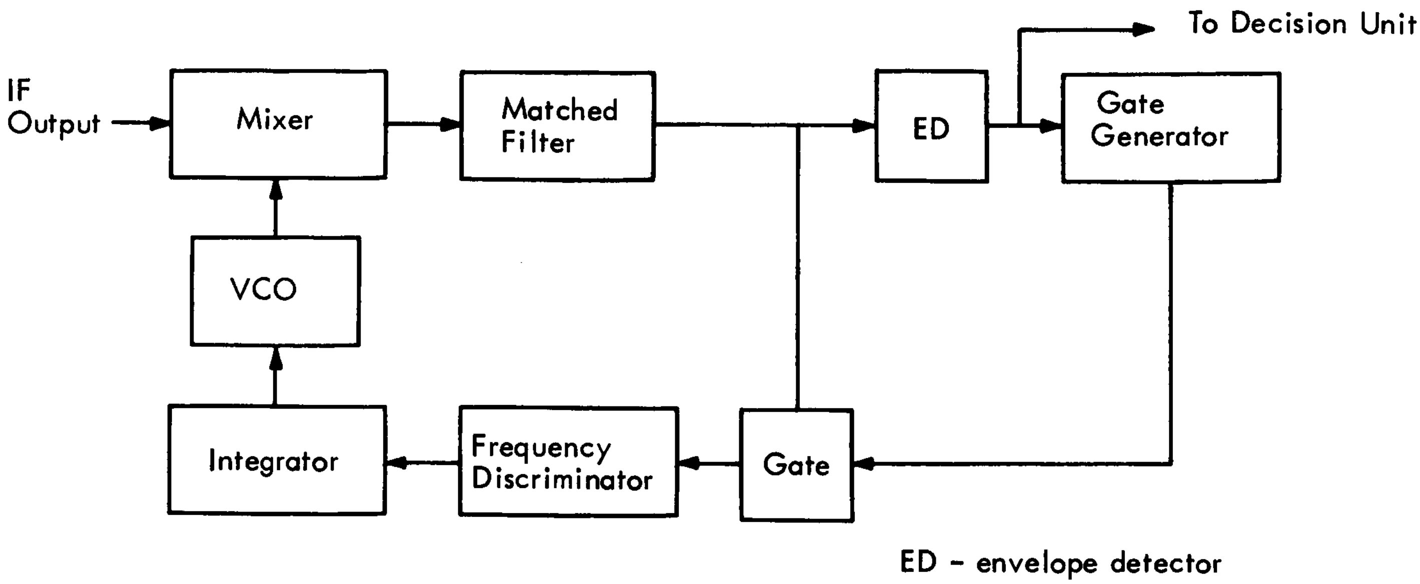


Figure 4-16. Doppler Tracking of Matched Filter Receiver

a given application is generally determined by the signal bandwidth and the signal bandwidth time product.

The delay lines which are used for a matched filter structure can be digital (i.e., shift registers), analog, or combinations. The digital delay-line has the advantage that it is inherently tapped, the input is regenerated at each tap, and precise spacing of the taps is achieved by an accurate clock. In addition, different message rates can easily be accommodated by changing the shifting clock rate. The digital line, however, requires sampling and clipping which results in some loss in signal-to-noise ratio. A substantial amount of drive power is also required. It is bandwidth limited by the present state of the art in digital circuitry. The reliability is also limited; for example, if one trigger stage breaks down, the section of the matched filter which begins with this trigger will operate improperly.

The analog delay lines are limited in length and bandwidth as well as in the number of taps. However, the bandwidth time product of some lines is extremely large. Distortion is present and reflections at the transducers create added noise. Generally, these structures are characterized by a high insertion loss and hence substantial drive power is required. The reliability is good. Destruction of the transducer is required to prevent the line from operating.

In this section several correlation filter structures are discussed in detail.

4.2.3.1 Equivalent Bandpass Structures

Throughout the previous discussion it has been assumed that the filter used for passive signal generators and for receiver matched filters are actually bandpass structures. In practice, it is difficult to accurately realize a bandpass matched filter for signals of large dimensionality. In the Figure 4-17 a bandpass signal and its low-pass (baseband, or video) components are shown. As is evident from the figure, a slight displacement of the samples in the bandpass case will seriously affect any correlation or matching being performed on the samples. The spacing of the samples (corresponding to physical placement of taps on a delay line) is critical; and if a local oscillator is used to heterodyne the actual received signal down to an IF for matching, the phase of the local oscillator is also important.

A practical method of receiving bandpass signals with a large number of degrees of freedom is to use an equivalent bandpass structure which utilizes only the low-pass quadrature components and quadrature mixing techniques followed by an envelope detector. The advantages are that slower sampling is required, and the phase of the local oscillator does not affect the correlation.

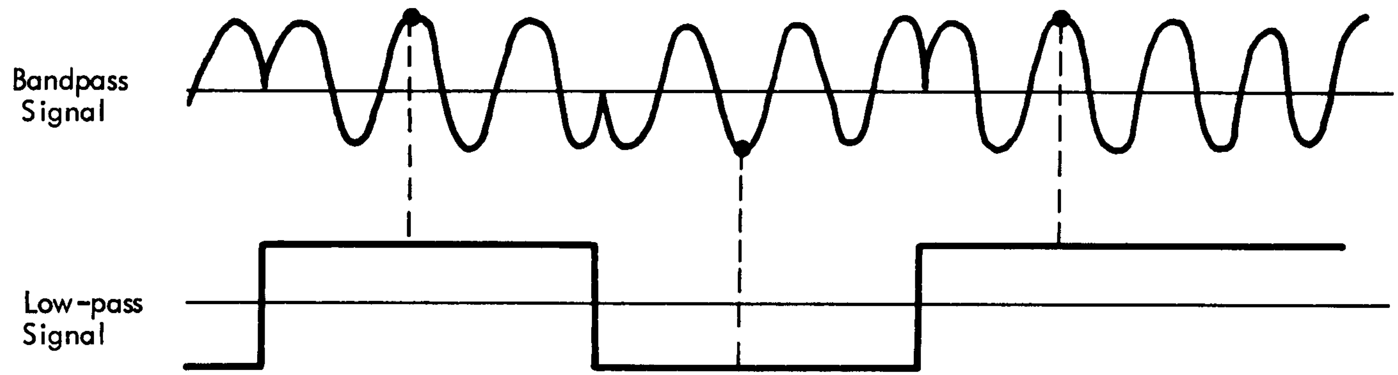
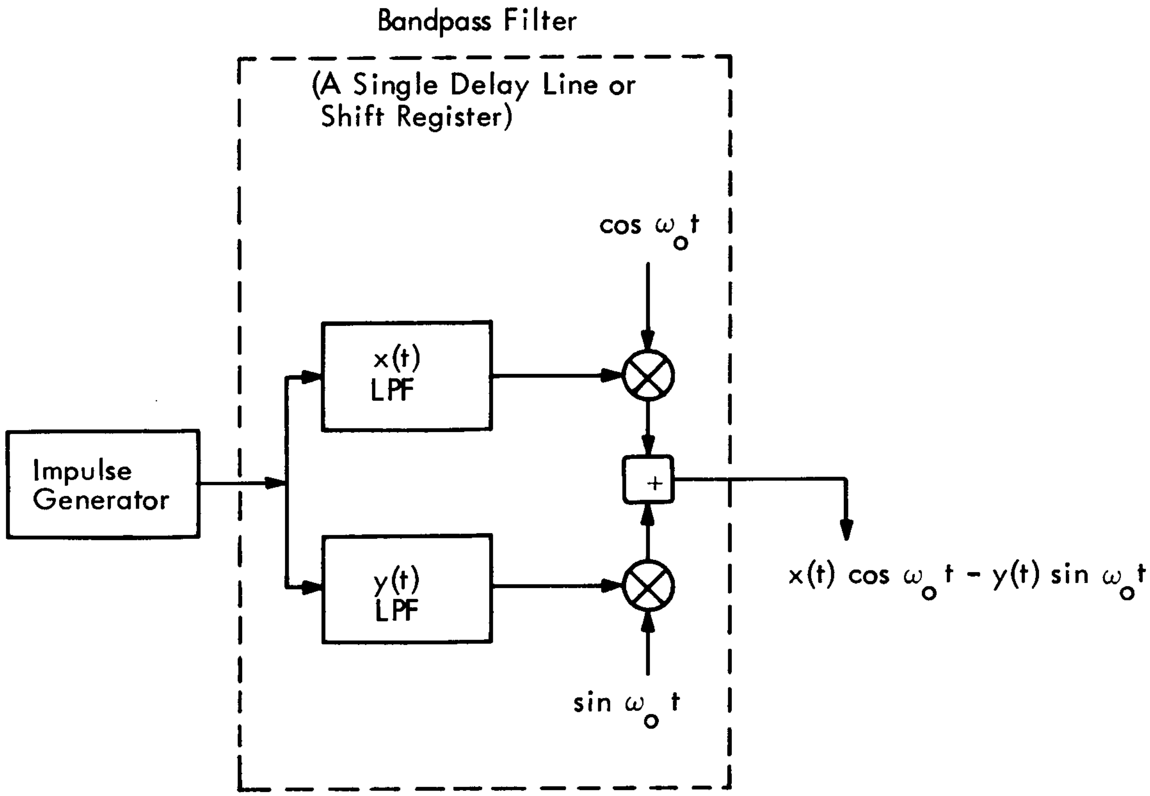


Figure 4-17. Example of Bandpass and Low-pass Signals

Waveform Generation. The bandpass equivalent for generating a desired waveform is shown in Figure 4-18. The impulse responses, x(t) and y(t), of the low-pass filter, are the quadrature components of the desired bandpass signals. These two impulse responses are mixed with quadrature carrier frequency signals (i.e., $\cos(\omega_o t + \theta)$, $\sin(\omega_o t + \theta)$ where θ is an arbitrary phase angle) and added to give the desired bandpass signal. Every impulse which excites the equivalent bandpass structure yields a corresponding bandpass output signal. A single delay line or shift register with appropriate weighting networks can be used for both quadrature components of several low-pass filters.

Equivalent Bandpass Receiver Filter. In Figure 4-19, the receiver filter for the above signal is shown. Each low-pass component is the time inverse of the corresponding transmitted low-pass component. Because the detection is incoherent (i.e., RF phase is unknown) four low-pass filters are required instead of two as in the waveform generator. After the signals are combined as shown, the resulting output signal is equivalent to bandpass matching to the received signal followed by envelope detection.

If the class of signals transmitted is restricted to only phase-reversal signals, then one of the quadrature components can be made



1. LPF - Low-Pass Filter
2. $\otimes$ - Multiplier

Figure 4-18. Equivalent Bandpass Filter (Waveform Generator)

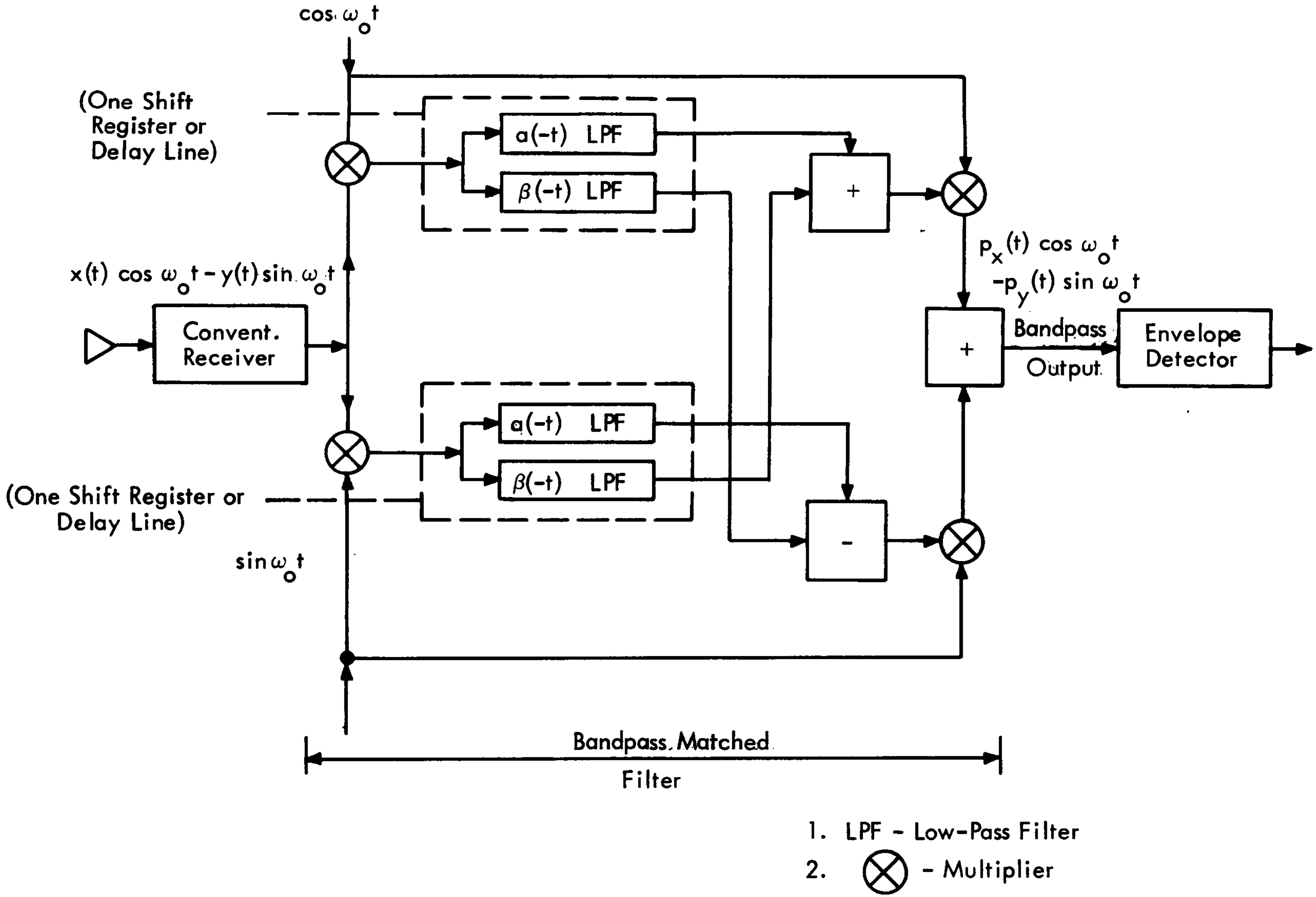


Figure 4-19. Equivalent Circuit of Bcndpass Correlation Receiver

zero. In this case, the signals can be represented by a single low-pass component, and only one low-pass filter is required for the waveform generator, and only two low-pass filters are required at the receiver.

The same number of delay lines or shift registers is required as before.

4.2.3.2 Analog Matched Filters

Delay lines of various kinds can be used to perform the correlation required for matched filters. Three types of systems are considered here: (a) tapped magnetostrictive delay lines, (b) a recirculating delay line, and (c) an optical correlator.

Tapped Delay Line Matched Filter System. Delay lines with multiple taps can be used as correlators for signals with large time-bandwidth products. It is difficult to obtain multiple taps on glass sonic delay lines, but magnetostrictive sonic delay lines and lumped-constant electrical delay lines are suitable for this purpose.

In the diagram of Figure 4-20 a single tapped delay line is connected to two banks of resistors. Each bank has one bus for taps weighted positively (+) and one for taps weighted negatively. Every impulse, as it propagates down the line, will cause positive or negative contributions to the difference amplifiers, the contribution at each tap being inversely proportional to the value of the resistor at that tap.

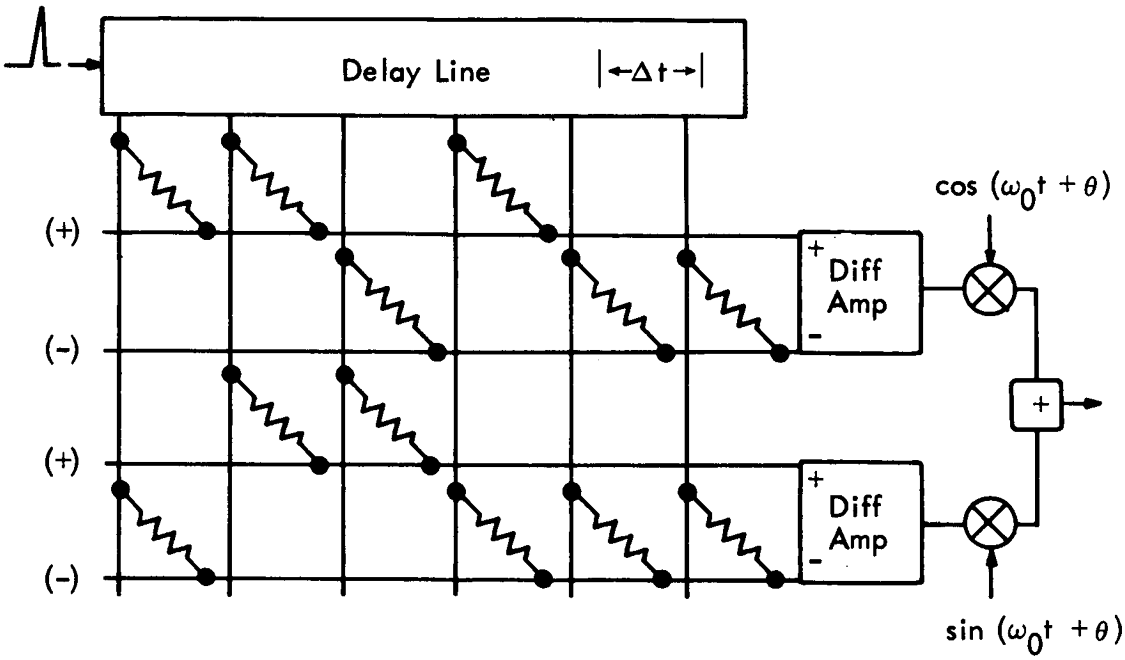


Figure 4-20. Tapped Delay Line Waveform Generator

Thus, x(t) and y(t) are generated, and after mixing with $\cos(\omega_o t + \theta)$ and $\sin(\omega_o t + \theta)$ the components are summed to give the desired bandpass impulse response. A bandpass receiver is shown in Figure 4-21. The received bandpass signal is mixed with the carrier frequency quadrature signals ($\cos \omega_o t$ and $\sin \omega_o t$) and the products are low-pass filtered. This produces two quadrature low-pass components, x(t) and y(t), which are the inputs to the tapped delay line. The resistor weighting networks are wired in reverse order from the corresponding network at the transmitter, so that the structure is "matched" to the signal produced at the transmitter. The difference amplifiers combine the positive and negative contribution from the delay line, which are combined and squared to produce the equivalent of a bandpass matched filter followed by an envelope detector.

In both the waveform generator and receiver, electronic gating circuits can be used to connect or disconnect any number of resistor weighting networks, thus making possible automatic address selection and address switching. For manual changing of addresses, a resistor weighting network could be mounted on a single pluggable unit.

<u>Recirculating Delay Line Correlation.</u> In a circulating delay line, the input is sampled and fed to the delay line; each new sample wipes out the oldest sample, and between samples the entire stored

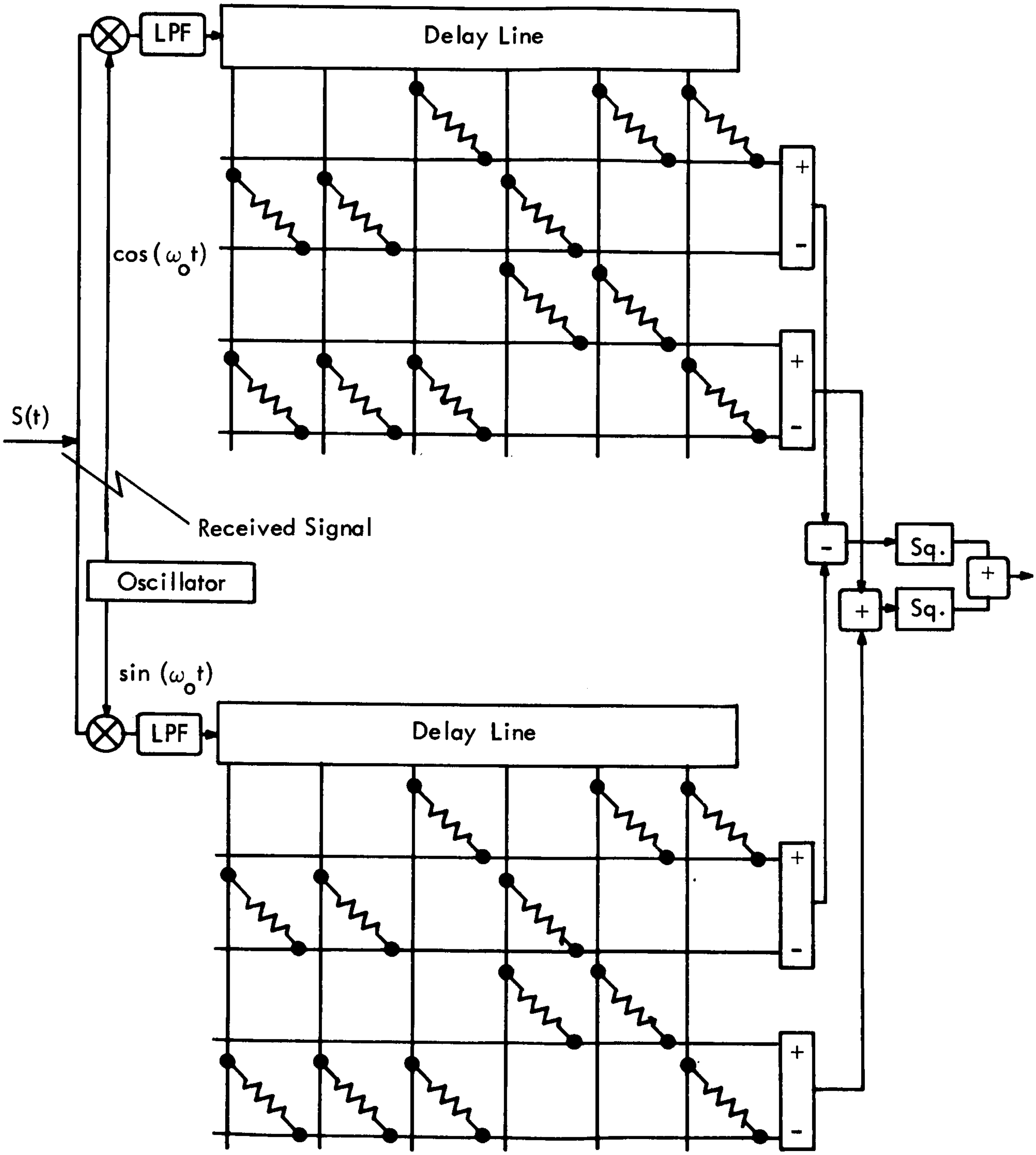


Figure 4-21. Tapped Delay Line Bandpass Matched Filter

contents of the delay line are available for processing. The basic device is called the DELTIC. The DELTIC is a polarity coincidence correlator, that is, it only senses the polarity of incoming signals. Instead of resistor weighting networks, DELTIC compared the last N samples of the input wave with a reference set of samples which is stored in another recirculating delay line. The input wave can be correlated with many different signals, by connecting to the DELTIC a line with a stationary stored reference for each signal of interest.

In the circuit of Figure 4-22, the upper DELTIC stores a "window" of a sample of the input; the lower DELTIC stores the reference samples. The output of the comparator is integrated over (n + 1) samples and then dumped. The output of the integrator is a sample of the polarity coincidence correlation every (n + 1) seconds.

Optical Matched Filters. Optical correlation based on the photoelastic effect in quartz offers a potentially simple technique for generating and detecting coded waveforms.

An optical correlator can be constructed using a quartz delay line and conventional optics. One configuration for such a correlator is shown in Figure 4-23. Here, the point light source is first converted to a plane wavefront covering the length of the device and then sent through a polarizer, quarter-wave plate, quartz delay line,

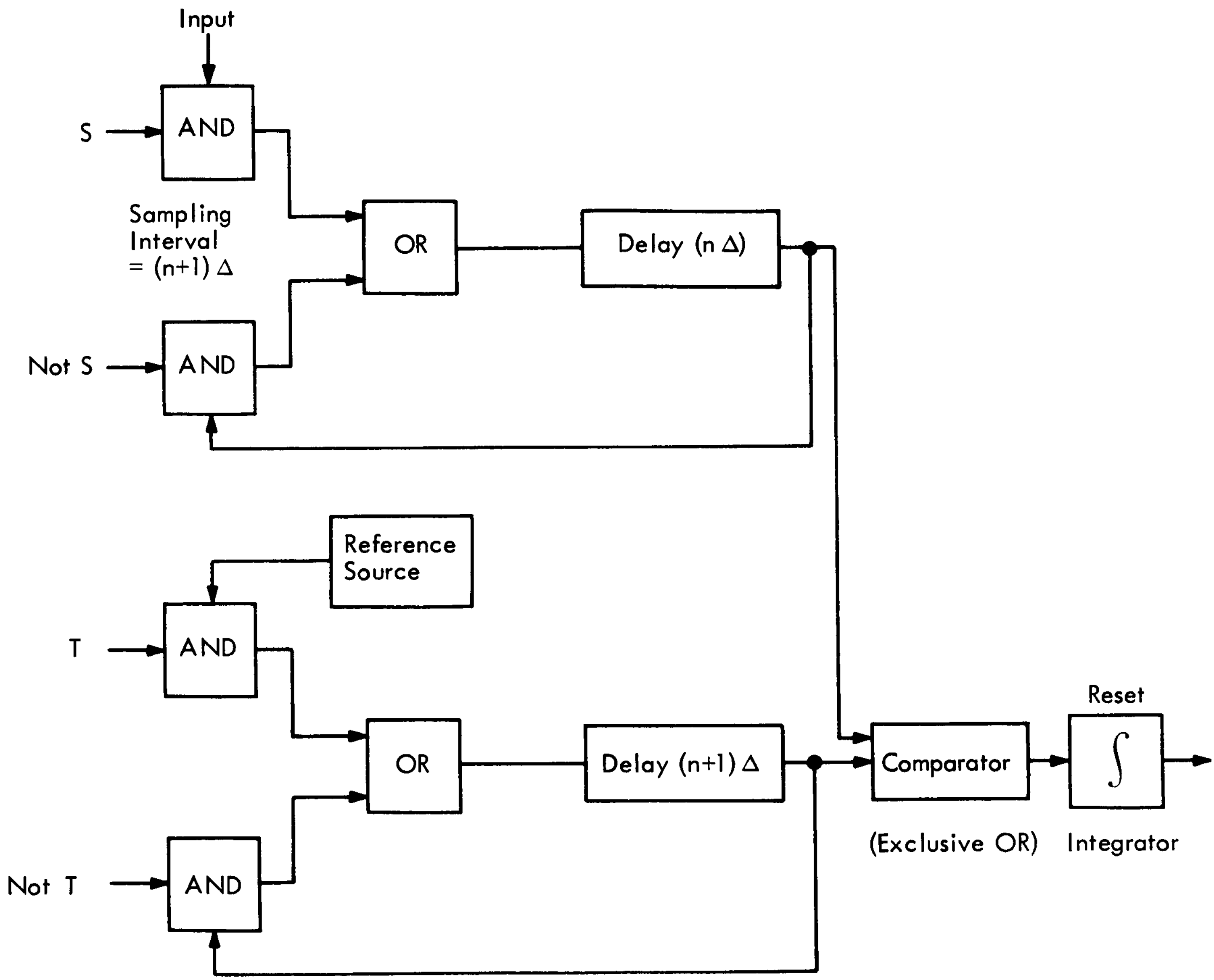


Figure 4-22. DELTIC Correlator Receiver

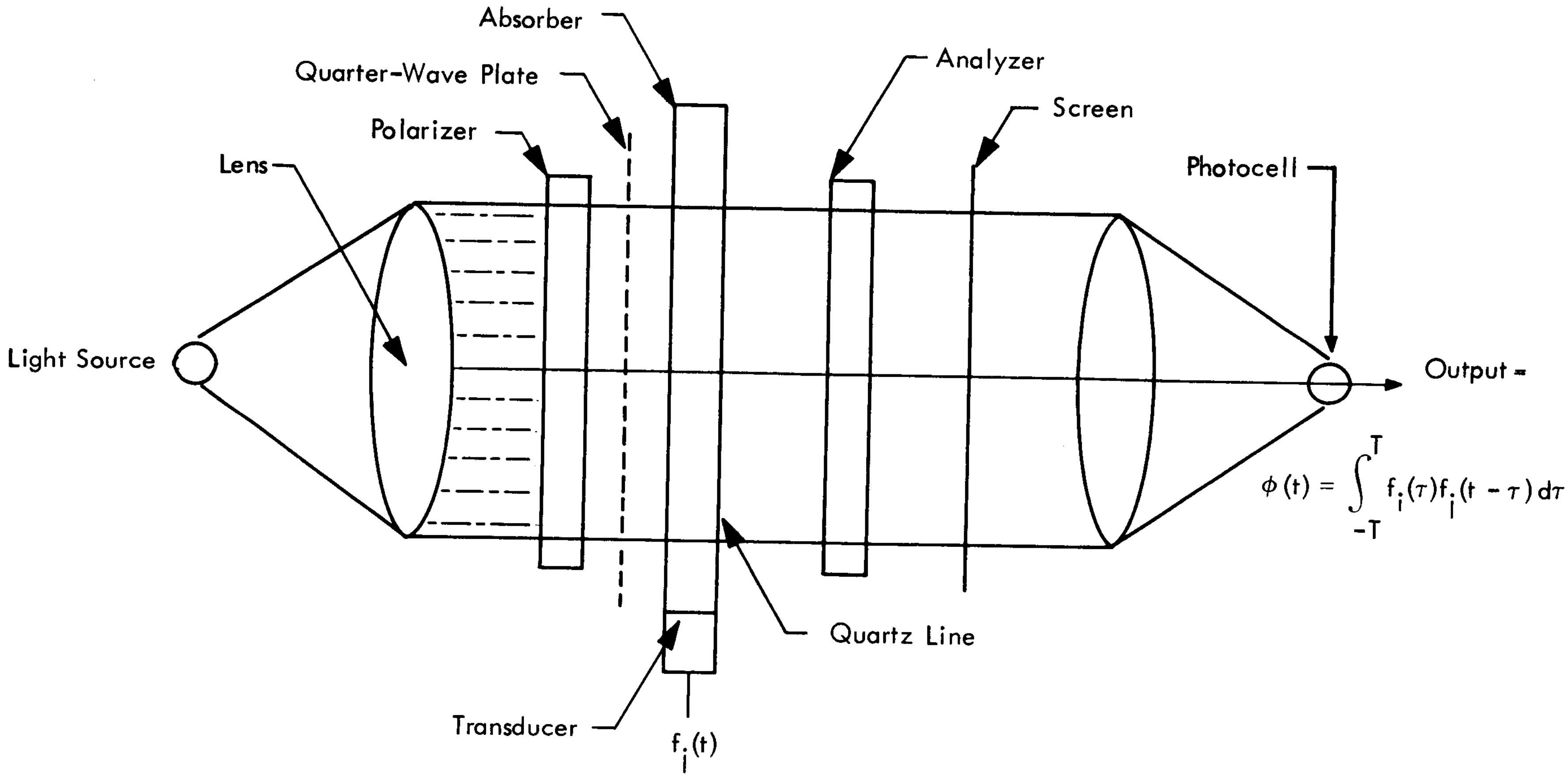


Figure 4-23. Optical Correlator

analyzer, screen, and finally a collection lens which refocuses the light onto a detector. The screen is composed of transparent and opaque increments which give a replica of the desired waveform, $f_i(t)$. The birefringence pattern, $f_j(t - \tau)$, set up by an incoming waveform in the quartz delay line at the time τ, is then multiplied point by point with the screen pattern since the extended light beam traverses both. The collection lens acts as an integrator giving an output proportional to the crosscorrelation function between the input signal and that stored on the screen. The device then acts as an ideal matched filter, and it has the important property of being as easily adapted to complex wave trains as to simple ones. Increasing the signal WT product in this system involves the addition of lines to a screen, whereas in an electronic system increasing WT requires additional circuitry. In the optical system, the entire characteristic of the filter response can be changed simply by substituting a new screen into the system.

In Figure 4-24 a simple block diagram illustrates the use of an optical correlator in a communications link. Note that the correlation is done at bandpass, in a continuous delay line, so that quadrature detection is unnecessary.

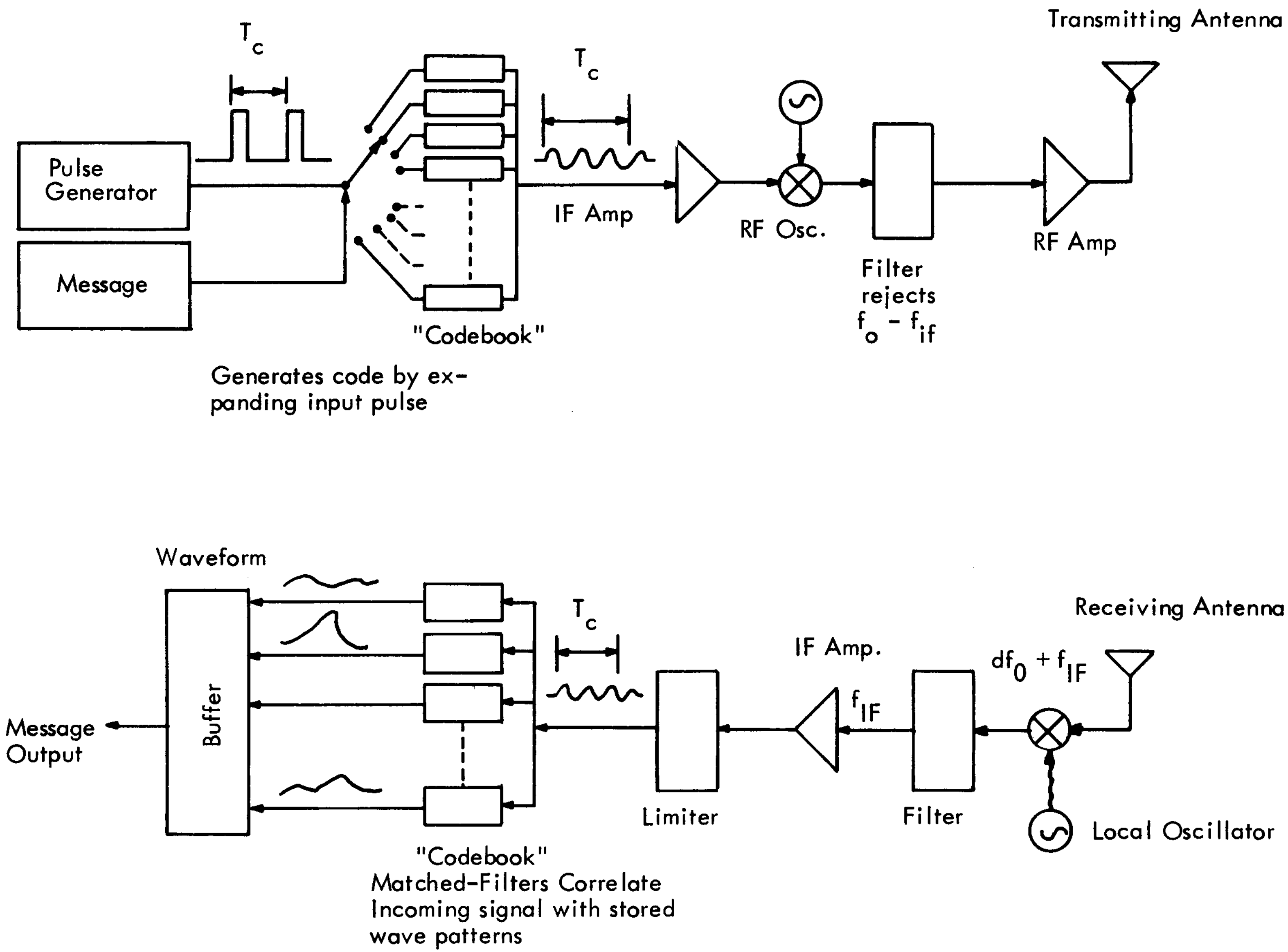


Figure 4-24. Asynchronous Communication System

4.2.3.3 Digital Matched Filter Structure

A digital matched filter can be constructed using a shift register as a time discrete delay line. The input signal must be bandpass limited and sampled to give a binary input to the shift register. It has been demonstrated that pseudo-noise signal alphabets having large time-bandwidth products can undergo severe limiting, such as infinite clipping, with little loss in signal-to-noise ratio. The binary signal resulting from the clipping then constitutes the input to the shift registers. Resistor weighting networks are fed in parallel by the shift register, as in the analog delay line. The greatest advantage of a digital shift register is that its shifting rate can be varied to change the transmission rate, and its length can be very long since the information samples are regenerated at each stage as they shift through the shift register. The timing of successive "taps" can be precise compared to the taps on a delay line, due to digital clocking techniques.

Figure 4-25 shows a digital bandpass filter matched to the set of frequency shifted waveforms shown in Equation (4-16). There are two shift registers, one for each quadrature component. Since the waveforms used are frequency shifted, two filters per quadrature component per signal address are required.

In Figure 4-25 the a-bus sums terms which have positive

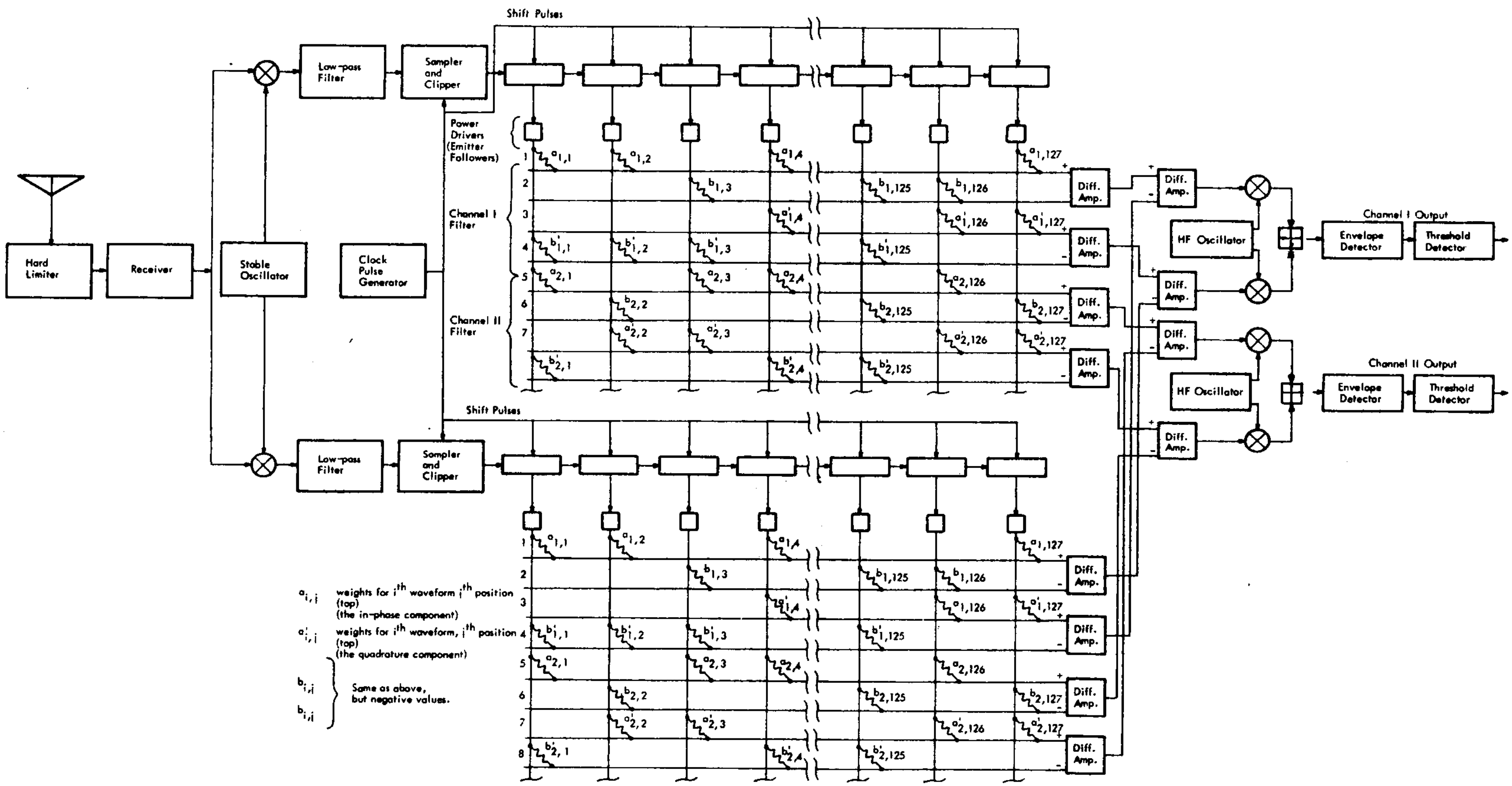


Figure 4-25. Random Access Matched Filter Receiver

polarity and the b-bus sums those that are negative. The polarity is obtained in a difference amplifier. In order to obtain the bandpass output, the buses are combined according to Figure 4-25. The oscillator at the output of the filter injects a cosine wave and sine wave at IF bringing the multiplier output down to video. The oscillator at the output of the filter, which may be the same as the input oscillator, injects a cosine and a sine signal into the respective buses and translates the signal back up to IF where a conventional envelope detector is used. This mixing operation can be eliminated by feeding each output quadrature component into a square-law device and adding.

4.2.4 Computer Simulation Results

A number of important measurements have been made by means of the computer simulator. Printouts of the output signals of the matched filter--with and without hard limiting--were obtained for a mixture consisting of 2, 5, 10 and 15 FSK, PN signal addresses for random time shifts. The ambiguity function was measured. From these signals, measurements were made of; (1) the clutter power, (2) the maximum sidelobe, (3) the peak signal-power-to-mean-clutter power power ratio, (4) the peak signal power to peak clutter power ratio and (5) the probability density of the clutter. Simulations thus far performed have been limited to the equal power talker case.

4.2.4.1 Ambiguity Function

Figures 4-26 and 4-27 show sections through the ambiguity function parallel to the τ-axis. Figure 4-26 is the ambiguity function for a single RF pulse (aperiodic case) phase reversal modulated by a 127-bit m-sequence. Figure 4-27 is for the periodic case, i.e., where the modulated RF pulses are contiguous. In both cases, the scale has been expanded to amplify the sidelobes.

4.2.4.2 Matched Filter Receiver Outputs

A mixture of FSK, PN signal addresses having equal power with random delays was generated and fed into the matched filter. In one set of measurements the input to the matched filter was preceded by a hard limiter while in the other set it was not. Runs were made for 2, 5, 10 and 15 equal power talkers. Figures 4-28 through 4-30 represent the signal outputs of a square-law detector for 5, 10 and 15 talkers. For hard limiting, note that the sidelobes are bounded and for all practical purposes independent of the number of interfering signals.

4.2.5. Clutter Power Measurements

Figure 4-31 shows the normalized mean-square clutter as a function of the number of equal power talkers. These measurements were obtained from 20 such realizations. The 20 runs contain an

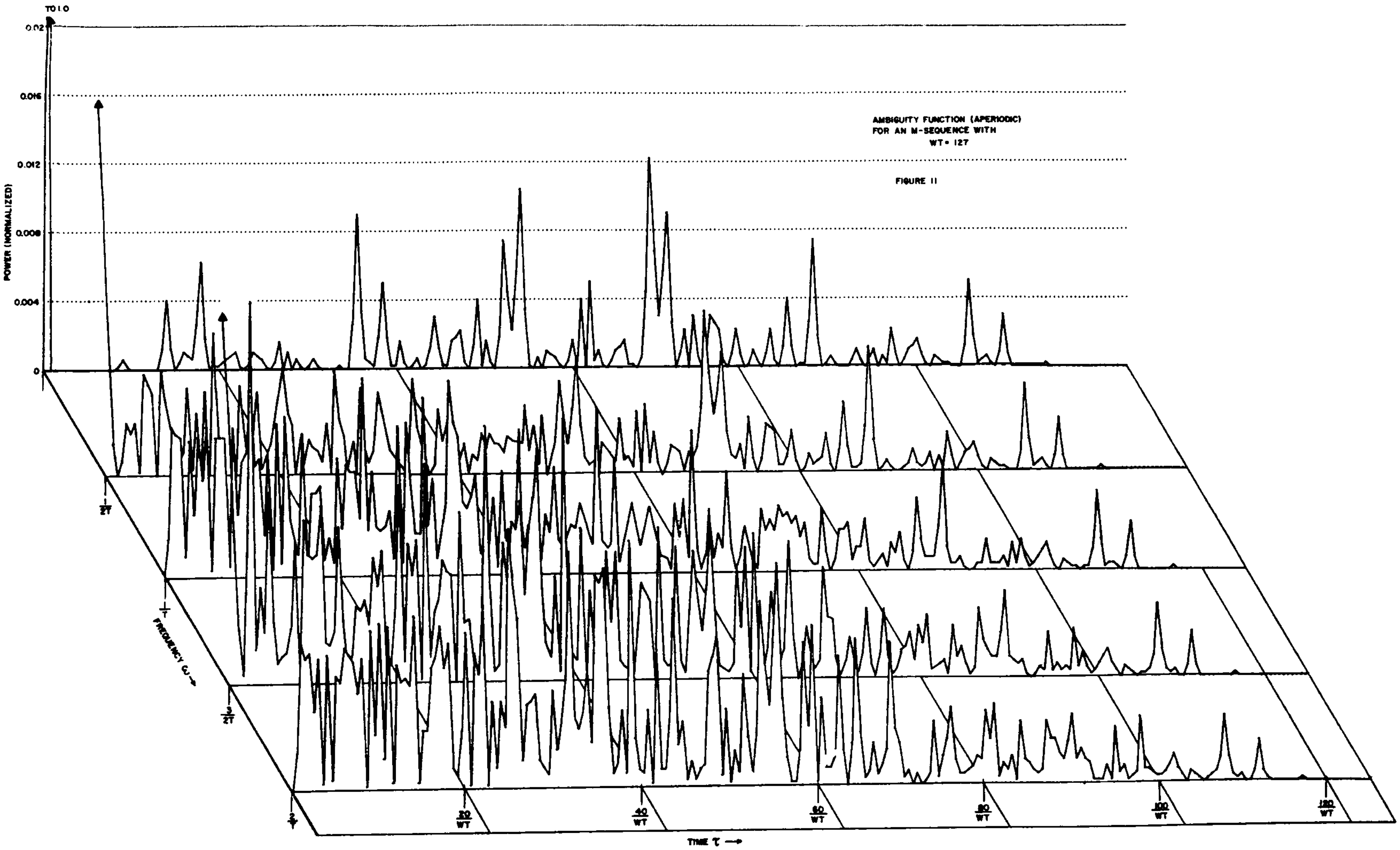


Figure 4-26. Ambiguity Function (Aperiodic) for an M-Sequence with WT = 127

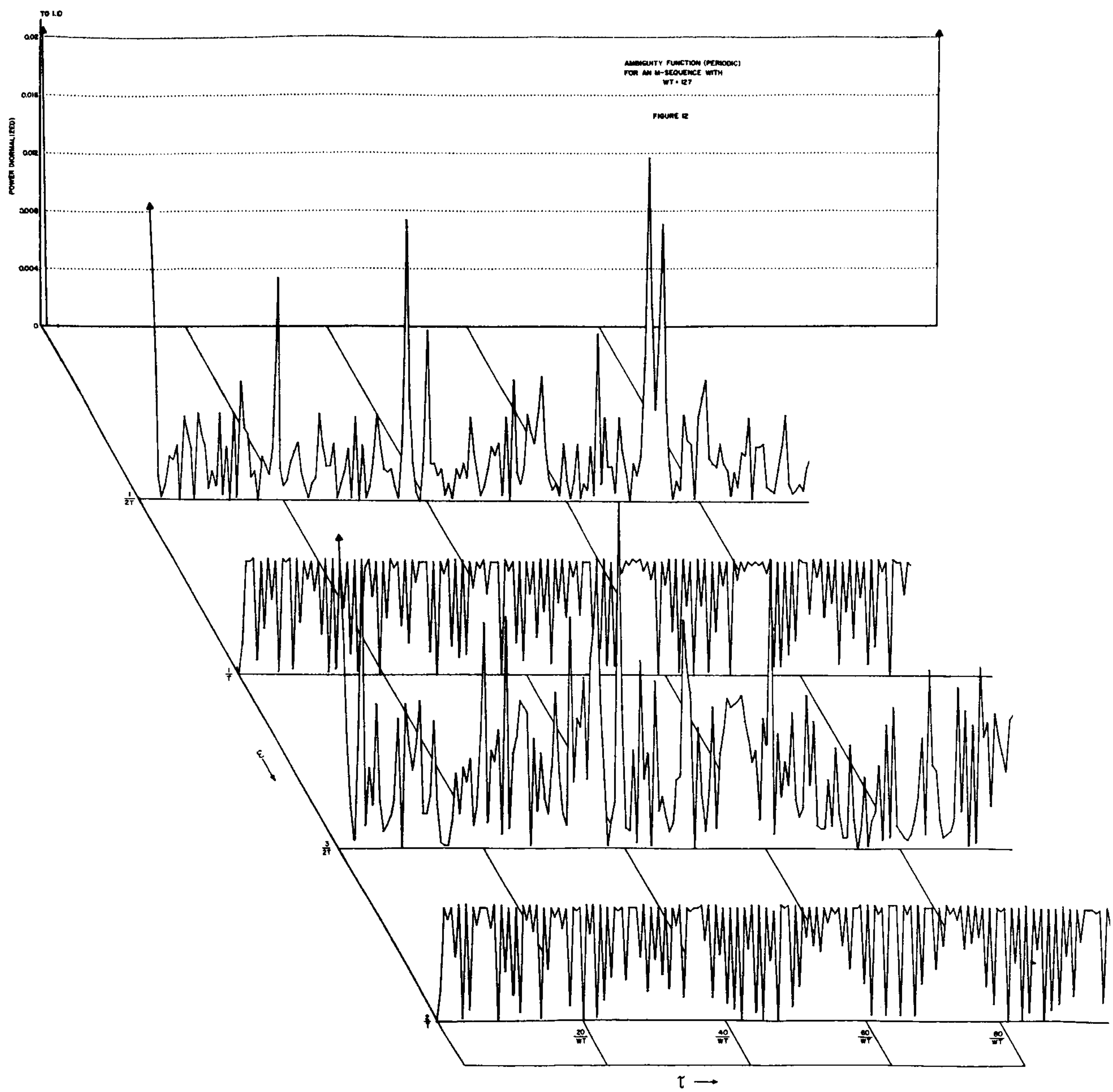


Figure 4-27. Ambiguity Function (Periodic) for an M-Sequence with WT = 127

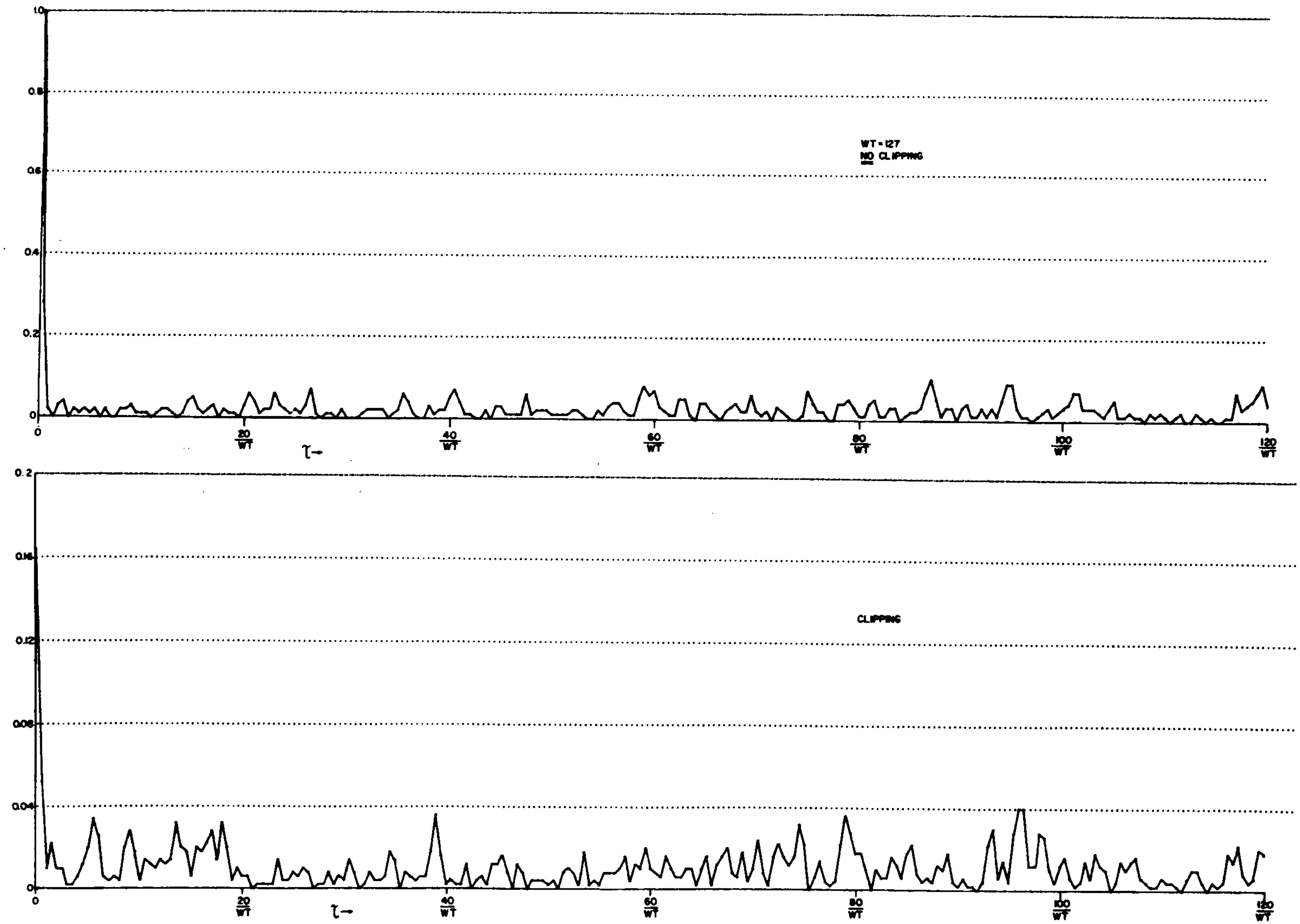


Figure 4-28. Typical Matched Filter Output Signal for 4 Clutter Sources (Periodic Case)

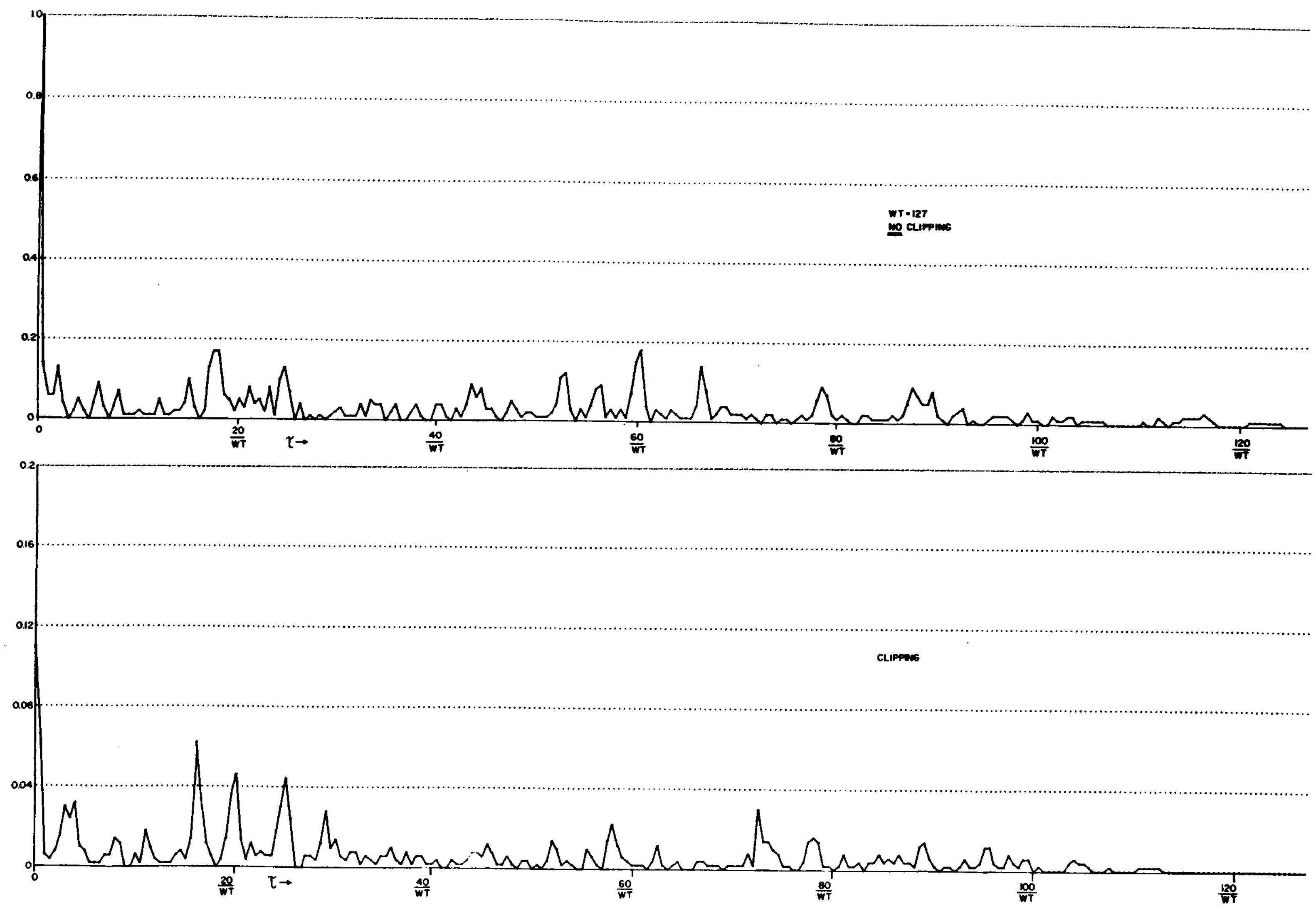


Figure 4-29. Typical Matched Filter Output Signal for 9 Clutter Sources (Periodic Case)

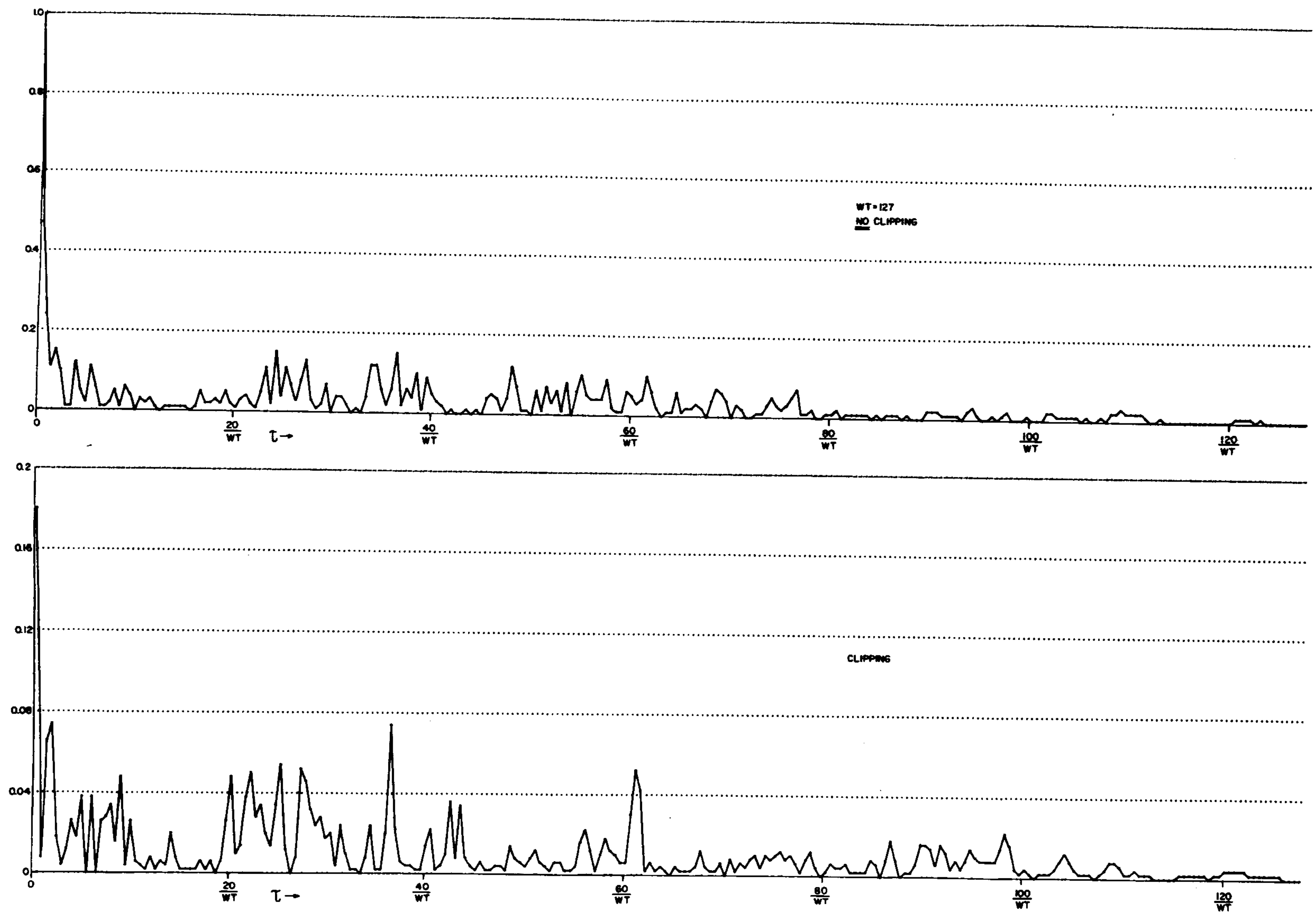


Figure 4-30. Typical Matched Filter Output Signal for 14 Clutter Sources (Periodic Case)

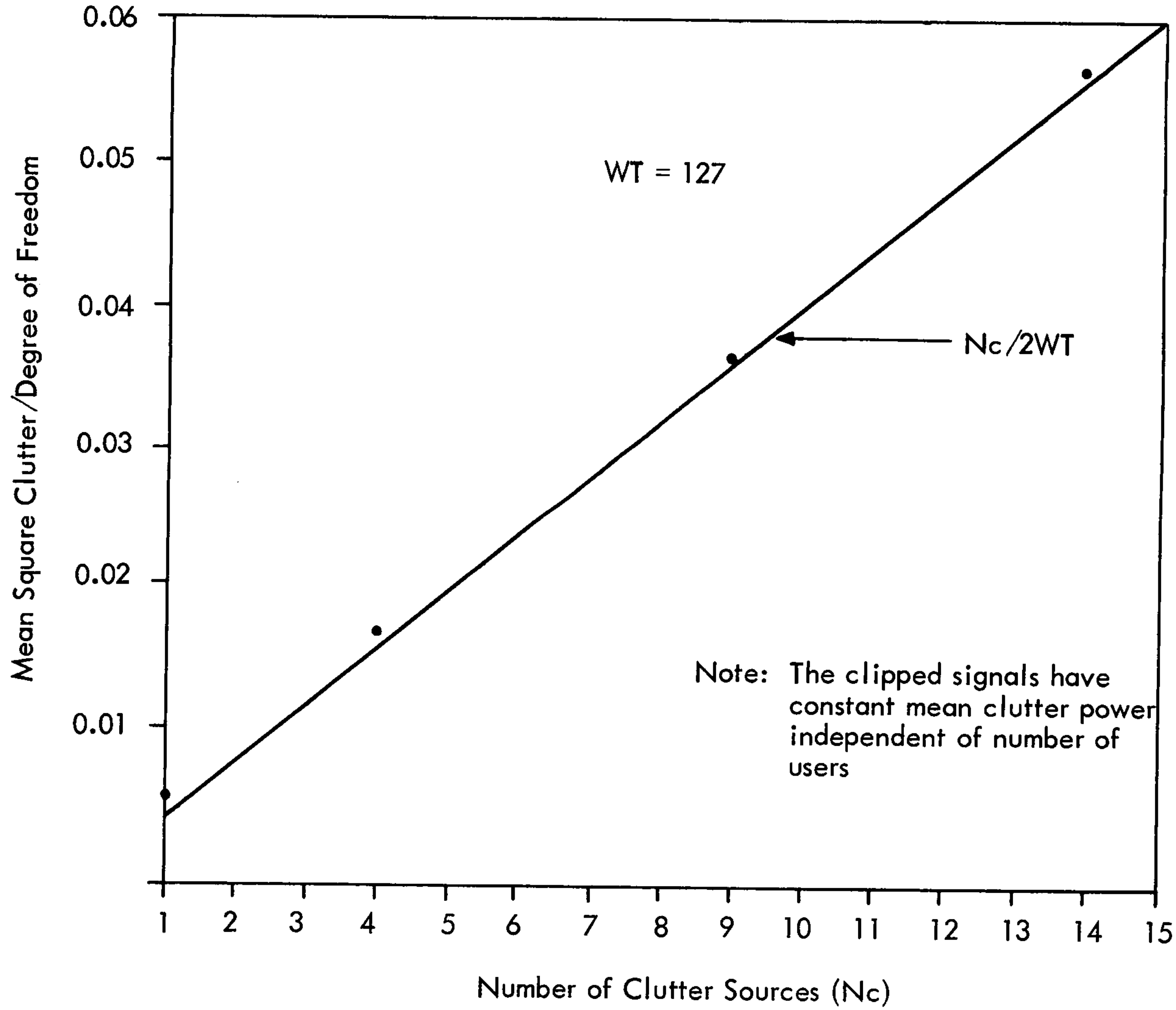


Figure 4-31. Mean Square Clutter Power vs Number of Active Users

extremely large number of samples for measuring the mean-square clutter while the average of the maxima is obtained from 20 samples.

The mean-square clutter in the presence of hard limiting is a constant independent of the number of signals, as expected.

Figure 4-32 represents curves of the peak signal power to mean-square clutter and peak signal power to maximum sidelobe power as a function of the number of equal power talkers with and without hard limiting. The data was averaged over an ensemble consisting of a mixture of periodic and aperiodic signals. Hard limiting causes a loss in signal-to-noise ratio of 1.4 to 2.0 db as predicted from theoretical considerations. The greater the number of signals in the mixture the smaller the relative loss in signal-to-noise ratio. The latter results are also expected from theory.

Figures 4-33 and 4-34 show a comparison of the measured probability density of clutter with a Rayleigh density. The measured mean-square power was substituted into the Rayleigh density and the empirical and theoretical curves were plotted in the same paper. It is seen that with and without hard limiting the clutter density is Rayleigh. This indicates that the predetection clutter is a Gaussian process.

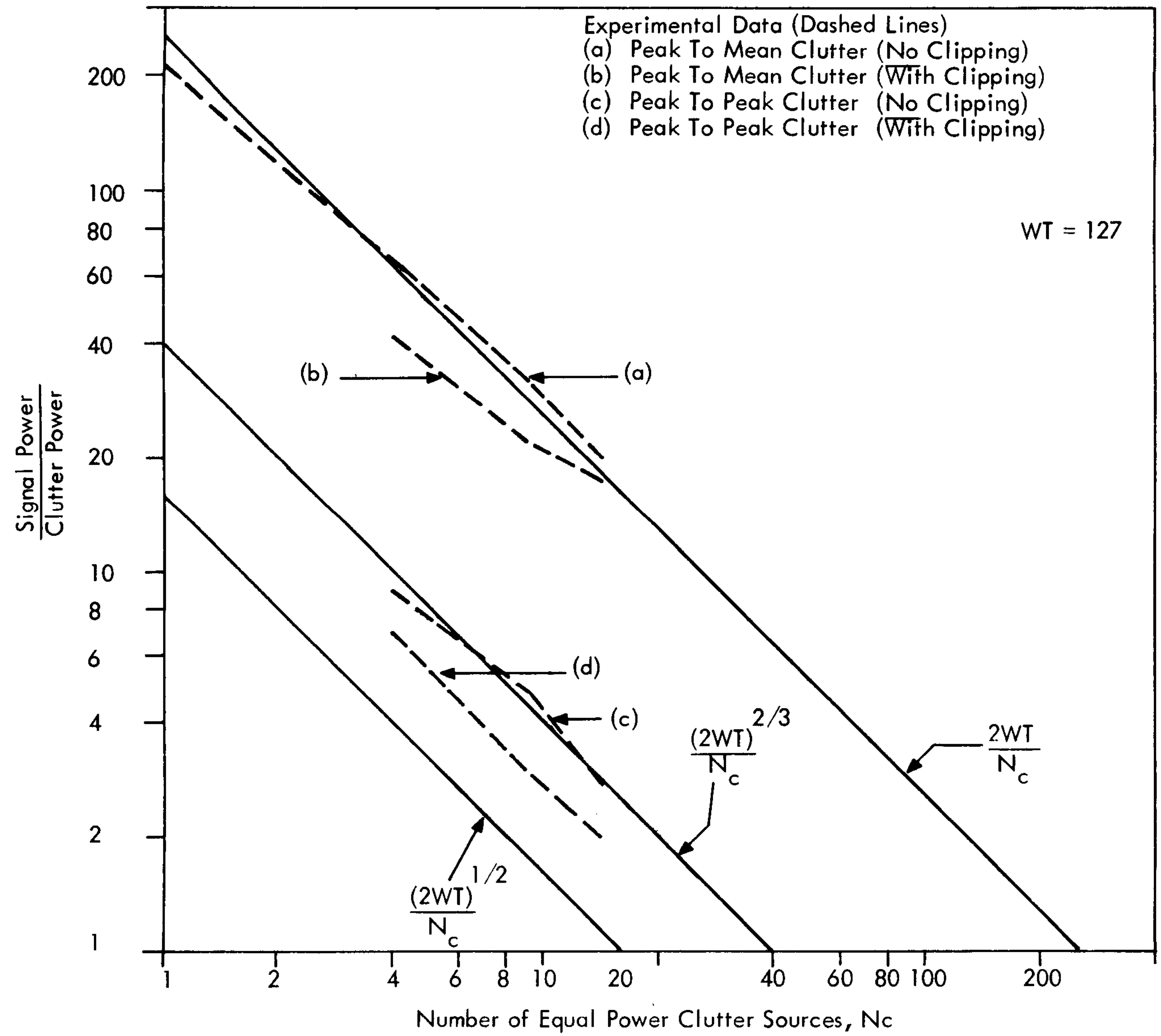


Figure 4-32. Ratio of Peak Signal Power to Peak and Mean Clutter Power vs the Number of Clutter Sources

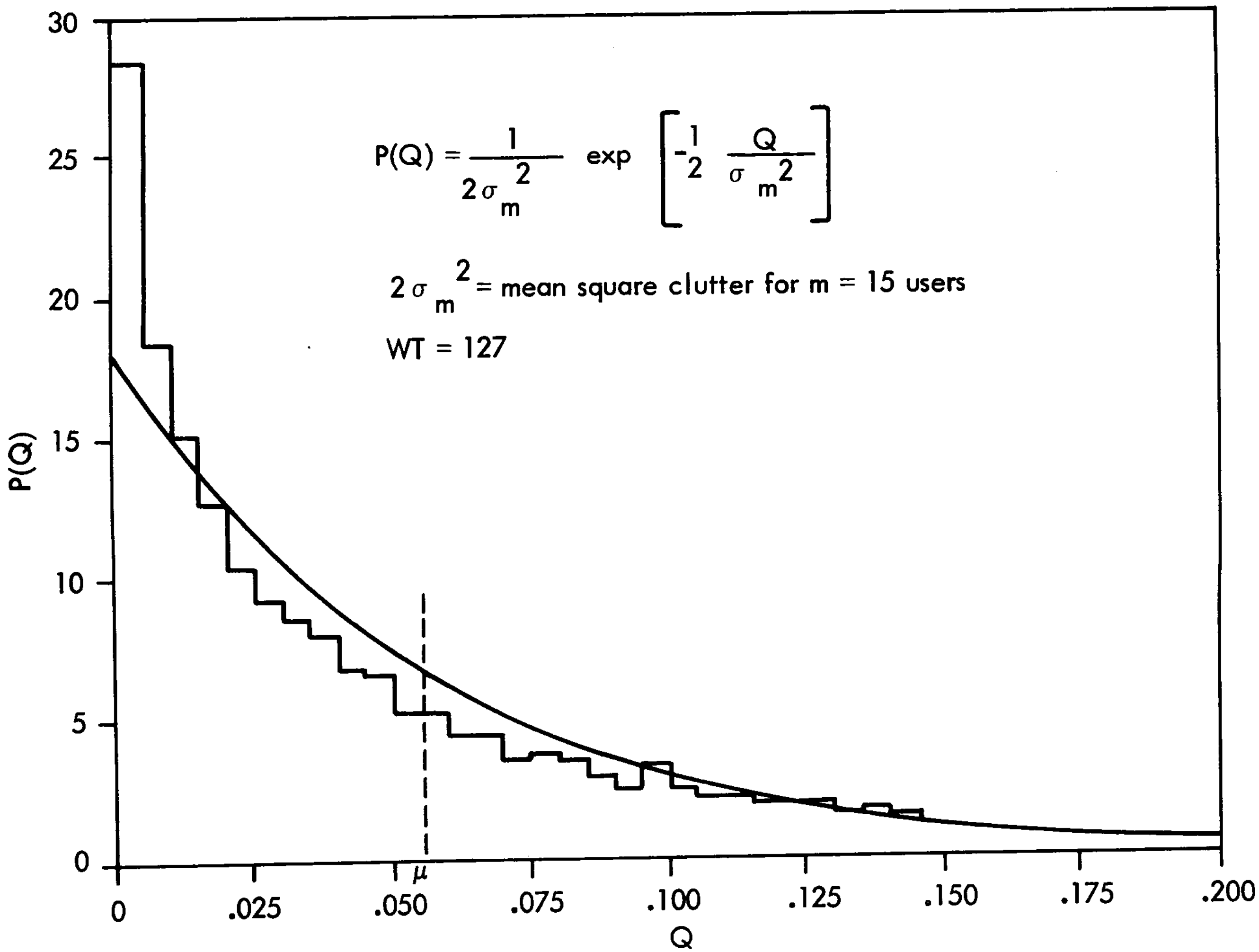


Figure 4-33. Probability Density of Clutter for 15 Users at Output of Square Law Detector (No Hard Limiting)

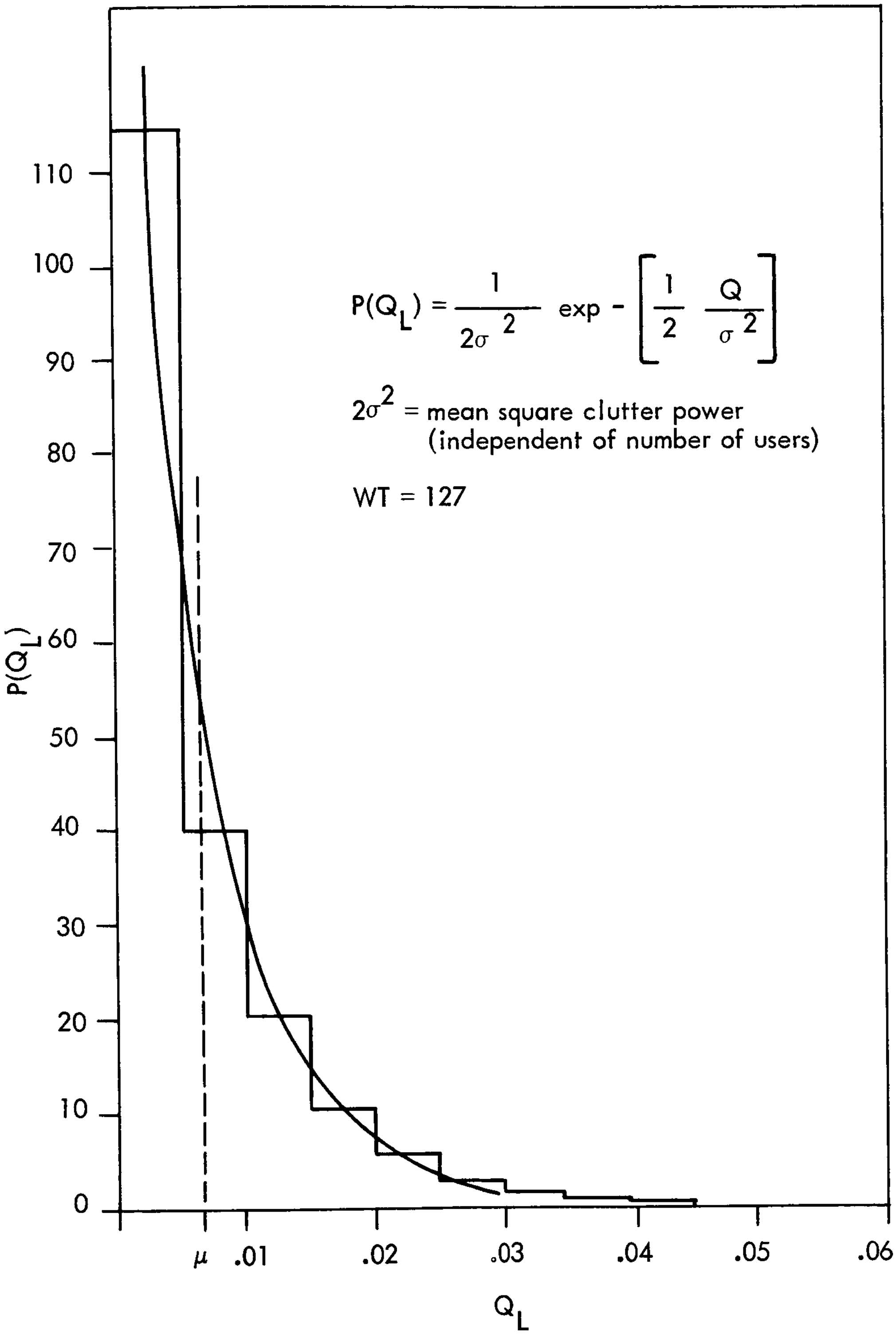


Figure 4-34. Probability Density of Clutter for 15 Users at Output of Square Law Detector (with Hard Limiting)

4.2.6. Measured Characteristics of the Ambiguity Function (127- Bit m-Sequence)

Figure 4-35 is a table of the mean-square clutter and peak power of the clutter for several frequency shifts of the ambiguity function for the periodic and aperiodic cases shown in Figures 4-26 and 4-27. The mean-square clutter is the same for orthogonal (integral multiples of 1/T) and non-orthogonal frequency shifts (i.e., midway between orthogonal shifts). The maximum sidelobe, however, is significantly greater in the non-orthogonal case than in the orthogonal one. In the periodic case the peak clutter power is approximately twice the mean-square clutter in the orthogonal case and approximately six times the mean-square clutter power in the non-orthogonal case. In the aperiodic case the orthogonal signals are only slightly better than non-orthogonal ones as far as maximum sidelobes are concerned. The aperiodic case is the more realistic one in this paper. Thus, orthogonal FSK, PN signal addresses are superior to non-orthogonal ones as far as peak sidelobes are concerned but not any better in terms of mean-square clutter.

ω	mean clutter power	peak clutter power (aperiodic)	peak clutter power (periodic)	peak power
0	0.0046	0.0122	0.0001	1.0000
$^{1}/2T$	0.0039	0.0093	0.0238	0.4023
$^{1}/_{T}$	0.0036	0.0112	0.0079	0
$^{3}/2T$	0.0039	0.0225	0.0250	0.0449
$^{2}/T$	0.0039	0.0150	0.0079	0
$^{5}/2T$	0.0039	0.0192	0.0258	0.0162
$^{3}/T$	0.0038	0.0126	0.0079	0

Figure 4-35. Summary of Mean and Peak Data of M-Sequence Ambiguity Function for WT = 127

4.3 Correlation-Locked Techniques

4.3.1 Description of Transmitted Messages

PN signals used for matched filter reception, as described in Section 4.2, are pulsed. In correlation-locked systems, PN subcarriers are continuous in time and behave much like conventional pulse or sinusoidal subcarriers. Let $Z_{sc}(t)$ be the complex subcarrier. Then,

$$Z_{sc}(t) = a_{sc}(t) \exp\{ j\phi_{sc}(t) \} \tag{4-24}$$

If the signal is used to modulate a complex RF carrier, then

$$Z_c(t) = a_{sc}(t) \exp\{ j(\phi_{sc}(t) + \omega_c t) \} \tag{4-25}$$

In most spread spectrum applications of interest the transmitted RF carrier will be angle modulated both by message and subcarrier. Then,

$$Z_c(t) = A \exp\{ j[\phi_M(t) + \phi_{sc}(t) + \omega_c t] \} \tag{4-26}$$

where $\phi_M(t)$ is the angle modulation produced by the message.

In particular, if $\phi_{sc}(t)$ is an N-bit sequence which takes on the values $(\pi/2, -\pi/2)$ in the time interval $T = N\Delta T$ and then repeats; and if the message is also binary, such that $\phi_M(t)$ takes on the values $\pi/2$ or $-\pi/2$ every T seconds, then Equation (4-26) becomes

$$Z_c(t) = A \cos \phi_M(t) \cdot \cos \phi_{sc}(t) \cdot \{ \exp\ j\,\omega_c t\} \tag{4-27}$$

Equation (4-27) represents a double sideband suppressed carrier signal which results from bilevel amplitude modulating a carrier at the rate $1/\Delta T$ bits per second.

A block diagram of a random access system transmitter corresponding to Equation (4-27) is shown in Figure 4-36.

Just as in the matched filter case, the address selector chooses the called party's address by presetting the sequence generator to the proper code segment. The PN generator operates so that it will repeat the sequence every N bits. The modulo-2 (mod-2) addition of the PN subcarrier and binary message results in an output PN sequence which is biphase modulated by the message. The output is fed into a balanced modulator and then to the transmitter.

Another important type of modulation is where $\phi_M(t)$ is a narrow deviation phase modulation by an analog message, and $\cos \phi_{sc}(t)$ is phase reversal PN modulation. Then,

$$Z_c(t) = A \cos \phi_{sc}(t) \cdot \exp\{ j(\phi_M(t) + \omega_c t)\} \tag{4-28}$$

The PN subcarrier $\cos \phi_{sc}(t)$ spreads the small deviation angle-modulation over a much broader frequency band. Figure 4-37 shows a block diagram of a transmitter which applies narrow-deviation analog phase modulation to a carrier wave for random access system.

As in previous block diagrams, the address selector sets the

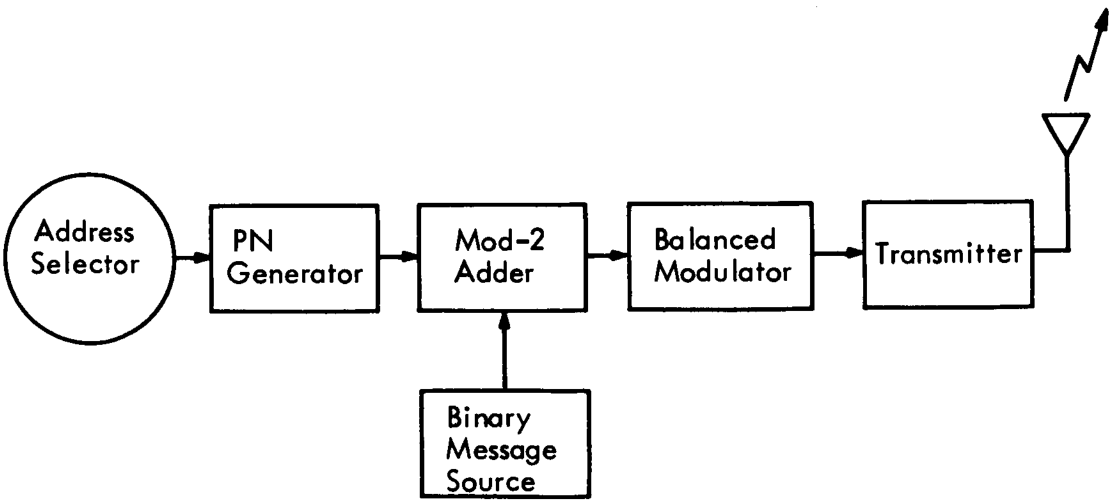


Figure 4-36. Binary Message Modulation of a PN Subcarrier

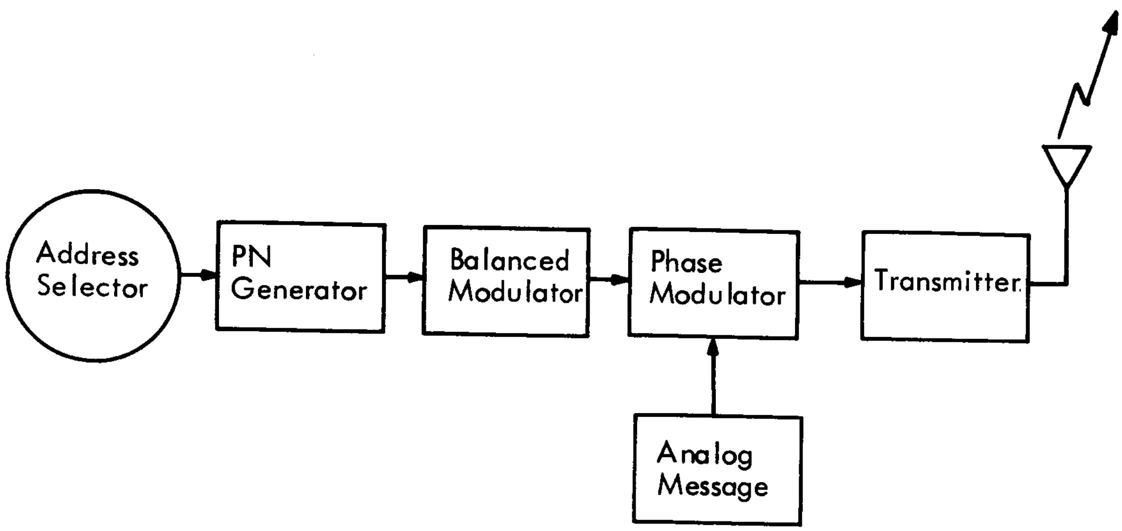


Figure 4-37. Analog Phase Modulation of a Binary PN Subcarrier

PN generator to the called party's address. The PN phase reversals modulate a carrier in the balanced modulator. The analog message is applied to the bandpass PN subcarrier by a narrow-deviation phase-modulation process and is transmitted over the RF link.

Another interesting type of analog modulation was discovered by Dr. F. Corr of the IBM Communications Systems Department (i.e., PN subcarrier rate modulation - PNRM). In this approach the bit rate of the PN subcarrier is varied directly by the analog message source much as doppler would vary it. The message can be extracted by tracking the received subcarrier; the output analog message is the correction voltage fed to the tracking device. This will be discussed in more detail in the section on correlation-locked reception.

Figure 4-38 is a block diagram of the PN subcarrier modulated system. The analog message modulates the pulse repetition rate of the clock which drives the PN sequence generator. The clock rate in turn controls the shifting rate of the sequence generator. This expansion and compression of the PN bit rate represents the analog message.

4.3.1.1 Transmission of Higher-Order Signal Alphabets

If T_o is the duration of a message bit, W; the channel bandwidth, $(WT_o >> 1)$; then the number of orthogonal signals that can be transmitted (one at a time) is

$$2^m = 2mWT_o \qquad (4\text{-}29)$$

where m is the number of message bits which are transmitted with each orthogonal signal. Thus, each orthogonal signal occupies the channel bandwidth W and time duration $T = mT_o$. If the same set of signals is assigned to each subscriber, it is essential to modify the set by means of PN signal addresses. For example, if orthogonal binary signals are used, the mod-2 addition of each orthogonal signal with a distinct PN binary signal address which is assigned to each subscriber will give each subscriber a unique set. The waveform set of each subscriber still remains orthogonal. However, the waveforms among different subscribers are only quasi-orthogonal. Since the mutually interfering signals are asynchronous relative to the desired signal, the orthogonal property among subscriber signal addresses is not advantageous (i.e., it is the crosscorrelation function that must be low everywhere). However, the orthogonal property among elements of the same alphabet is desirable.

Mathematically, the orthogonal set is given by

$$Z_n(t) = \exp\{j(\omega_o t + \emptyset_{on}(t))\};\ n = 1, 2, \ldots, 2mWT_o \qquad (4\text{-}30)$$

In addition, each subscriber is assigned a periodic PN binary signal address which is distinct from all others in the sense that the mean square distance among these signals exceeds some positive number. After modulo-2 addition with the orthogonal signals, the quasi-orthogonal set for the k^{th} subscriber is

$$Z_n^{(k)}(t) = \exp \{ j(\omega_o t + \emptyset_{on}(t) + \emptyset_k(t)) \} \tag{4-31}$$

$$n = 1, 2, \ldots, 2mWT_o$$

The orthogonal signal set $\{ \emptyset_{on}(t) \}$ can be frequency shifted. Then,

$$\emptyset_{on}(t) = \Delta \omega t - \frac{2 \pi}{mT_o} t \tag{4-32}$$

If binary orthogonal signals are used, then

$$Z_n^{(k)}(t) = \cos \emptyset_{on}(t) \cdot \cos \emptyset_k(t) \cdot \exp (j \omega_o t) \tag{4-33}$$

where the product $\cos \emptyset_{on}(t) \cdot \cos \emptyset_k(t)$ is another binary sequence.

The set of signals $Z_n^{(k)}(t)$ are orthogonal. That is

$$\frac{1}{mT_o} \int_0^{mT_o} Z_n^{(k)}(t)(Z_n^{(k)}(t))^* dt = \frac{1}{mT_o} \int_0^{mT_o} Z_{on}(t)dt = \begin{cases} 1 ; & n = r \\ 0 ; & n \neq r \end{cases} \tag{4-34}$$

As previously stated, however, the elements of $Z_n^{(k)}(t)$ $k = 1, 2, \ldots.$ are not orthogonal. That is

$$\rho_{kp} = \frac{1}{mT_o} \int_0^{mT_o} Z_n^{(k)}(t) \left(Z_r^{(p)}(t) \right)^* dt$$

$$\leq \frac{1}{mT_o} \left| \int_0^{mT_o} \exp\{ j(\emptyset_{on}(t) - \emptyset_{or}(t)) \} \exp \{ j(\emptyset_n(t) - \emptyset_p(t)) \} dt \right| \tag{4-35}$$

where $\rho_{kp} < 1$ for all $k \neq p$.

When the sequences are binary the first term in Equation (4-35) is a binary sequence as well as the second exponential. The first term is an orthogonal binary sequence while the second one is not. Thus, the absolute value of the crosscorrelation coefficient is not zero.

In the case of higher-order alphabet signal addresses the concept of modulating a PN subcarrier is still applicable. For sinusoidal signals the PN subcarrier has a broad spectrum relative to the bandwidth of the orthogonal components, while in the case of the binary orthogonal signals the components have the same band as the PN subcarrier.

Although we have concentrated here on orthogonal higher-order alphabets it should be clear that this discussion is also applicable to quasi-orthogonal signals. Figure 4-39 is a block diagram showing a

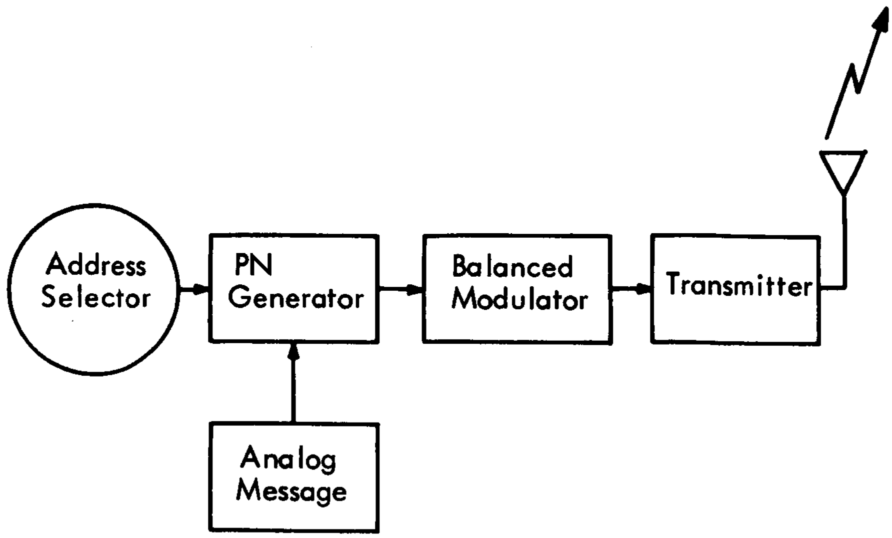


Figure 4-38. PN Rate Modulation by an Analog Message

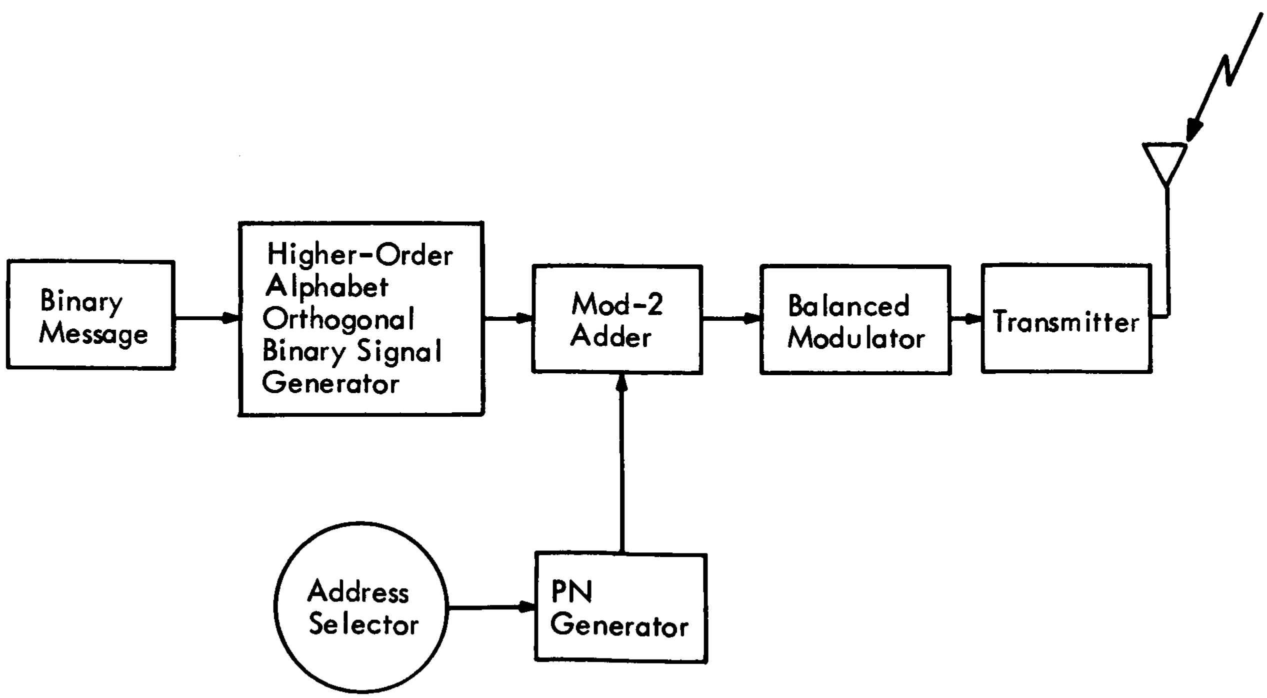


Figure 4-39. Binary Higher-Order Alphabet Generator

binary higher-order alphabet transmitter.

The higher-order alphabet signal generator is capable of generating 2^m orthogonal binary signals corresponding to the number of message sequences that are m-bits long. Each m-bit sequence selects one of the 2^m orthogonal binary waveforms. The PN generator is set to the called party's address. Each orthogonal signal is encoded into the called party's address and then transmitted. Each subscriber has the same higher-order alphabet generator. The address selector encodes the alphabet so that it will be recognized at only one receiver.

4.3.1.2 PN Multiplexing of Many Subcarriers at a Transmitter Terminal

Multiplexing Using Delay Resolution. Matched-filter random access systems, exploiting the delay resolution properties of the signals, lead to significant practical results. When correlation-locked techniques are used the delay resolution property of the PN subcarrier can be used effectively in practice for multiplexing many modulated subcarriers at a transmitter terminal much like conventional frequency or time division systems. The PN subcarriers here are, however, non-orthogonal; in fact the spectra of the subcarriers are strongly overlapping. However, the autocorrelation function has low sidelobes and hence the signals can be time-multiplexed. At the receiver each channel can be separated provided transmitter and receiver are correlation-locked. Figure 4-40 is a block diagram of a multiplexer using the delay resolution property. The multiplexer shown assumes that the messages and subcarriers are binary.

The address selector chooses the called party's address by "dialing" the desired code on the sequence generator. The sequence generator produces a periodic sequence which drives a digital (delay line) shift register. The register is tapped as shown. At each tap the input PN subcarrier is delayed τ seconds. The output of each tap is modulated in the mod-2 adder by the message sequence. Each mod-2 adder is multiplexed in the majority logic device. (This is equivalent to summing sequences algebraically (1, -1) and feeding the sum into a hard limiter.) The output of the majority logic is a binary sequence at the subcarrier rate which is fed into a balanced modulator. The synchronizing channel is the PN sequence generator output that is used to modulate balanced modulator BM_0, which is coherent and orthogonal to the message balanced modulator BM_1. The output of the balanced modulator is summed and fed to the transmitter. The synchronization channel is used to lock in the locally generated sequence at the intended receiver. The weight $a \leq 1$ in the synchronization channel permits more power in the message channel at the expense of synchronization power.

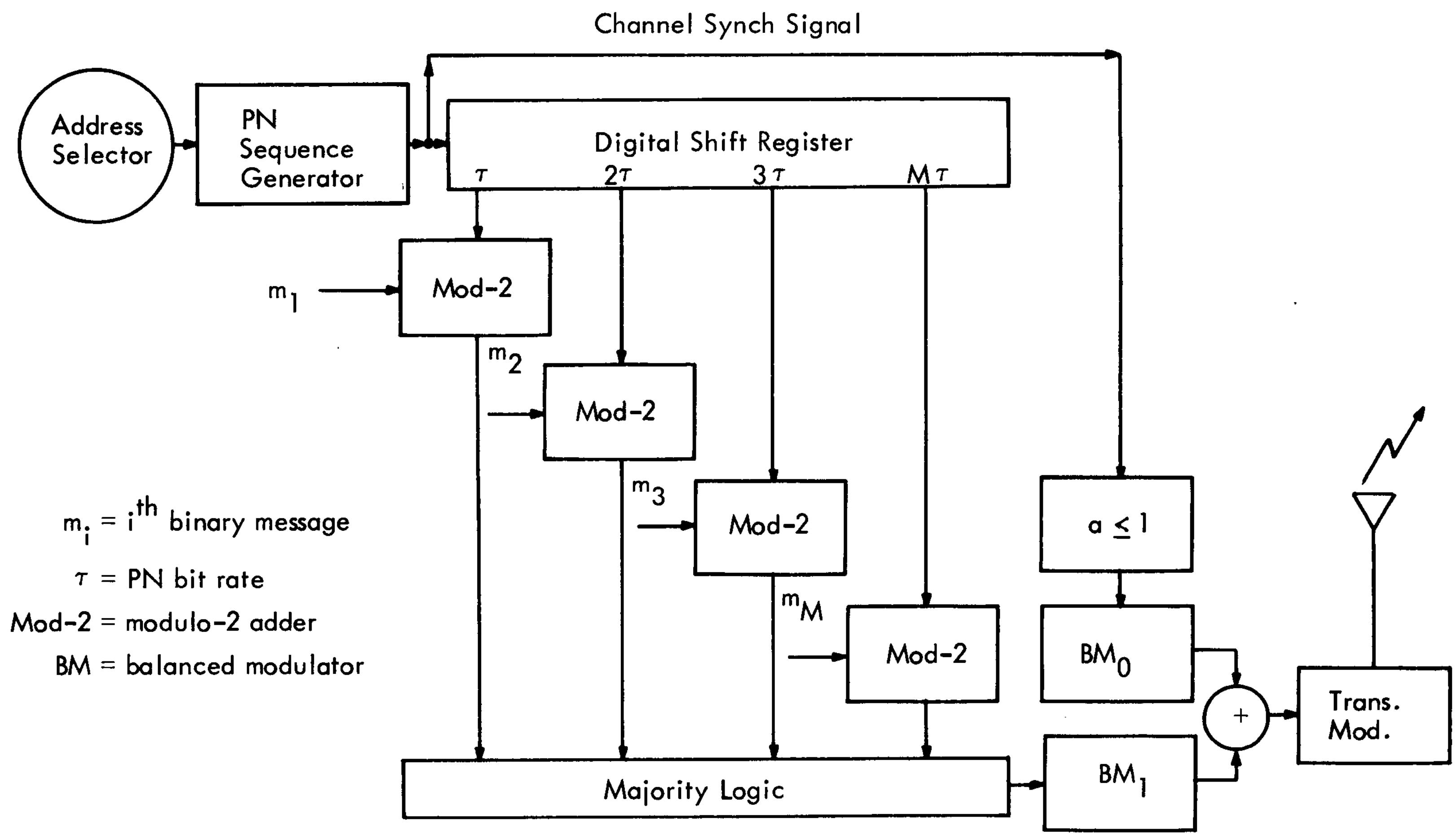


Figure 4-40. Multiplexer of PN Subcarriers Using Delay Resolution

The output from this multiplexer is addressed to one station. The multiplexed subcarrier will generate some mutual clutter at the receiver. Many more stations will use the same medium and will have the same multiplexer except that a different sequence will be used. Thus, additional clutter will be generated in each receiver.

It should be recognized that the same multiplexer can be used to generate delayed PN subcarriers, which are then frequency shifted by the message. In this case, the mod-2 adder is replaced by a balanced modulator which is then angle modulated by the message and the majority logic is replaced by a bandpass hard limiter.

General PN Subcarrier Multiplexer. A more general multiplexer is shown in Figure 4-41. The operation of this multiplexer is the same as that in Figure 4-40 except here the subcarriers are distinct rather than delayed versions of each other. In practice the subcarrier can be generated simultaneously by means of logic rather than having to use a different sequence generator for each.

Conventional Frequency Division Multiplexing with PN Sub-Carrier Addressing. Conventional frequency division multiplexing can also be used in correlation-locked techniques. Here the multiplexed signal can (at based-band) be used to angle-modulate a band-pass PN subcarrier. At the receiver the PN subcarrier is removed;

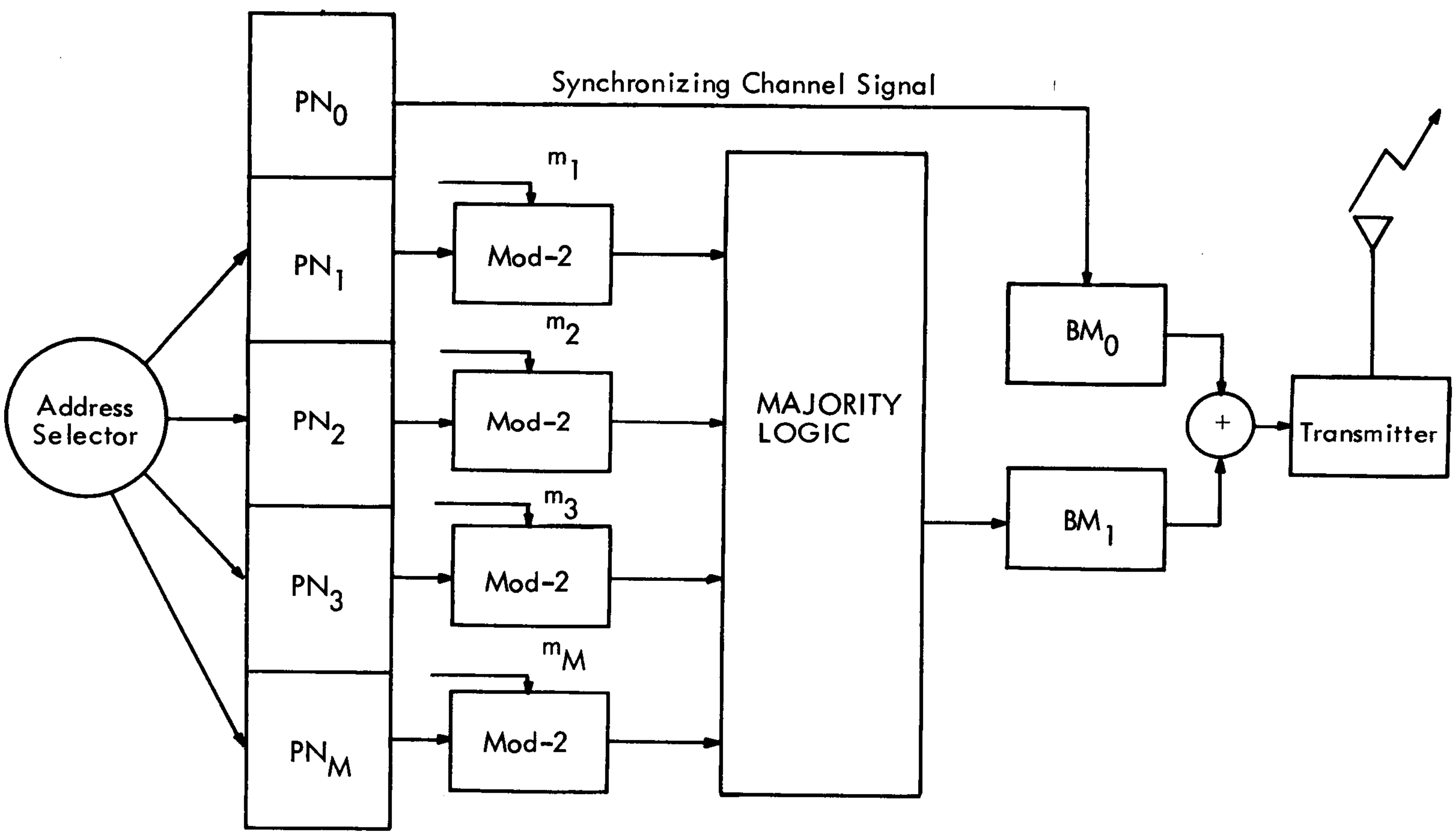


Figure 4-41. PN Subcarrier Multiplexer Using Different PN Signals

the output being the multiplexed signal. Here, conventional terminal equipment can be used. The PN operation can then be considered a MODEM.

Figure 4-42 shows a single-sideband (SSB) multiplexer for voice channels which is used to angle-modulate a PN subcarrier address.

4.3.1.3 Techniques for Transmitting Correlation Reference Signals

Orthogonal Synch Signal Combinations. Correlation-locked systems must be synchronized. It was previously mentioned that a correlation-locked system can be synchronized by transmitting a PN signal pulse which is received by the matched filter. The matched filter output pulse starts the receiver signal address generator.

In order to maintain lock it is possible for the receiver to derive correlation lock information either from the received signal or from a special unmodulated signal which is transmitted for this purpose. Figure 4-43 is a block diagram of one such technique which transmits along a reference for maintaining correlation lock at the receiver.

The address selector chooses the called party's address. We assume that synchronization has been established by a matched filter random access system which was used for establishing a connection.

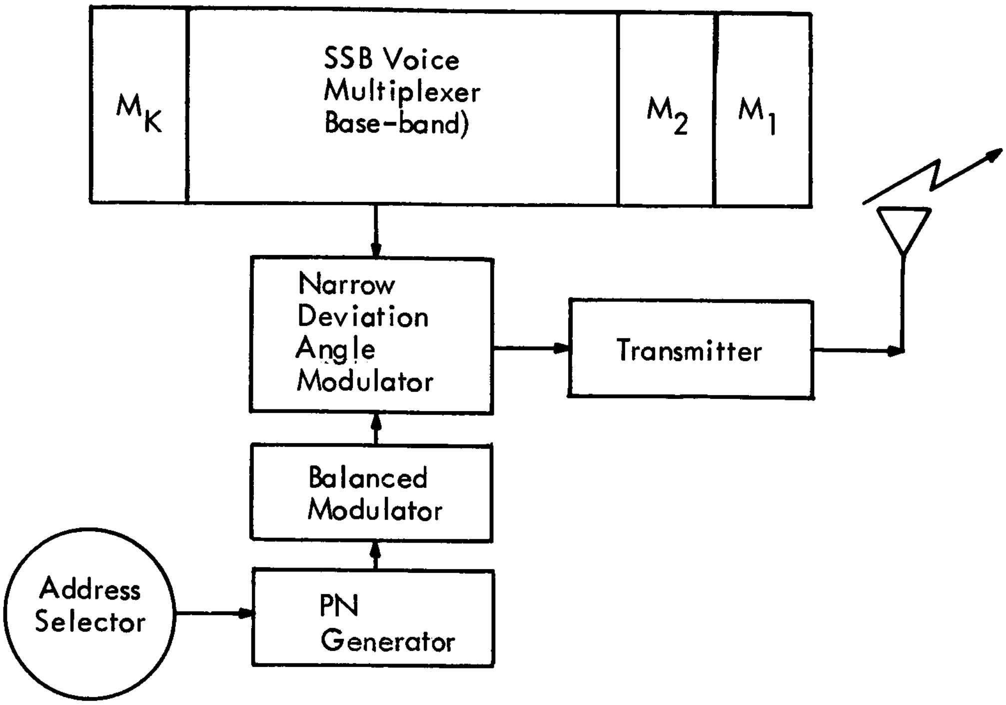


Figure 4-42. Conventional SSB Voice Multiplexer with PN-Subcarrier Addressing

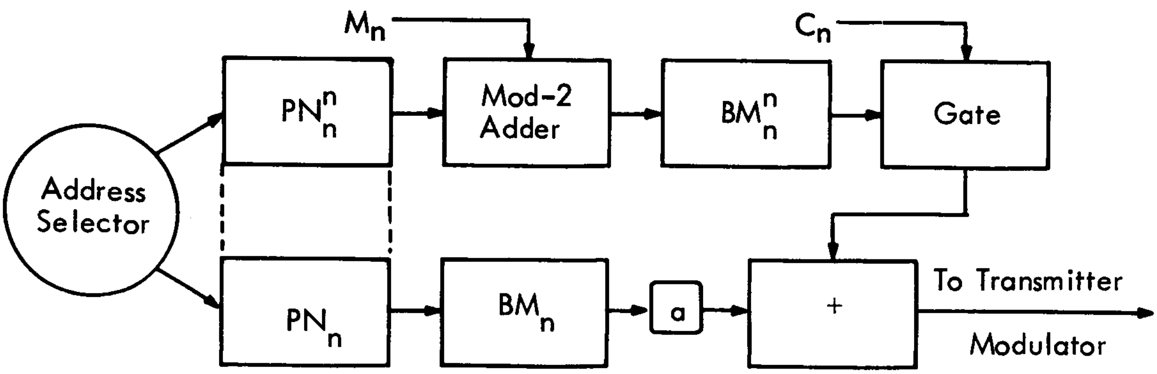


Figure 4-43. Modulators Combined With Orthogonal Synchronous Signal Combination

The two sequence generators at the transmitter are precisely in step. The generator PN_n^n is the message subcarrier while PN_n is the synchronizing subcarrier. When message transmission is interrupted, the control signal C_n gates out the message subcarrier, reducing the clutter which is generated in the signal environment. The synchronizing channel is active as long as the channel is in use. In this manner it is not required to reestablish synchronization.

The balanced modulators BM_n^n and BM_n are orthogonal to each other. Thus, at the receiver there is an absence of mutual interference between the message signal and the synchronizing one of the same channel. The mathematical expressions for the message and synch signal are given, respectively, by

$$Z_{on}^n(t) = \exp\left\{ j(\omega_o t + \frac{\Delta\omega}{2} t + \emptyset_M(t) + \emptyset_n(t) \right\}$$

$$Z_{on}(t) = \exp\left\{ j(\omega_o t - \frac{\Delta\omega}{2} t + \emptyset_n(t) \right\} \qquad 0 \leq t \leq T \tag{4-36}$$

where $\emptyset_M(t)$ is the message modulation and $\phi_n(t)$ is the periodic PN subcarrier. If T is the duration of a PN subcarrier bit, then

$$\Delta\omega = \frac{2\pi}{\Delta T} .$$

At this frequency shift, the two signals in Equation (4-36) are orthogonal.

In a system of this type all message subcarriers are transmitted at the same frequency and all synch signals at another frequency. Thus, a received signal contains a mixture of both channels. The spectra of the mixture of synch signals and message signals will have some overlap which will result in some mutual clutter generated in the synch and message channel. However, the mutual interference between the message and synch signals of a called party will generate zero clutter since these are orthogonal.

Quasi-Orthogonal Synch Signal Combinations. At the transmitter the synchronizing signal can be combined with the message signal by means of "majority" logic. Since only two signals are used, a majority will exist only when the pair (1, 1) occurs and when (0, 0) occurs. For the special case (0, 1) and (1, 0) it is necessary to establish a special rule such as $(0, 1) \rightarrow 0$ and $(1, 0) \rightarrow 1$. If the PN sequences are sufficiently long such a rule will suffice. A convenient way of transmitting synch is to send along an unmodulated version of the PN subcarrier. In Figure 4-44a an arbitrary synch signal is the delayed subcarrier.

The configurations in Figure 4-44 are very similar to Figure 4-42 except for the majority logic combiner. The combiner provides a weighting γ which (if desired) would favor the message sequence,

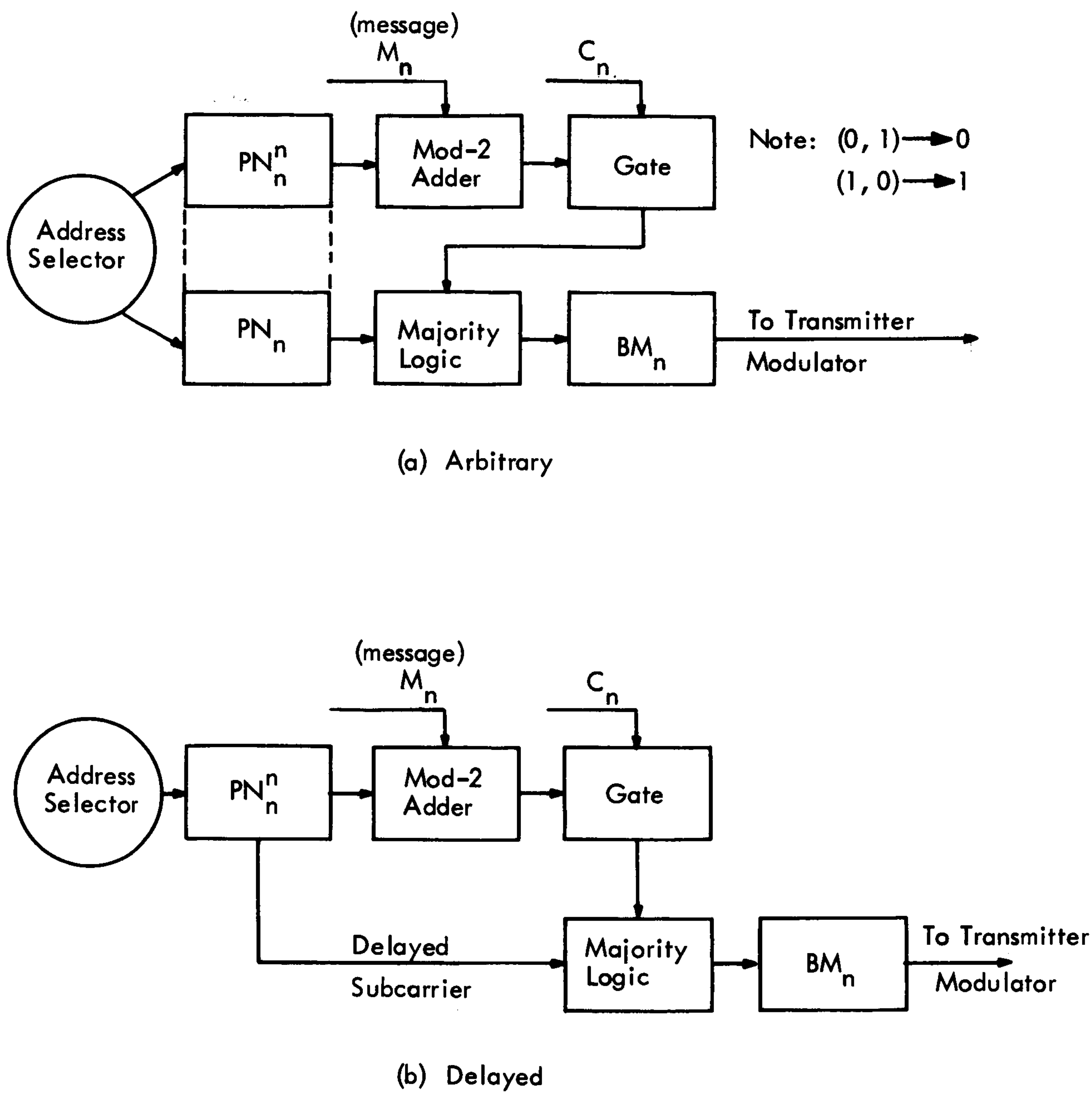


Figure 4-44. Modulator Combined With Quasi-Orthogonal Synchronous Signal

since the synch sequence can be smoothed at the receiver with a narrower band filter. The weighting rule can be such that each time the ambiguous sequences (0, 1) and(1, 0) occur, the rule for converting to zero or one can be made to favor the message sequence.

4.3.2 Description of Reception and Demodulation

Once correlation lock is established and maintained, the reception operation is simple. Figure 4-45 is a block diagram which shows the receiver operation. The received signal contains the message signal plus clutter. The output clutter power is attenuated by the ratio of the message filter bandwidth to the received PN signal bandwidth.

4.3.2.1 Technique for Calling and Establishing Synchronization

We will assume here that the technique which is used for establishing synchronization uses a matched filter for reception. In particular the first B-bits ($B > 50$) of an address signal are used for locking in the locally generated PN sequence. This is part of the calling procedure. Just as in the matched filter system, here the called party's address is dialed and a signal address is transmitted which is received by the called party's matched filter. In addition, the calling party's code is also transmitted. At the end of this message the sequence generator is started at the receiver so that it will correlate precisely with the transmitted correlation reference. The synchronization portion of the receiver is shown in Figure 4-46.

The received signal is fed into a matched filter receiver. At the appropriate time the sequence generator is triggered so that it will be synchronous with the received correlation reference. If desired, the matched filter output can be inhibited after acquisition. The received signal and the local reference are then fed into the correlation lock unit which maintains transmitter and receiver coherency.

It should be clear that any one of the asynchronous techniques discussed in Section 4.2 can be used for establishing synchronization.

4.3.2.2 Techniques for Maintaining Correlation Lock

Digital Delay-Lock Discriminator. The delay-lock discriminator is a technique for maintaining correlation lock between a received PN signal and a locally generated replica at the receiver. Once locked on, the device will track the received signal in the presence of doppler, noise, clock drift or other disturbances.

Figure 4-47 is a block diagram of a digital delay-lock discriminator. The shift register sequence generator is tapped at the output of the nth stage and the (n-2) stage. Thus, we have an "early-late" correlator. The received signal is multiplied by the early reference and the late reference. The difference in the resultant output is the

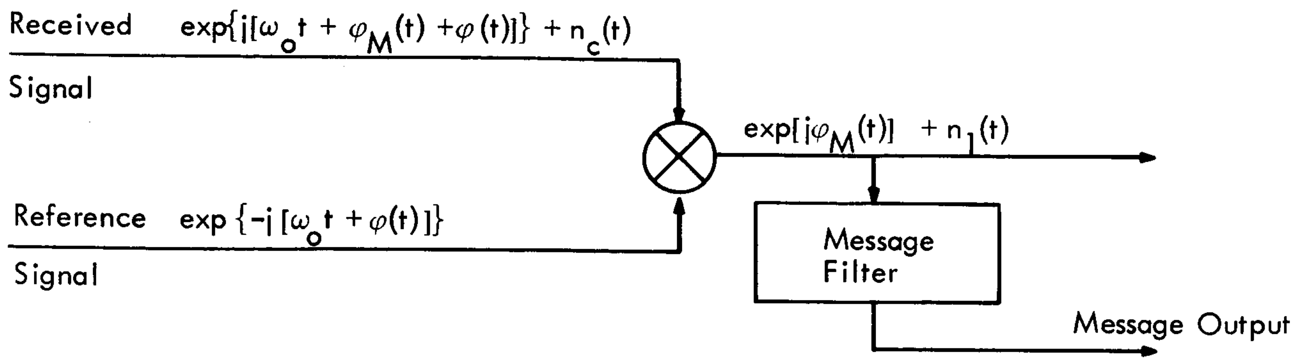


Figure 4-45. Correlation-Locked Receiver

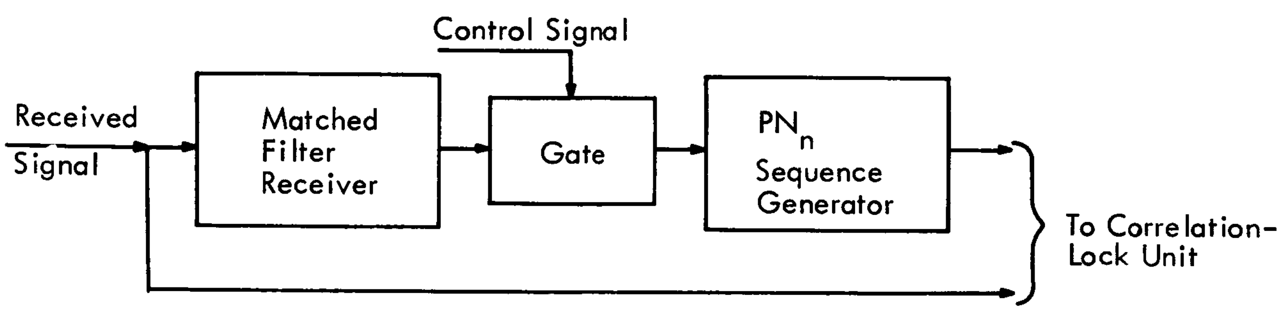
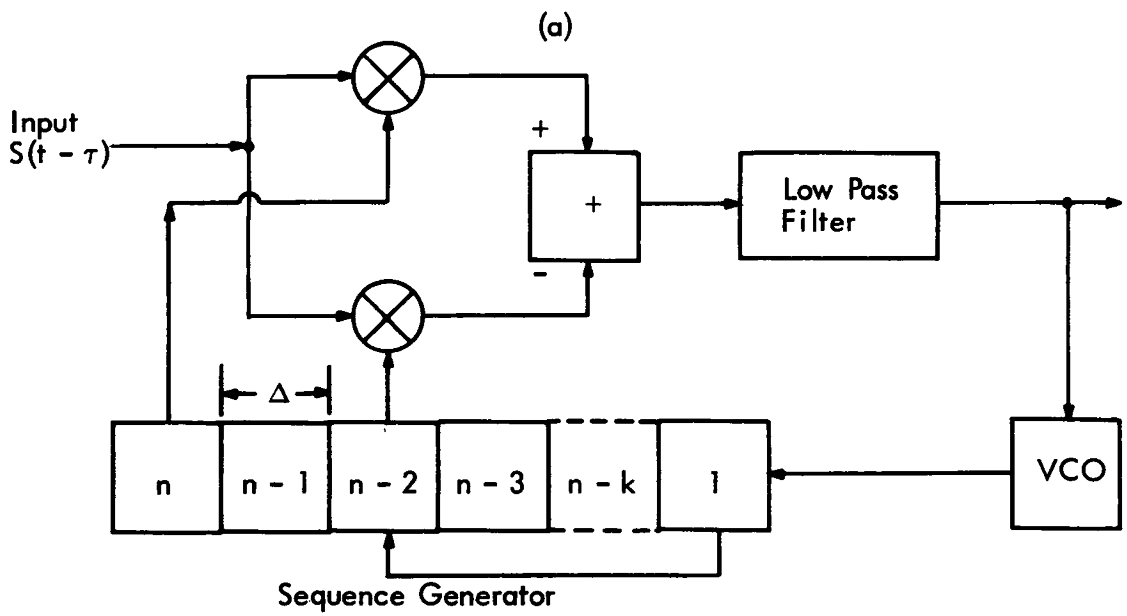
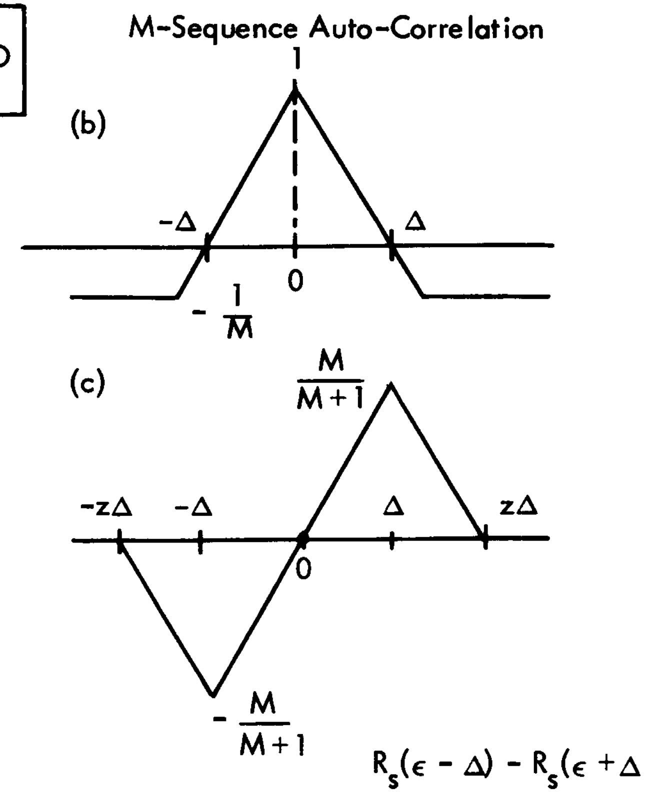


Figure 4-46. Synchronization of PN Sequence Generator





1. Δ = time between clock pulses
2. E = expected value operation
3. R_s = auto-correlation function

$$R(\epsilon) = E\{S(t - \tau)[S(t - \tau - \Delta) - S(t - \tau + \Delta)]\} = R_s(\epsilon - \Delta) - R_s(\epsilon + \Delta)$$

Figure 4-47. Digital Delay-Lock Discriminator the error signal. After smoothing the error, the output of the low-pass filter controls the clock rate so as to minimize the error. (This circuit is a more general version of the well known "early-late" gate range tracking in fire control radar.)

Figure 4-47, item (c), shows the discriminator characteristics of the delay-lock device when an M-sequence generator is used. This characteristic is given by

$$D(\varepsilon) = R_s(\varepsilon - \Delta) - R_s(\varepsilon + \Delta) \qquad (4\text{-}37)$$

where $R_s(\varepsilon)$ is the autocorrelation function at the point $\tau = \varepsilon$. When the signal has an autocorrelation function that is differentiable, then the derivative is the discriminator characteristic rather than the difference.

4.3.2.3 Correlation-Lock Reception Using Delay-Lock Discriminator

Analog Band-Pass Delay-Lock Discriminator and Correlation Receiver[14, 15] (Orthogonal Signals). The delay-lock discriminator can operate at bandpass provided a variable bandpass delay line can be used. In this case the device is the broad band equivalent of the well known narrow-band phase-locked loop. Short delay lines that are voltage controlled can be used or, where long delay lines are required, a servo-controlled mechanically movable sliding tap can be used.

The autocorrelation function of the bandpass PN signal having the envelope shown in Figure 4-47, item (b), is shown in Figure 4-48. The envelope function shown modulates a cosinusoidal wave. The mathematical expression for the bandpass autocorrelation function is

$$\phi_b(\tau) = \phi(\tau) \cos \omega_o \tau \tag{4-38}$$

where $\phi(\tau)$ is the autocorrelation function of the PN signal and ω_o is the IF frequency. The discriminator characteristic is obtained by introducing a 90^o phase shift at ω_o. Hence

$$\phi_d(\tau) = - \phi(\tau) \sin \omega_o \tau \tag{4-39}$$

This characteristic is shown in Figure 4-48. It should be clear from Equation (4-36) that the bandpass characteristic has ambiguity in that the device can lock on to any of the negative sloping portions of the sine wave. An adjustment must therefore be introduced to make certain that the discriminator locks on to the major portion of the characteristic which also gives the maximum signal-to-noise portion.

Figure 4-49 is a block diagram of a PN modulator of the type discussed previously and its receiver employing a bandpass delay-lock discriminator. The PN modulator, Figure 4-49, item (a), transmits a reference for synchronization. The reference source of the receiver, Figure 4-49, item (b), is the same as the transmitter except for the mod-2 adder. The received signal is fed into a bandpass limiter,

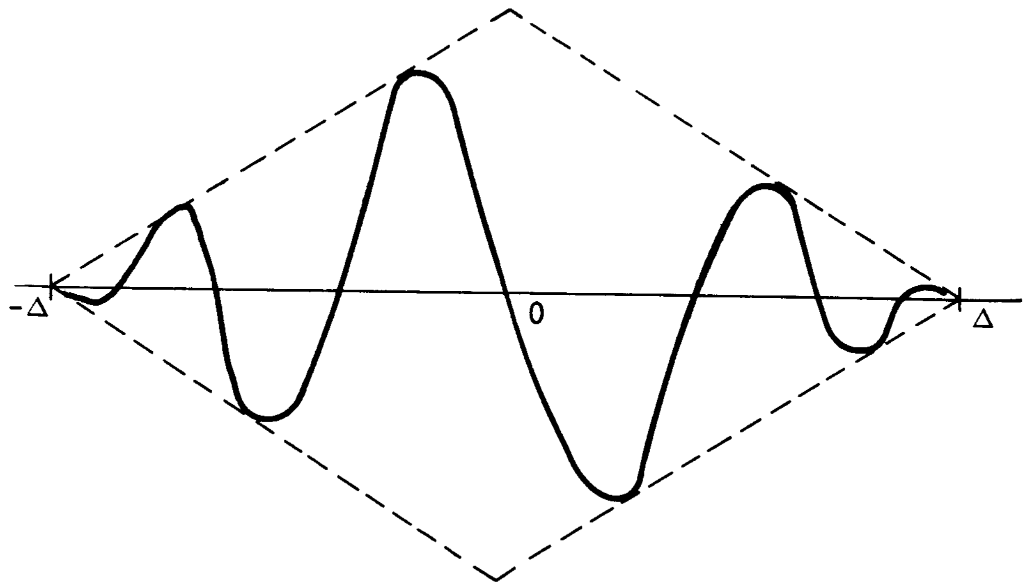


Figure 4-48. Bandpass Delay-Lock Discriminator Characteristic

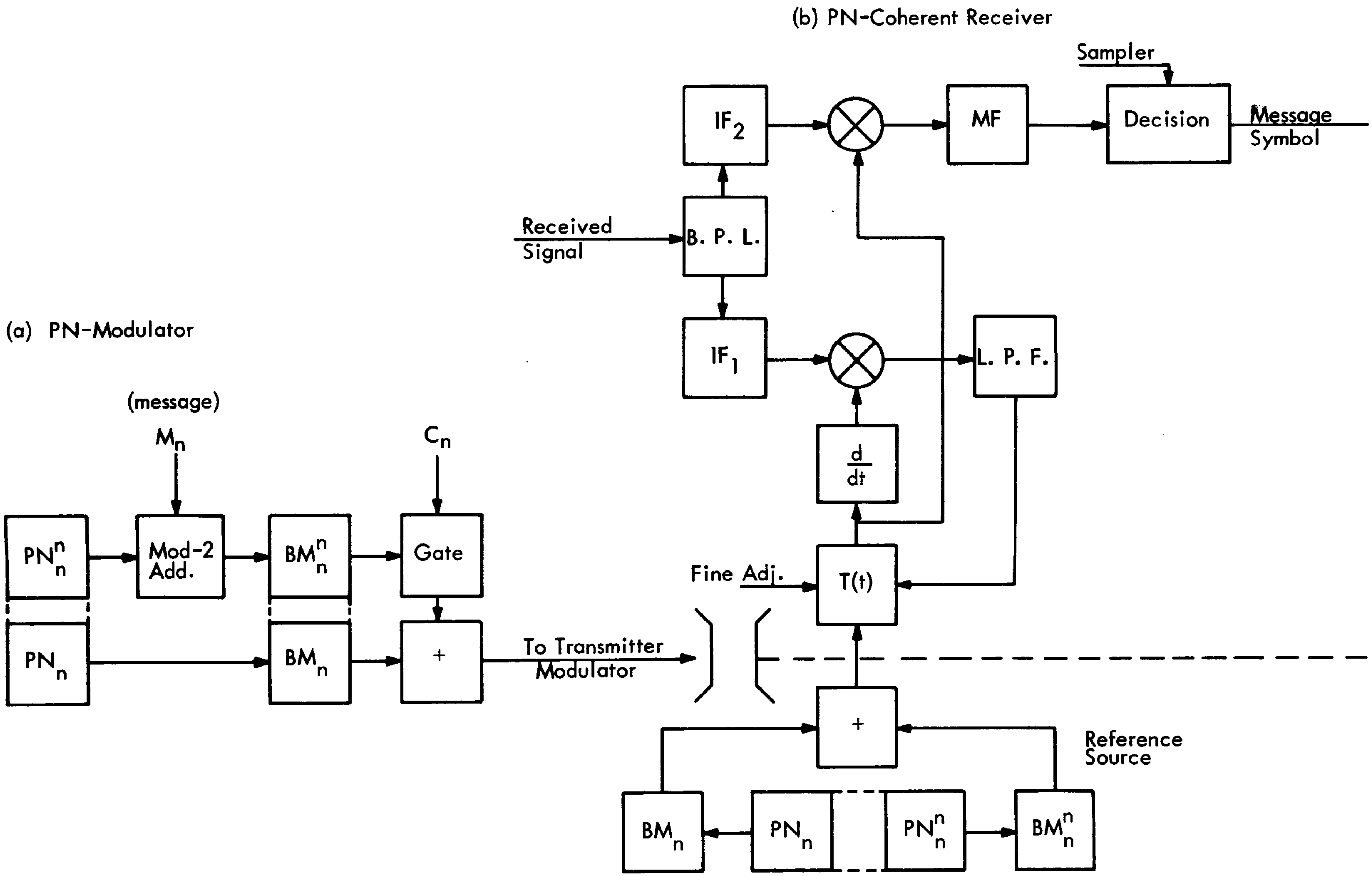


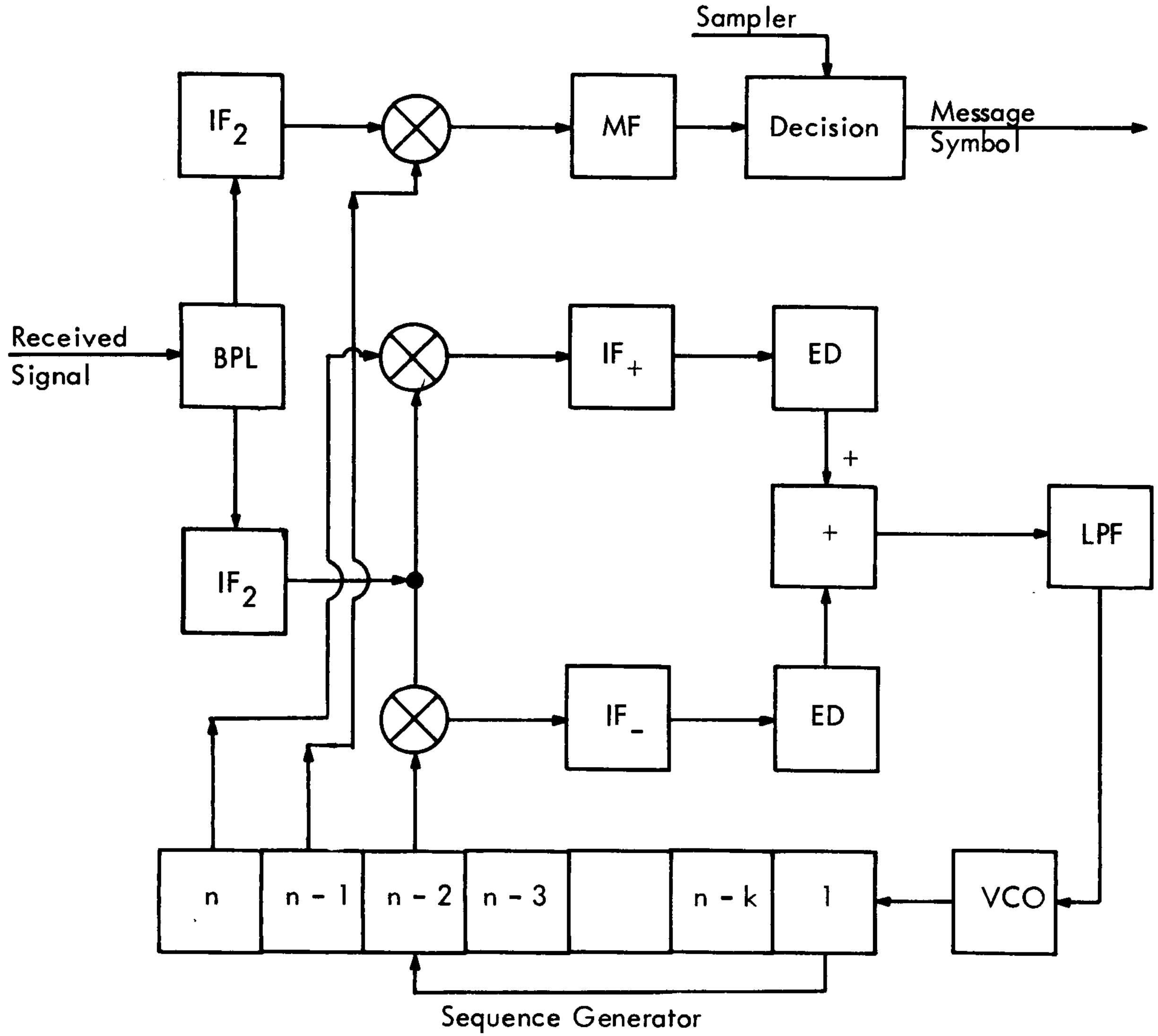
Figure 4-49. PN Modulator and Coherent Receiver

BPL, and then to IF_1 and IF_2. The filter IF_1 selects the synch channel. This output is fed into the bandpass delay-lock discriminator having the characteristic shown in Figure 4-49. The delay-lock discriminator will lock on to the received synchronizing signal. The variable delay line, T(t), introduces a variable of delay which brings the reference in time phase with the received signal. This delayed reference is then multiplied into the output of IF_2, the message channel. The message is then fed into the message filter and subsequently recovered. Message recovery can be accomplished either coherently or by means of a matched filter "detect and dump" circuit. The fine adjustment on the delay-lock discriminator is for the purposes of removing delay ambiguity. This receiver can also be used to recover a message from the continuously angle-modulated PN subcarrier shown in Figure 4-37.

Delay-Lock Discriminator and Correlation Receiver (Orthogonal Signals). The delay-lock discriminator proposed by Dr. Frank Corr of IBM, is directly applicable to problems where severe doppler shift is absent but where it is required to maintain precise synchronization between receiver and transmitter PN signals due to inherent clock instabilities and external disturbances. It also has the important advantage that the delay ambiguity inherent in the bandpass device is eliminated. Figure 4-50 is a block diagram of the envelope delay-lock discriminator.

The discriminator characteristic of envelope delay-lock discriminator is essentially the same as in Figure 4-47, item (c). IF_+ and IF_- are narrow bandpass integrators which are fed into envelope detectors, ED. The output of the different amplifiers represents the envelope delay errors and is independent of the RF phase. This error signal controls the VCO. This device is applicable to the transmitter configuration of Figure 4-49. If the same PN-generator is used in the message and synch channels then the output at tap (n-1) of the sequence generator is fed into the multiplier at the output of IF_2. The bandpass output is then recovered in a conventional manner. If the message has a different PN signal, then it is essential either to derive it from the synchronizing PN signal by a suitable operation or to drive another sequence generator in step with the synch reference by the same VCO. With this technique the message can be recovered from a continuously angle-modulated PN subcarrier. This receiver therefore corresponds to the transmitter of Figure 4-37 as well.

Envelope Delay-Lock Discriminator and Correlation Receiver (Quasi-Orthogonal Case). We will discuss reception of signals transmitted by the modulator in Figure 4-44 by means of the envelope delay-



IF_+ = narrow band IF
IF_- = narrow band IF
ED = envelope detector
LPF = low pass filter
VCO = voltage controlled oscillator

Figure 4-50. Bandpass Delay-Lock Discriminator and Correlation Receiver lock discriminator. The signals can also be received by means of the bandpass delay-lock discriminator discussed previously. Figure 4-51 is a block diagram showing how the message is extracted when the synchronizing and message signals are multiplexed by means of majority logic. This receiver is relatively simple. A single IF is required since the message and synch signal occupy the same frequency band. Since the synch signal is a delayed version of the PN subcarrier, it is only necessary to tap the PN generator to obtain the local PN reference for message recovery.

PN Rate Correlation Reception and Demodulation. Figure 4-38 is a block diagram of a transmitter using PN rate modulation where the PN bit is varied in accordance with a narrow-band analog message. The coherent demodulator for such a signal is shown in Figure 4-52.

The most interesting part of Figure 4-52 is that the transmission of a reference is not required; the delay-lock discriminator itself is the correlation receiver. Here the delay-lock discriminator tracks the narrow-band analog modulation. The tracking bandwidth is the message band, IF_M. The "error" signal represents the recovered analog message. The delay-lock, once locked on, will track the desired signal. The mutually interfering signals will not correlate. The fact that a reference is not required makes this technique attractive.

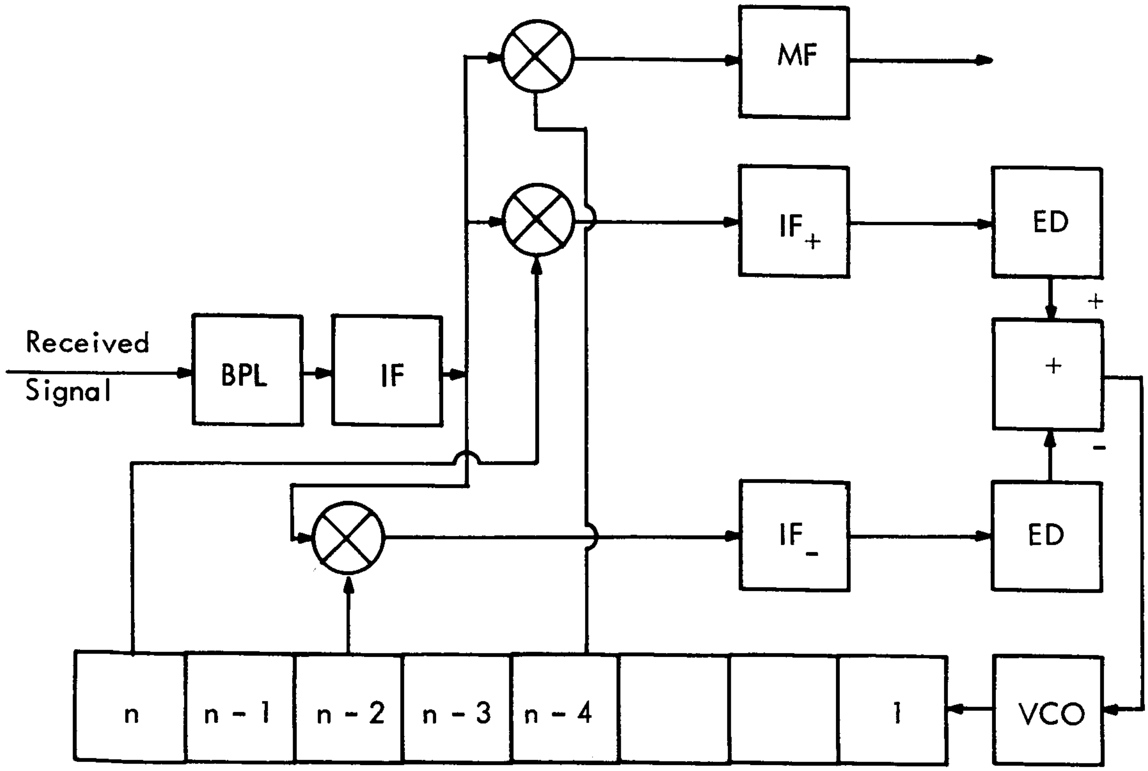


Figure 4-51. Envelope Delay-Lock Discriminator
and Correlation Receiver

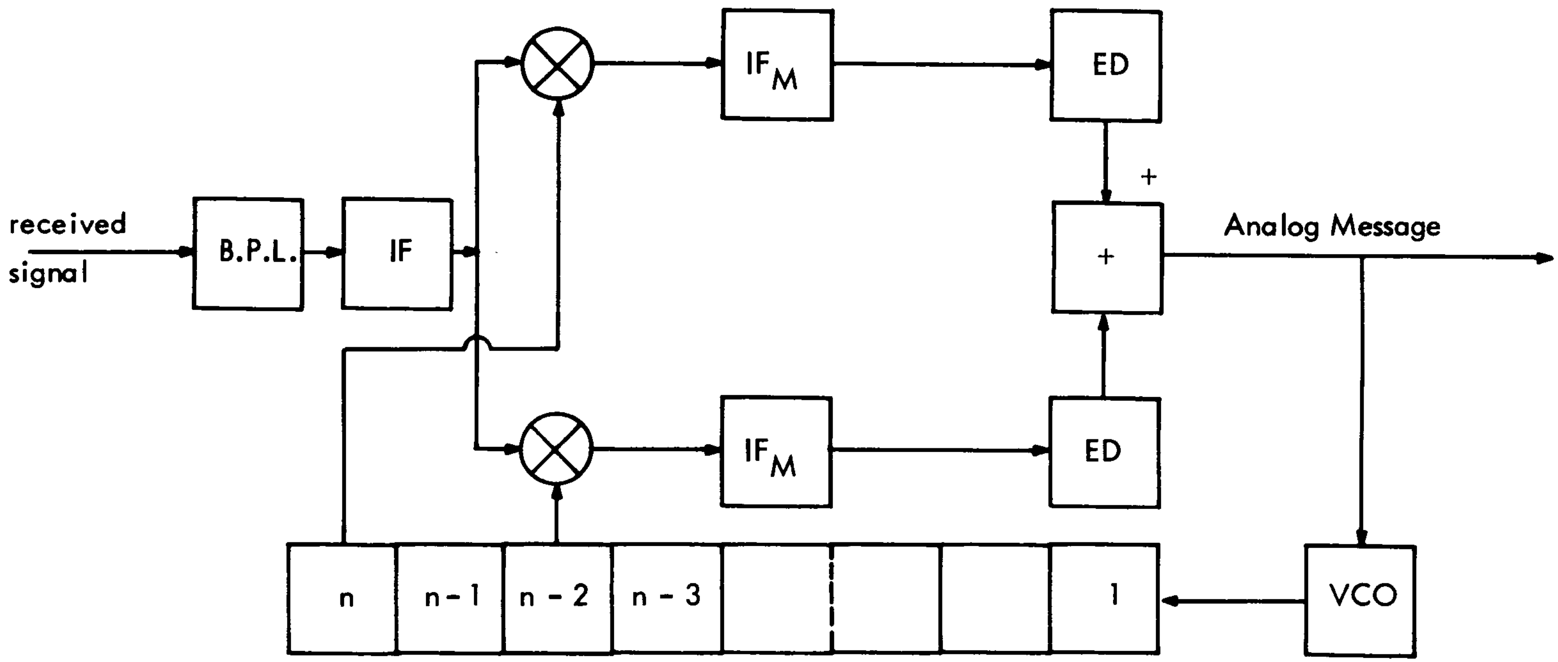


Figure 4-52. PN Rate Correlation Receiver

This technique of demodulation is the delay-lock discriminator equivalent to narrow-band phase detection by means of a conventional phase-locked loop.

PN MODEMS. It should be evident that a PN modulator and synchronized delay-lock discriminator represent a MODEM (i.e., modulator-demodulator pair). The MODEM establishes a "connection" between two terminals; the transmitter and receiver. Such a "connection" is equivalent to a wire line between the two terminals. Message transmission can now be digital or analog, synchronous or asynchronous. This property, combined with the fact that for large WT products the number of "good" signal addresses is very large, makes this technique attractive for random access communications via a stationary satellite.

4.3.2.4 Reception of Higher-Order Signal Alphabets

As shown in Section 4.3.1, a higher-order alphabet can be used in a correlation-locked system. In Figure 4-53 a synchronizing signal is assumed to be transmitted along with the message. If an orthogonal synchronizing signal is used then it must be added to the output of the balanced modulator. In this case the orthogonality among elements of the alphabet is preserved. However, if a binary synch signal is combined with the message signal at the output of the mod-2 adder by

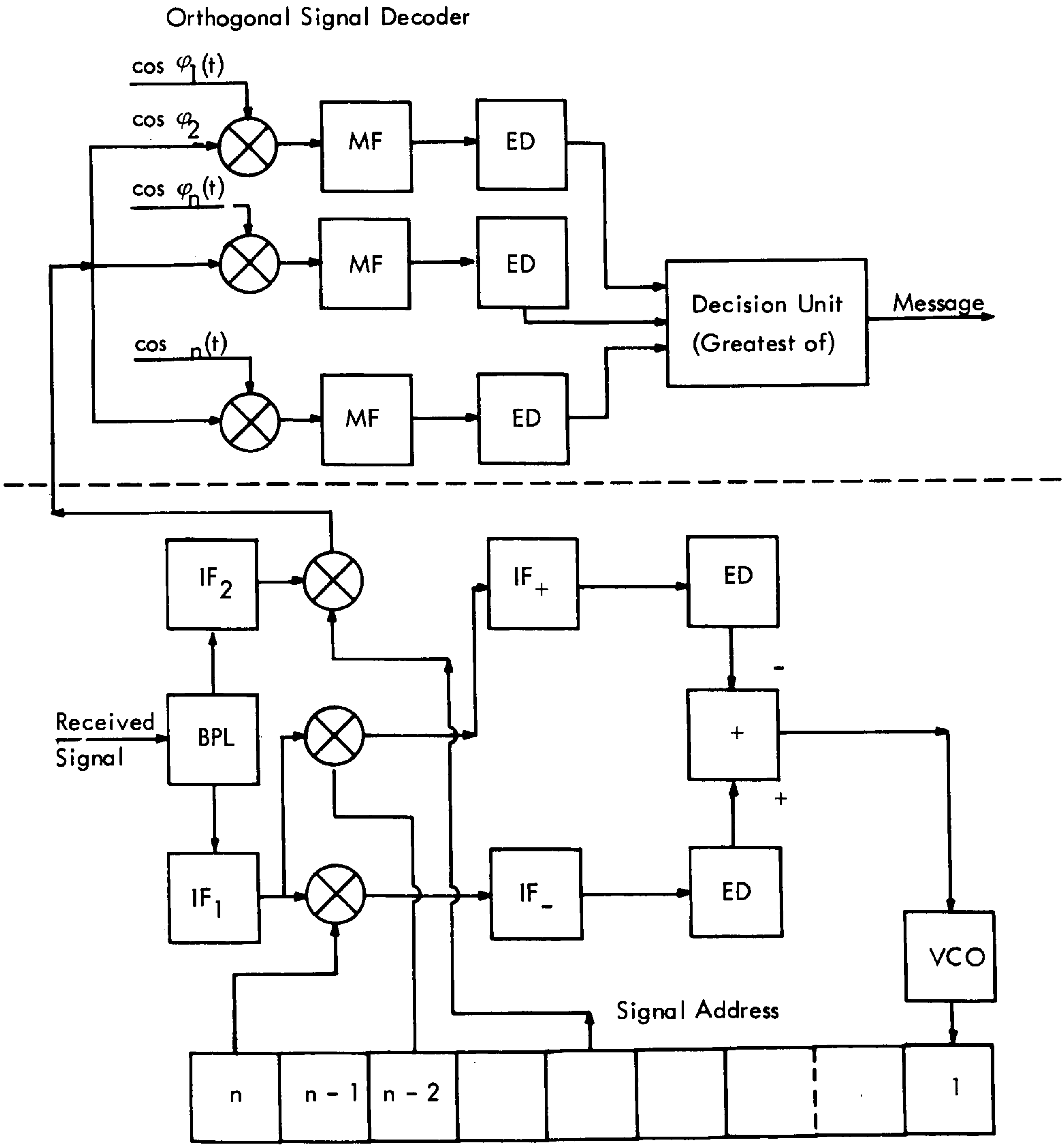


Figure 4-53. Correlation-Locked Receiver for Higher Order Orthogonal Binary Signals

means of majority logic, then the orthogonality is lost.

In Figure 4-53, the portion of the circuit at the top of the dashed line is the same as in the single channel case. The signal address code which is multiplied into the output of IF_2 removes the address code. Beyond this point the receiver of each subscriber is precisely the same. The locally generated references $\{\cos \phi_n(t)\}$ are all synchronous with the VCO and the locking PN sequence generator. Since the outputs of the array of multipliers are at bandpass, the message filters, MF, are all narrow-band IF filters which feed envelope detectors. The decision circuit decides that $\{\cos \phi_k(t)\}$ was transmitted if the kth channel has the largest output.

The codes $\{\cos \phi_k(t)\}$ can be binary or frequency shifts. If they are frequency shifted, then $\phi_k(t) = \omega_k t$. In this case the reference carriers are frequency shifted with respect to each other. This can be accomplished so that the bandpass message filters are all the same. On the other hand the multipliers and local references can be eliminated if the bandpass filters, MF, are tuned to the received signals in the alphabet. In this case the orthogonal signal decoder consists of a bank of narrow-band receivers tuned to different center frequencies.

4.3.2.5 Demultiplexing Many PN Subcarriers

Demultiplexing Using Delay Resolution. The multiplexer using delay resolution is shown in Figure 4-40. In all discussions which follow in this section, we will assume that a delay-lock discriminator is locked on to the transmitted reference signal. The techniques which have been discussed previously for a single message are just as applicable here. The reference signal can be orthogonal to the message signals or quasi-orthogonal. We will therefore concentrate only on the demultiplexing logic.

Figure 4-54 is a block diagram of the delay resolution demultiplexer. (The multiplexer is shown in Figure 4-40.) The message channel output IF_2 is fed into a parallel array of multipliers, one for each channel. The delay-lock discriminator which is coherent with the received reference feeds a digital shift register (delay line). The output from each tap is applied to the corresponding multiplier. The multiplier output is then fed into a message filter, MF, and then to a decoder. The PN subcarriers introduce mutual clutter in the channels. The amount of clutter in each channel is the same.

It should be recognized that the same system can be used for the case where the reference signal is quasi-orthogonal to the message. In this case the reference is just another binary signal in the majority-

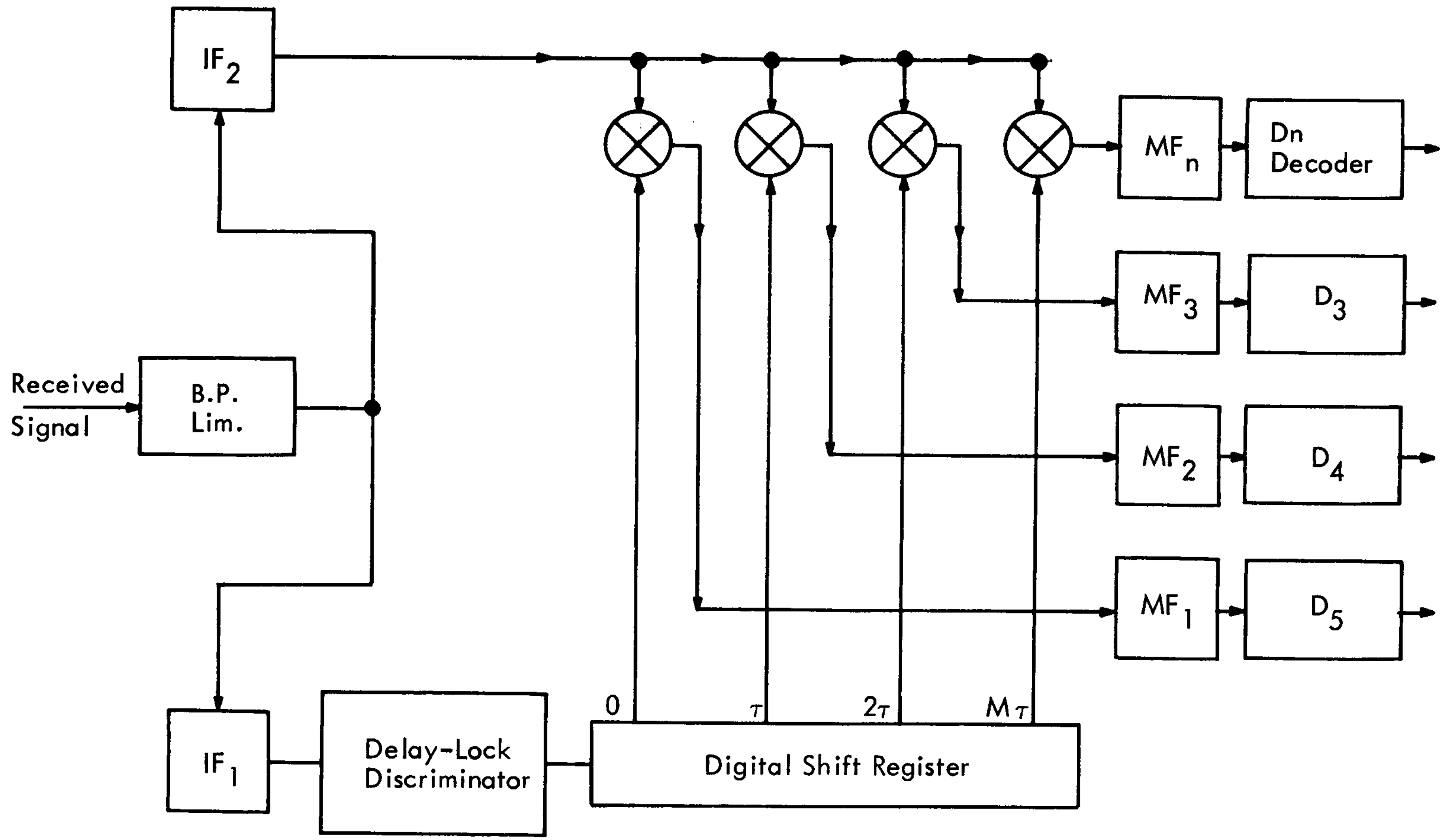


Figure 4-54. Demultiplexer Using Delay Resolution

logic multiplexer. Since it is unmodulated, however, a filter of much narrower bandwidth than the message filter can be used. Hence, for equal weighting the synchronization channel will have a substantially greater signal-to-noise ratio, a desirable situation.

General PN Subcarrier Demultiplexer. In order to separate the signals when multiplexed according to Figure 4-41, it is necessary to have a corresponding synchronous array of PN subcarriers at the receiver. The demultiplexing operation is the same as shown in Figure 4-54 for delay resolution except that the tapped shift-register can be replaced by a parallel array of PN reference signals.

Reception of Angle Modulated PN Subcarriers. We will now consider reception of an angle-modulated PN subcarrier signal. The transmitter is shown in Figure 4-42. In Figure 4-55, the delay-lock discriminator locks on to the reference signal. The received signal and the locally generated PN reference are fed into a balanced modulator where the PN subcarrier is removed from the received message. The narrow deviation angle demodulator is a conventional phase-locked loop. The loop has a filter whose bandwidth is equal to the SSB multiplexer band. The "error" voltage at the output of the message filter, MF, is the multiplexed SSB message. The SSB signal is then fed into a conventional SSB demultiplexer where each message is channelized.

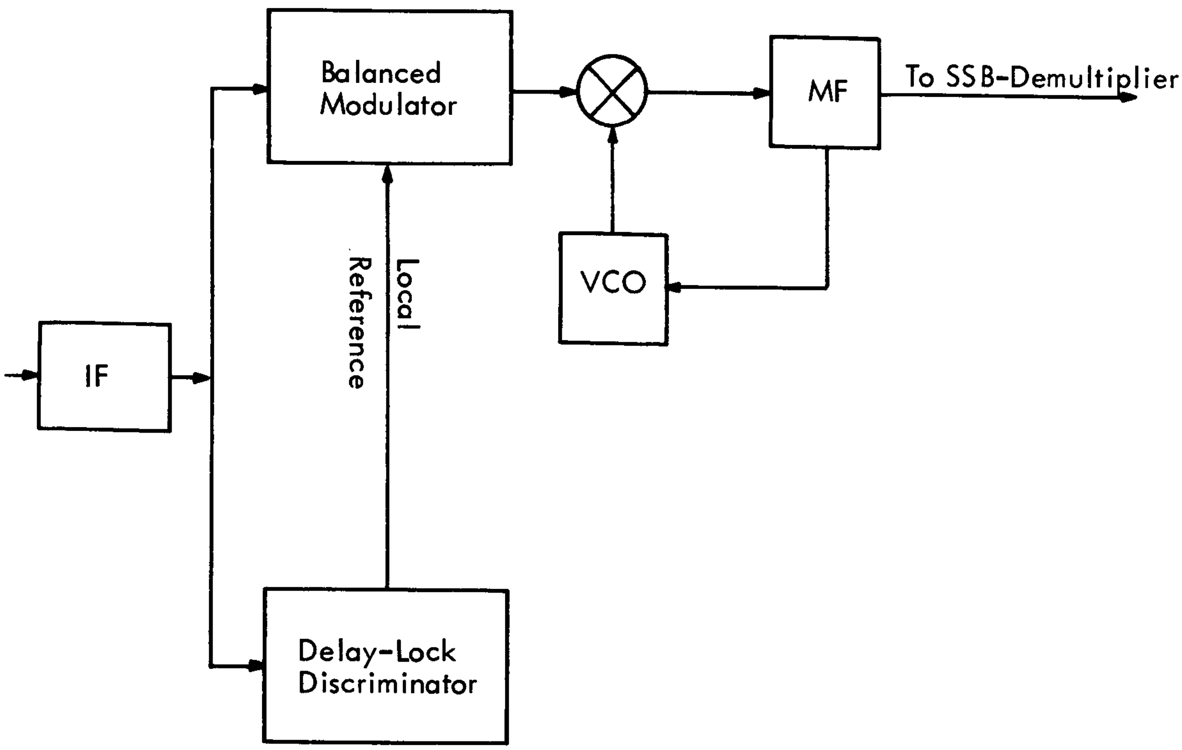


Figure 4-55. Receiver for Narrow Deviation Angle-Modulated PN Subcarrier

This receiver once again demonstrates that the PN subcarrier modulator and the delay-lock discriminator pair form a MODEM.

The transmitter of Figure 4-41 and the receiver discussed here represent an extremely useful configuration for a gateway type of stationary satellite communication system (and perhaps Telstar types as well), since conventional terminal equipment can be used. The PN subcarrier merely addresses the analog message so that it will be received at the intended station.

A large number of addresses exist which can be used to steer traffic to the intended receiving station. This technique will supply the routing of traffic through a repeater satellite network.

slots while the other portion selects the time slots. In particular, VCO-I and pulse position modulator PPM-I select the (f, t) coordinate which would correspond to, say, message symbol binary one. The complements select the (f, t) coordinate which would correspond to message symbol binary zero. A comparison and decision is made just as in the time-hopping case. This technique is more complex than the other two but also more flexible. Once again, synchronization must be maintained.

4.4.4 F-T Matrix Techniques

In a common channel system using f-t matrixing, each user is assigned an f-t pattern or signal address much like in the matched filter case. A connection is established when an address is received at the intended destination. The receiver samples the regions of the f-t plane where the desired signal is expected. If the signal is present in this portion of the f-t plane, a message element is detected. Binary messages or analog pulse position messages can be transmitted this way.

Figure 4-65 is a frequency matrixing transmitter and Figure 4-66 is the receiver. A signal address is selected by choosing an f-t pattern. This is accomplished by selecting, at the transmitter, a set of oscillator frequencies and a set of delay-line taps are chosen

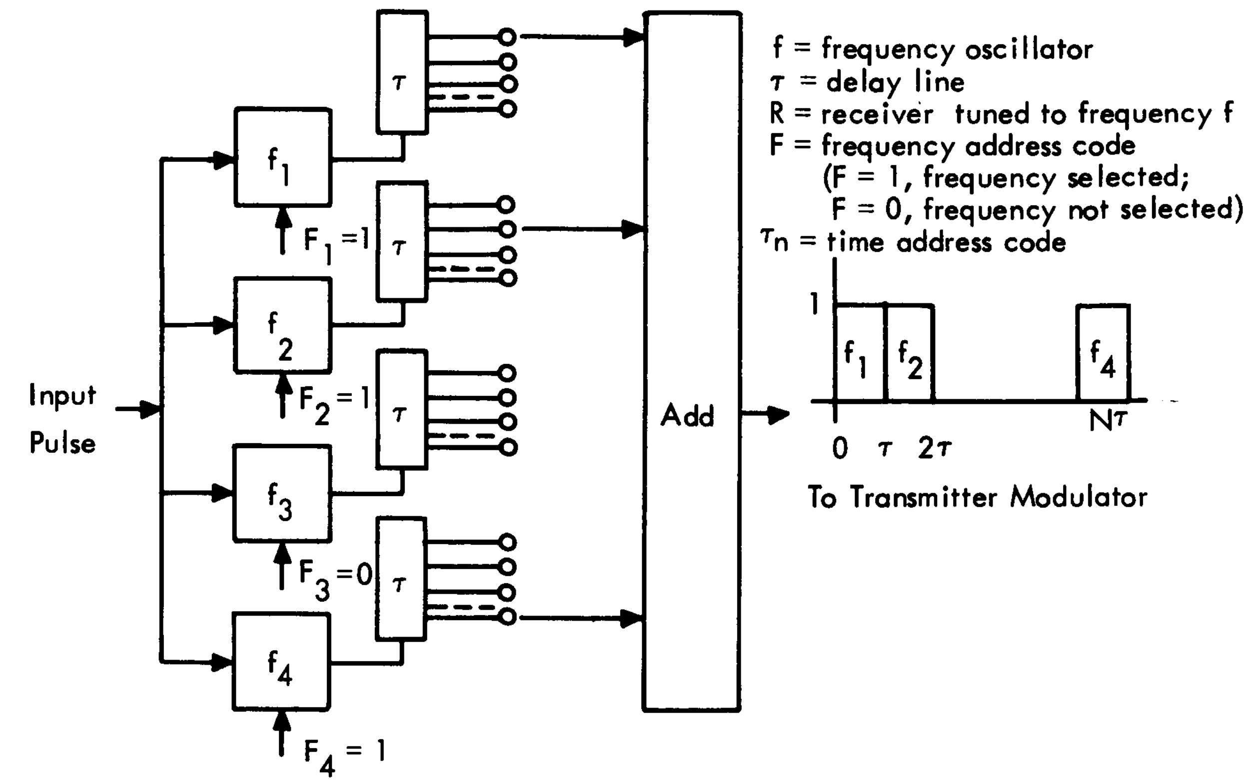


Figure 4-65. Frequency Matrixing Transmitter

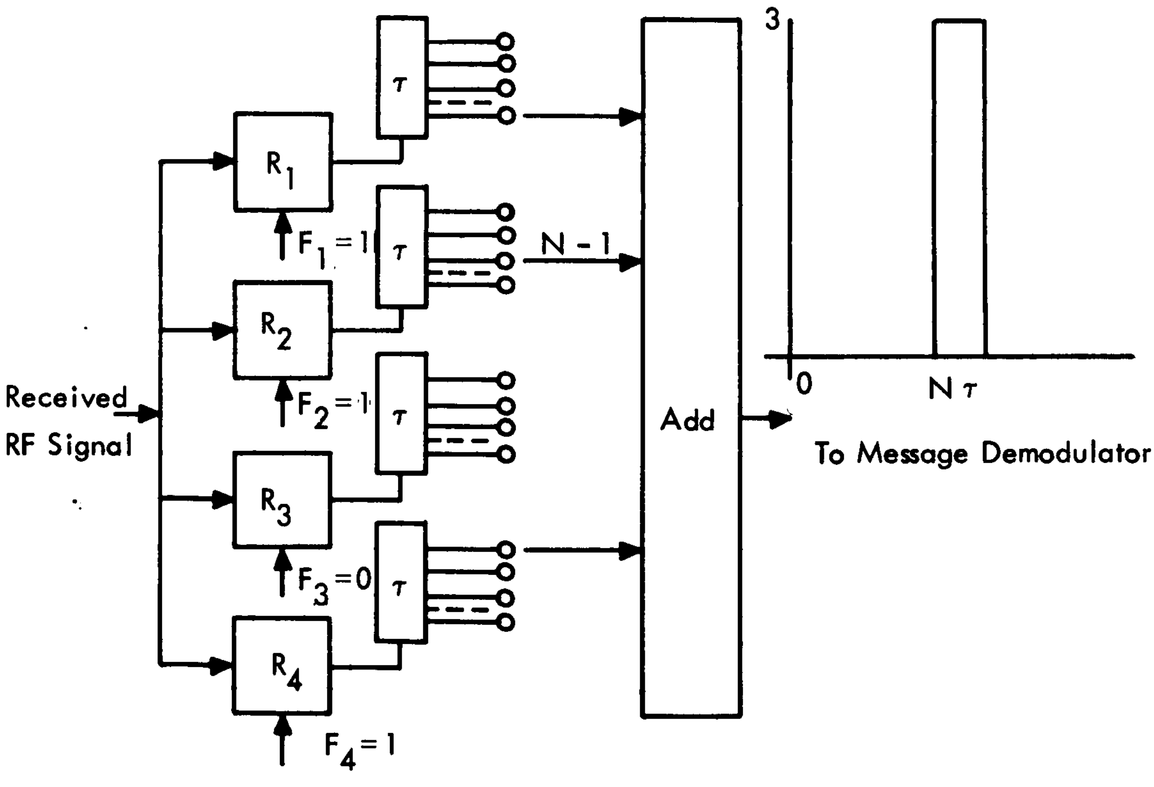


Figure 4-66. Frequency Matrixing Receiver

so that there is no time overlap. However if desired, more than one tap can be selected for a particular frequency. The message pulse keys the selected oscillators which drive, in turn, their respective delay lines. The output of the taps is summed and transmitted.

The receiver has an IF filter and envelope detector for each frequency and the same set of tapped delay lines as the transmitter. The receiver is connected to respond to a particular address. The tuned IF filters select the received frequencies. The IF is removed by the envelope detectors in the receivers. The delay line taps at the receiver are complementary to those at the transmitter; for example, the pulse which is transmitted first is delayed at the receiver so that it will arrive at the summing device in-phase with the pulse transmitted last, etc. The received pulse signal is shown in Figure 4-66. Of course, other addresses will cause some of the filters to respond, introducing interfering signals. The interfering signals, however, will not add in-phase; when they do, errors can occur.

4.5 Theory of Asynchronous* Multiplexing Using PN-Signals

In this sub-section we will develop the theory of asynchronous multiplexing using PN signals. Statistical methods of choosing almost orthogonal signal address alphabets of extremely large size will be developed and the significant results will be presented. The latter results were obtained in 1962 in an IBM sponsored program on the multiple-access problem.[16] The significant results of this work will be summarized. This work is fundamental to the signal design problem.

The signal-to-noise ratio at the output of a random access receiver will also be obtained here. In addition channel capacity theorems and error rates are obtained. The fundamental assumption here is that the clutter generated by mutual signal interference is a white Gaussian process. The inherent randomness characteristics of the random access signals combined with the receiver operations justify this assumption. Simulation results which have been obtained further support the assumption.[17]

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* The word "asynchronous" refers here to the random nature of the voice links relative to each other. It does not refer to the method of modulation on each individual voice link.

The preliminary results which are presented here are applicable to both matched filter and correlation-locked random access systems. A more complete set of performance measures will be obtained later in the program (Phase III) for different detection criteria. Performance curves will be presented at that time and these will be compared with the characteristics that are obtained from computer simulation.

4.5.1 Almost Orthogonal Signal Addresses

The mathematically strict concept of orthogonality is not entirely useful in common channel systems. That is, the mathematical relationship

$$\rho_{ij} = \frac{1}{T}\int_{-T/2}^{T/2} S_i(t)\, S_j(t)\, dt \begin{array}{l} = 1 \;;\; i = j \\ = 0 \;;\; i \neq j \end{array} \tag{4-51}$$

is not entirely satisfactory in common channel systems. The criterion on the mutual interference of the signals which must be satisfied in a common channel system is

$$\rho_{ij}(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} S_i(t)\, S_j(t+\tau)\, dt \tag{4-52}$$

with

$$\begin{aligned} |\rho_{ij}(\tau)| &\leq \rho_o \geq 0 \quad \text{for all} \begin{array}{ll} i \neq j & ; \quad \tau = 0 \\ (i,j) & ; \quad \tau \neq 0 \end{array} \\ &= 1 \;;\; i = j \;;\; \tau = 0 \end{aligned} \tag{4-53}$$

This requirement is far stronger than orthogonality. It requires that the autocorrelation and crosscorrelation functions be small everywhere except at the instant of match. Equivalently, Equation (4-53) is a condition on the cross-spectrum of the signals; that is, the absolute value of the cross-spectrum must be bounded.

$$|\rho_{ij}(\tau)| = \left| \int_{-\infty}^{\infty} F_i(\omega) F_j^*(\omega) e^{-j\omega\tau} d\omega \right| \leq \rho_o \geq 0 \qquad (4\text{-}54)$$

Conventional asynchronous "orthogonal" systems are of the frequency division type. The signals overlap strongly in the time domain but are almost disjoint in frequency. This property is achieved by having the cross-spectral intensities of the signals take on a small value. In Equation (4-54) this is shown as equivalent to having

$$|\rho_{ij}(\tau)| < \int_{-\infty}^{+\infty} |F_i(\omega)| \, |F_j(\omega)| \, d\omega < \rho_o \qquad (4\text{-}55)$$

Thus, frequency division systems are asynchronous multiplexed systems which achieve low crosscorrelation by imposing a constraint on the cross-spectral intensity; the phase spectrum is arbitrary.

In signals of large dimensionality, low crosscorrelation can be achieved by randomizing the phases of the signal components while the cross-spectral intensity may be strongly overlapping. Our ability to communicate through noise is due to the fact that the received signal component correlates completely only with the desired reference signal while the noise component does not. A major advantage here is that an extremely large number of such signals can be obtained having "good" correlation properties for signals having large dimensionality. The number of orthogonal signals, by comparison, is negligibly small. It is well known from modern communication theory that to communicate efficiently a large number of signals must be used that, in addition, are "far apart from each other."

As an example, let us consider signals having 100 bits per address. There are 2^{100} distinct signal addresses available. Of these, there are only 200 that are orthogonal in the strict mathematical sense; that is, the autocorrelation function and the mutual crosscorrelation function can have extremely large sidelobes. One can certainly expect that out of a possible 2^{100} signals there will be an extremely large subset that will exhibit random-like properties and hence guarantee (at least with a high probability) that the pair-wise RMS distance between them will be at least some acceptable number. This is one of the problems which is studied in an IBM report entitled "Synchronous and Asynchronous Multiplexing by Means of Almost Orthogonal Noisy Signal Alphabets."

4.5.1.1 Signal Address Selection -- Gaussian Case

The following question is considered here: What must the bandwidth-time product be to ensure that the random selection of a signal alphabet consisting of Gaussian sequences will contain elements that are at least a prescribed distance apart?

<u>Formulation of the Problem.</u> We assume that our waveform generator is a white Gaussian noise source of zero mean, limited to the low-pass bandwidth, W. Then, at the time points $t = \frac{n}{2W}$, the sample values are statistically independent. We will now choose from this source an ensemble of waveforms each of which contains 2WT samples. This particular waveform is given by the sequence of numbers

$$S_j = \{x_{ij}\} \quad \begin{matrix} i = 1, 2, \ldots, 2WT \\ j = 1, 2, \ldots, M \end{matrix} \tag{4-56}$$

The distance between any pair of waves is given by

$$d_{jk} = \sqrt{\frac{1}{2WT} \sum_{i=1}^{2WT} (x_{ij} - x_{ik})^2} \tag{4-57}$$

There will be

$$A(M) = \frac{M(M-1)}{2} \tag{4-58}$$

distinct pairs of distance measures where M is the number of waveforms. With an alphabet of M waveforms we can transmit

$$H = \log_2 M \quad \text{bits per waveform} \tag{4-59}$$

if each message sequence is equally probable.

We postulate that the random noise source has a mean square power σ^2 and, hence, an entropy

$$h_{max} = \log_e \sqrt{2 \pi e}\, \sigma \tag{4-60}$$

It is well known that for a given mean square power a Gaussian random noise source has the maximum entropy.[18]

Referring to Equation (4-57), if $\{x_{ij}\}$ is normal of mean zero and variance σ^2, then $\{x_{ij} - x_{ik}\}$ is also normal of mean zero and variance

$$\sigma_o^2 = 2\sigma^2 \tag{4-61}$$

and, hence, the distance measure has the chi distribution of (2WT) degrees of freedom. Thus, the probability density function of the distance, d, for any pair of waveforms is[19]

$$P_{2WT}(d) = \frac{2(WT)^{WT}}{\sigma_o^{(2WT)}\Gamma(WT)} d^{(2WT-1)} \exp\left[-\frac{WT}{\sigma_o^2} d^2\right] \quad \text{for all } d \geq 0 \tag{4-62}$$

$$= 0 \quad \text{for } d < 0$$

The probability that the distance, d, is at least as large as d_o, after suitable normalization, is given by[20]

$$P_{2WT}(d \geq d_o) = \frac{2}{\Gamma(WT)} \int_{\left(\frac{d_o}{\sigma_o}\right)\sqrt{WT}}^{\infty} y^{(2WT-1)} e^{-y^2} dy \tag{4-63a}$$

$$= \frac{2}{(WT-1)!} \int_{\left(\frac{d_o}{\sigma_o}\right)^2 WT}^{\infty} Z^{(WT-1)} \exp(-Z)\, dZ \tag{4-63b}$$

$$= \exp\left[-\left(\frac{d_o}{\sigma_o}\right)^2 WT\right]\left[1 + \left(\frac{d_o}{\sigma_o}\right)^2 WT + \frac{1}{2!}\left(\frac{d_o}{\sigma_o}\right)^4 (WT)^2 + \ldots + \frac{1}{(WT-1)}\left(\frac{d_o}{\sigma_o}\right)^{2(WT-1)} (WT)^{(WT-1)}\right] \tag{4-63c}$$

It is of particular interest to examine Equation (4-63) for large WT products. For a large number of degrees of freedom, the chi-distribution tends to normality. For $2WT \geq 30$, the normal distribution is within 1% of the chi.[20] For large WT, the expected value of d is[19]

$$E(d) = \sqrt{2}\ \sigma = \sigma_o \tag{4-64a}$$

and the variance is

$$E(d - \sigma_o)^2 = \frac{2\sigma^2}{4\,WT} = \frac{\sigma_o^2}{4\,WT} \tag{4-64b}$$

Then the probability density of the distance, d, is given by

$$P_{2WT}(d) = \frac{1}{\sqrt{2\pi}\left(\frac{\sigma_o^2}{4\,WT}\right)^{1/2}} \exp\left[-\frac{1}{2}\frac{(d-\sigma_o)^2}{\left(\frac{\sigma_o^2}{4WT}\right)}\right] \tag{4-65}$$

If we let

$$y = \frac{d-\sigma_o}{\left(\frac{\sigma_o}{2\sqrt{WT}}\right)} \tag{4-66}$$

then y is the normal of mean zero and unit standard deviation.

Probability that the distance between the waveforms is at least d_o is given by

$$P_{2WT}(d \geq d_o) = 1 - \int_{-\infty}^{2\sqrt{WT}\left[\frac{d_o-\sigma_o}{\sigma_o}\right]} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}y^2\right] dy \tag{4-67a}$$

$$\geq 1 - \frac{\exp\left[-2WT\left(\frac{d_o}{\sigma_o}-1\right)^2\right]}{2\sqrt{2\pi WT}\left|\frac{d_o}{\sigma_o}-1\right|} \tag{4-67b}$$

The probability that all M waveforms will be at least d_o apart is approximately given by

$$\equiv P_{2WT}^{(M)}(d \geq d_o) = \left[P_{2WT}(d \geq d_o)\right]^{\frac{M(M-1)}{2}}$$

$$\geq \left\{ 1 - \int_{-\infty}^{2\sqrt{WT}\left[\frac{d_o - \sigma_o}{\sigma_o}\right]} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}y^2\right] dy \right\}^{\frac{M(M-1)}{2}} \tag{4-68a}$$

$$\geq 1 - \frac{M(M-1)}{2} \frac{\exp\left[-2WT\left(\frac{d_o}{\sigma_o} - 1\right)^2\right]}{2\sqrt{2\pi WT}\left|\frac{d_o}{\sigma_o} - 1\right|} \tag{4-68b}$$

Case 1: $d_o > 1$ "Negative Correlation Case"

For the purpose of the following discussion we will assume that $\sigma_o = 1$, without any loss of generality.

Let us now study the behavior of Equation (4-68). If $d_o > 1$, then the upper limit in Equation (4-68) is positive and the

$$P_{2WT}^{(M)}(d > 1) \to 0$$

extremely rapidly as $WT \to \infty$. This case is equivalent to the deterministic case of requiring that all waveforms be negatively correlated. It is well known that in the deterministic case the negative correlation between the waveforms tends to zero as $1/M$. Hence for large WT we cannot achieve such a set from this statistical mode. Case 1 is shown graphically in Figure 4-67.

Case 2: $d_o = 1$ "Orthogonal Case"

The case where $d_o = 1$ corresponds to the orthogonal deterministic case (zero crosscorrelation coefficient). From Equation (4-68) it is seen that for $d_o = 1$, the probability is independent of WT for large WT and approaches a finite value. That is

$$P_{2WT}^{(M)}(d \geq 1) = \exp_2 \left[-\frac{M(M-1)}{2} \right]$$

where

$$\exp_2 x \equiv 2^x$$

It is clear from Equation (4-69) that this probability goes to zero exponentially with the number of waveforms when WT is large. Thus, for all practical purposes, large signal alphabets cannot be chosen which are orthogonal. Orthogonality is a deterministic property rather than a statistical one. This situation is illustrated in Figure 4-68.

Case 3: $0 < d_o < 1$ "Almost Orthogonal Case"

The interesting, almost orthogonal case, $d_o < 1$, has a negative upper limit and, hence,

$$P_{2WT}^{(M)}(d > d_o) \rightarrow 1 \text{ as } WT \rightarrow \infty$$

for any finite value of M. This is shown graphically in Figure 4-69.

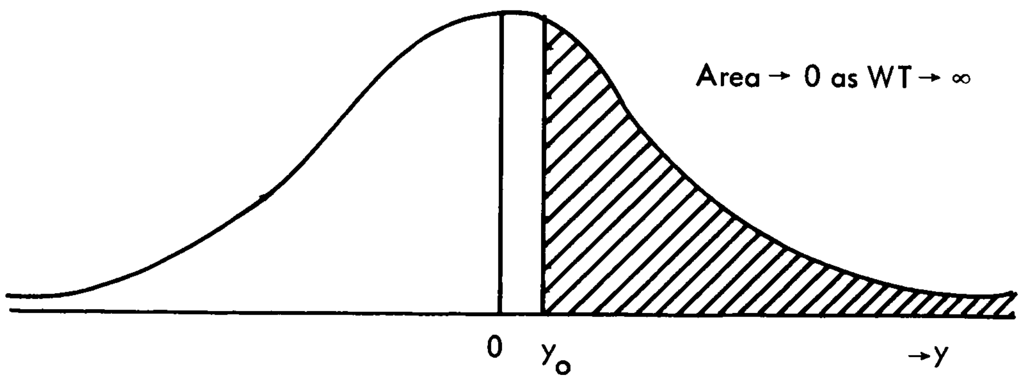

Figure 4-67. Graphical Illustration of Case 1

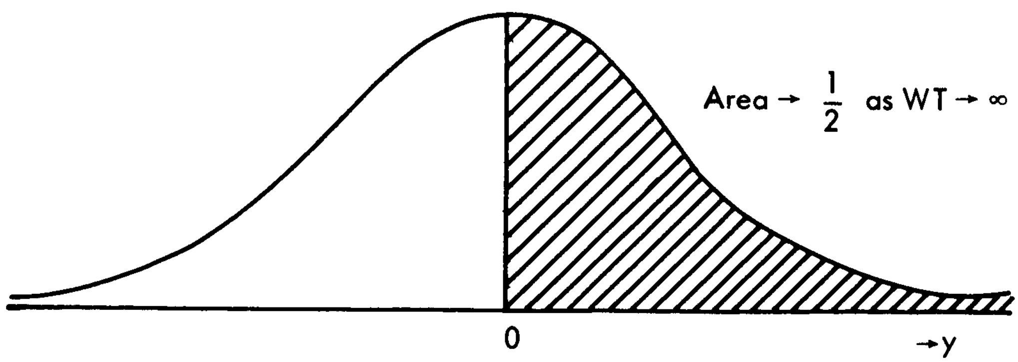

Figure 4-68. Graphical Illustration of Case 2

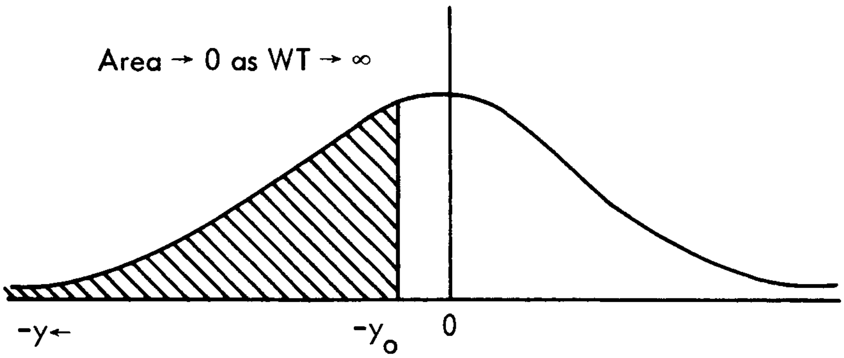

Figure 4-69. Graphical Illustration of Case 3

If we take logarithms of Equation (4-68) we have

$$\log p^{(M)} = \frac{M(M-1)}{2} \log \left\{ 1 - \int_{-\infty}^{2\sqrt{WT}\left(\frac{d_o}{\sigma_o} - 1\right)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) dy \right\} \tag{4-69}$$

For $\frac{d_o}{\sigma_o} < 1$ and WT large we have

$$\log p^{(M)} \approx - \frac{M(M-1)}{2} \int_{-\infty}^{2\sqrt{WT}\left(\frac{d_o}{\sigma_o} - 1\right)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) dy \tag{4-70}$$

Making use of the asymptoic expression for the normal integral, we have,

$$\log p^{(M)} \approx - \frac{M(M-1)}{2} \frac{\exp\left[-2\,WT\left(\frac{d_o}{\sigma_o} - 1\right)^2\right]}{2\sqrt{2\pi WT}\,\left|\frac{d_o}{\sigma_o} - 1\right|} \quad ; \quad \frac{d_o}{\sigma_o} < 1 \tag{4-71}$$

Solving Equation (4-71) for M where $M >> 1$ gives

$$\begin{aligned} M &\approx 2\,(2\pi WT)^{1/4} \left|\frac{d_o}{\sigma_o} - 1\right|^{1/2} \left(\log \frac{1}{p^{(M)}}\right)^{1/2} \exp\left\{WT\left(\frac{d_o}{\sigma_o} - 1\right)^2\right\} \\ &\approx 2\,(2\pi WT)^{1/4} \left|\frac{d_o}{\sigma_o} - 1\right|^{1/2} \gamma_M^{1/2} \exp\left\{WT\left(\frac{d_o}{\sigma_o} - 1\right)^2\right\} \end{aligned} \tag{4-72}$$

where $p^{(M)} = 1 - \gamma_M$ and $\sigma_M << 1$.

It is seen from Equation (4-27) that as long as $\frac{d_o}{\sigma_o} < 1$, the almost orthogonal requirement, the number of signal waveforms grows exponentially with WT. The number of message bits per signal waveform is from Equation (4-59)

$$H(T) = \log_2 M(T) = \log_2 \{2(2\pi WT)^{1/4} \left|\frac{d_o}{\sigma_o}\right|^{1/2} (\log \frac{1}{p(M)})^{1/2}\} + (\log_2 e) WT \left(\frac{d_o}{\sigma_o} - 1\right)^2$$

$$= \log_2 \{2(2\pi WT)^{1/4} \left|\frac{d_o}{\sigma_o}\right|^{1/2} \gamma_M^{1/2}\} + (\log_2 e) WT \left(\frac{d_o}{\sigma_o} - 1\right)^2 \qquad (4\text{-}73a)$$

where H(T) is the number of message bits in time T which must be mapped into the signal alphabet. Note that

$$h_s = \lim_{T \to \infty} \frac{H(T)}{T} = (\log_2 e)\, W \left(\frac{d_o}{\sigma_o} - 1\right)^2 \qquad (4\text{-}73b)$$

Given a source generating information at the rate h_s bits per second, the bandwidth required for the signal alphabet in order to encode the message source is

$$W = \frac{h_s}{(\log_2 e)\left(\frac{d_o}{\sigma_o} - 1\right)^2} = \frac{1}{T} \quad \frac{H(T)}{(\log_2 e)\left(\frac{d_o}{\sigma_o} - 1\right)^2} \qquad (4\text{-}74)$$

Equation (4-74) is, therefore, the channel bandwidth required to transmit h_s. Note that the channel bandwidth required for the almost orthogonal waves is directly proportional to the source rate. The encoding is therefore efficient as far as channel bandwidth utilization is concerned. In fact, we can send at the rate W if

$$\left(\frac{d_o}{\sigma_o} - 1\right)^2 = \frac{1}{\log_2 e} = \log_e 2 \tag{4-75}$$

Since $\left(\frac{d_o}{\sigma_o} - 1\right)^2 < 1$, Equation (4-75) is valid. It is still necessary to find the error rate as a function of the parameters of Equation (4-73). However, prior to doing this, let us briefly consider the bandwidth required when orthogonal waves are used.

Assume that the source rate is h_s bits per second. In time T, the total number of bits generated is $h_s T$. If it is desired to encode $h_s T$ bits, then $\exp_2[h_s T]$ waveforms are required. The waveforms can be made orthogonal over the interval T requiring a bandwidth per waveform approximately $\Delta f = \frac{1}{T}$. Thus, the total bandwidth required for encoding into orthogonal waveforms is

$$W_o = \frac{\exp_2[h_2 T]}{T} \quad \frac{\exp_2[H(T)]}{T} \tag{4-76}$$

Thus, the bandwidth required to encode a message source of rate h_s grows exponentially with the sequence length while in the almost orthogonal case the bandwidth is directly proportional to the sequence length for large T. In the almost orthogonal case, the number of possible waveforms grows at the same rate as the number of possible message sequences as a function of waveform duration.

We have found a simple method of obtaining almost orthogonal waveforms, the number of which grows exponentially with the bandwidth time product. With this set it is possible to encode longer and longer message sequences efficiently as far as bandwidth is concerned since the waveform set grows at the same rate with time duration as the message sequence set. In order to achieve a waveform set of the required size, with probability $p^{(M)}$ and distance $\frac{d_o}{\sigma_o} < 1$, it is only essential to choose a sufficiently large number, $2\,WT$, of independent samples from a white Gaussian noise source. We are certain, with probability $p^{(M)}$ arbitrarily close to unity, that the entire set of M waveforms will be at least d_o units apart.

Entropy Considerations. Let us examine for a moment the further implications of the inequality $\frac{d_o}{\sigma_o} < 1$. From Equation(4-60)

$$\sigma_o = \frac{1}{\sqrt{2\pi e}} \exp h_{max} \tag{4-77}$$

Hence,

$$\frac{d_o}{\sigma_o} = \sqrt{2\pi e}\; d_o \;\exp(-h_{max}) < 1 \tag{4-78}$$

Thus,

$$\log \frac{d_o}{\sigma_o} = \log \sqrt{2 \pi e} \; d_o - h_{max} < 0$$

or

$$h_{max} - h_o > 0 \tag{4-79}$$

when

$$h_o \equiv \log \sqrt{2 \pi e} \; d_o \tag{4-80}$$

Maximum Likelihood-Sequences (MLS). We will now attempt to obtain more mathematical and physical insight into the nature of the noise-like signals which permit random selection of these to yield, almost certainly, a "good" set. It is clear from Equation (4-57) that the distance, d, is a maximum likelihood estimator of the standard deviation $\sqrt{2}\,\sigma$ of the normal population from which the samples have been selected. From maximum likelihood estimation theory it follows that h_o, Equation (4-80) is a maximum likelihood estimator of the source entropy h_{max}. The source in this case is Gaussian of variance $2\,\sigma^2$ corresponding to the difference between two random variables. Thus, measuring the distance between the two randomly selected Gaussian variables is equivalent to obtaining a maximum likelihood estimate of the standard deviation of the equivalent normal population. A maximum likelihood estimator is obtained by finding the parameter of the distribution σ^* which maximizes the probability of occurrence of the observed sequence (hence the name Maximum Likelihood-Sequences (MLS)). Furthermore, since the estimator is consistent, it is certain to converge to the standard deviation $\sqrt{2}\ \sigma$ and hence to the maximum distance. Thus, the random Gaussian sequences reflect the statistics of the source, in this case the standard deviation or equivalently the source entropy, and are in fact the sequences which have almost all of the probability of occurrence associated with them. The subset of sequences which do not reflect the source entropy form a larger set whose total probability is much less than that of the MLS. Our results are mathematical verifications of the physical arguments. We conjecture at this point that we need not measure the pairwise distance between the selected sequences which require $\frac{1}{2}M(M-1)$ measurements; it is sufficient to measure the estimator of the variance or equivalently the entropy of each sequence. If the estimate of the standard deviation of each sequence is $d \geq \frac{d_o}{2}$, we conjecture that the entropy $h \geq h_o$ of each sequence is such that the pairwise distance measure, Equation(4-57), will almost surely be satisfied. If this conjecture is valid, only M measurements are required.

The random sequences have certain other properties which are of interest and which may shed some light on the general problem of finding signal alphabets that have a good crosscorrelation property. Except for end effects, because of the finite duration of the sequences, statistically the same results will be obtained under autocorrelation as under cross-correlation. In communications, this property is desirable.

When random-noise waves are used, it is clear that the distance measure operation on each element of the alphabet has the same statistics as the distance measure operation between pairs. This appears to be some form of closure property. (Geometrically, the distance to each signal measured from the origin has the same functional form for the statistics as the distance between signals.)

For binomial sequences (discussed later) a similar property is observed if the measure of distinguishability is the absolute value of the crosscorrelation coefficient. Of course, zero-crosscorrelation coefficients represent maximum distinguishability while unity represents zero distinguishability. When two binomial sequences are multiplied, another binomial sequence is obtained having the same properties. The distinguishability is merely the absolute value of the difference between the number of ones and the number of minus ones. The same measure makes sense when applied to each sequence or to the result of multiplying two binomial sequences. Here too, we observe a closure property. The insight gained from the random sequences concerning crosscorrelation properties should shed some light on possible deterministic procedures. However, in light of the results presented here, there is a valid question if a search for deterministic procedures is even justified. The results show that for, say $2WT \geq 30$, random selection procedures yield excellent results. For $2\ WT \leq 10$, orthogonal alphabets are adequate. For some applications it is possible that deterministic procedures for choosing almost orthogonal signal alphabets will be useful.

4.5.1.2 Signal Address Selection - Bernoulli Case

The theory thus far developed assumed that the alphabet of signals was normal random variables . For the WT>> 1, we will now find equivalent relationships for the case where the alphabets are selected from a Bernoulli random source. Such a source can be derived from a Gaussian noise source by the operation,

$$x_i = R[f_i] \begin{cases} = 1 ; & f_i \geq 0 \\ = 0 ; & f_i < 0 \end{cases} \tag{4-81}$$

The distance between two binary sequences $\{x_i\}$, $\{y_i\}$ $i = 1, 2, \ldots 2WT$ is defined just as previously

$$d = \left[\frac{1}{2WT} \sum_{i=1}^{2WT} (x_i - y_i)^2 \right]^{1/2} \tag{4-82}$$

where x_i, y_i take on the values (1, -1) with probability 1/2. Expanding Equation (4-82) gives

$$d = \sqrt{2}\left[1 - \frac{1}{2WT}\sum_{i=1}^{2WT} x_i y_i\right]^{1/2}$$

$$\approx \sqrt{2}\left[1 - \frac{1}{4WT}\sum_{i=1}^{2WT} x_i y_i\right] \text{ when } 2WT >> 1. \qquad (4\text{-}83)$$

For $2WT >> 1$, using Equation (4-83) d is normal of mean

$$E(d) = \sqrt{2} \qquad (4\text{-}84a)$$

and variance

$$E(d - \sqrt{2})^2 = \frac{1}{4WT} \qquad (4\text{-}84b)$$

From Equations (4-65), (4-66), and (4-67)

$$P_{2WT}(d) = \frac{1}{\sqrt{2\pi}\left(\frac{1}{2\sqrt{WT}}\right)} \exp\left[-\frac{1}{2}\left(\frac{d - \sqrt{2}}{\frac{1}{2WT}}\right)\right]^2 \qquad (4\text{-}85)$$

$$y = \frac{d - \sqrt{2}}{\frac{1}{2\sqrt{WT}}} \qquad (4\text{-}86)$$

and

$$P_{2WT}(d \geq d_o) = 1 - \int_{-\infty}^{2\sqrt{WT}(d_o - \sqrt{2})} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2} y^2\right] dy \; ; \text{ where } \frac{d}{\sqrt{2}} < 1. \qquad (4\text{-}87)$$

Hence, for $WT >> 1$, Equation (4-87) for the binomial case yields a result which is equivalent to Equation (4-67) for the normal case. From this point on, all the results previously developed for the normal case apply to the binomial case as well.

From Equation (4-72) the total number of binary signals at least d_o apart is

$$M = 2(2\pi WT)^{1/4} |d_o - \sqrt{2}| \left(\log \frac{1}{p(M)}\right)^{1/2} \exp_2 WT \left[(d_o - \sqrt{2})^2 \log_2 e\right] \tag{4-88}$$

The total number of binary sequences which can be obtained is

$$M_{max} = 2^{2WT} \tag{4-89}$$

Thus,

$$\frac{M}{Max} = 2 \; (2\pi WT)^{1/4} |d_o - \sqrt{2}| \left(\log \frac{1}{p(M)}\right)^{1/2} \exp_2 WT \left[(d_o - \sqrt{2})^2 \log_2 e - 2\right] \tag{4-90}$$

Since $\frac{M}{Max} \leq 1$, it is essential that

$$(d_o - \sqrt{2})^2 \log_2 e < 2$$

or

$$\left(\frac{d_o}{\sqrt{2}} - 1\right)^2 < \log_e 2 \tag{4-91}$$

Since $\frac{d_o}{\sqrt{2}} < 1$, we have

$$1 - \sqrt{\log_e 2} < \frac{d_o}{2} < 1 \tag{4-92}$$

It should be recognized that the lower limit comes about as a result of the normal approximation to the binomial case for large values of WT. One might expect such behavior since a normal random variable will yield more waveforms having a greater entropy per degree of freedom. Since we make the normal approximation, it is clear that M, as given in Equation (4-88), is an upper bound for the binomial case. Thus, Equation (4-91) merely gives the lower bound on d_o for which the approximation makes sense.

Let us assume that $\frac{d_o}{\sqrt{2}} = 1/2$. Then, from Equation (4-88)

$$M \sim 2^{(.7WT)} \tag{4-93a}$$

and, from Equation (4-90)

$$\frac{M}{Max} \sim 2^{-(1.30\ WT)} \tag{4-93b}$$

For WT = 100, $M \sim 2^{70}$ and $\frac{M}{Max} \sim 2^{-130}$. Thus, for large WT the set of desired waveforms, M, is extremely large but still only a negligibly small subset of all the possible waveforms. This example demonstrates that the total number of waveforms that satisfy a certain distance relationship is extremely large although only a negligible subset of the total number of waveforms possible.

4.5.2 Power Constraints for In-Satellite Power Amplifier

The TWT in the satellite is a complex broad band amplifier which constrains the total power transmitted down. The detailed properties of the TWT amplifier are discussed in Section 5. Here we will simply consider the power division characteristics of an ideal power amplifier.

In addition, we will consider the power division characteristics of a hard limiter. The hard limiter, too, is a complex device from the analytical point of view and will be studied in the section on special devices.

4.5.2.1 Multiplexing In a Linear Amplifier

The total number of signals entering the linear amplifier is

$$S_{1n}(t) = \sum_{i=1}^{K} S_i(t) \tag{4-94}$$

where

$$S_i(t) = A_i \cos [\omega_o t + \Theta_i(t)] \tag{4-95}$$

and K is the number of signals of amplitude A_i, phase $\Theta_i(t)$ and carrier frequency ω_o. The output of the amplifier is,

$$S_o(t) = \sqrt{G} \sum_{i=1}^{K} A_i(\cos \omega_o t + \Theta_i(t)) \quad ; G = \text{power gain} \tag{4-96}$$

The power output of the amplifier is

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} S_o^2(t)\, dt$$

$$= G \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left[\sum_{i=1}^{K} A_i \cos\left(\omega_o t + \Theta_i(t)\right) \right]^2 dt$$

$$= G \sum_{i=1}^{K} \sum_{j=1}^{K} A_i A_j \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos[\omega_o t + \Theta_i(t)] \cdot \cos[\omega_o t + \Theta_j(t)]\, dt \tag{4-97}$$

Now,

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos[\omega_o t + \Theta_i(t)] \cos[\omega_o t + \Theta_j(t)]\, dt = \frac{1}{2} \quad i=j$$

$$= 0 \quad i \neq j \tag{4-98}$$

Then

$$P = G \sum_{i=1}^{K} \frac{A_i^2}{2} \tag{4-99}$$

where P is the satellite power constraint. Let the average input power be

$$\overline{A^2} = \frac{1}{K} \sum_{i=1}^{K} \frac{A_i^2}{2} \tag{4-100}$$

Then,

$$P = G \cdot K \overline{A^2} \tag{4-101}$$

and

$$G = \frac{P}{K \overline{A^2}} \tag{4-102}$$

The i^{th} output signal of the amplifier therefore has the form,

$$S_{oi}(t) = \sqrt{\frac{A_i^2}{\overline{A^2}} \frac{P}{K}} \cos[\omega_o t + \Theta_i(t)] \tag{4-103}$$

If all signals have equal power going into the amplifier then from Equation (4-100),

$$\overline{A^2} = \frac{A^2}{2} \tag{4-104}$$

and a typical output signal has the form,

$$S_{oi}(t) = \sqrt{2\frac{P}{K}} \cos(\omega_o t + \Theta_i(t)) \tag{4-105}$$

and the output power in the signal is simply,

$$\overline{A^2} = \frac{P}{K} \quad \text{for all signals.} \tag{4-106}$$

The simplified analysis clearly illustrates the power division in the satellite. Thus, if more signals are added to the input each will emerge with a decrease in power output.

Let us assume that signals at the input to the satellite are pulsed of duty factor d. Then, the output power is

$$P = dG \sum_{i=1}^{K} \frac{A_i^2}{2} \tag{4-107}$$

and from Equation (4-101),

$$G = \frac{P}{dK_d \overline{A^2}} \tag{4-108}$$

where

$$K_d = \frac{K}{d} \tag{4-109}$$

The quantity K is the number of unity duty factor signals which will yield an output power P while K_d is the number of duty factor d which will yield the same output power. Thus, for the same power in the down link ($\frac{K}{d}$) pulsed signals can be accommodated.

The TWT problem is a far more complex device than this analysis indicates and will receive special attention in the Special Devices Section. These mathematical relationships are, however, adequate for the signal-to-noise ratio calculations in random access systems.

4.5.2.2 Multiplexing in a Hard Limiter

It is desirable to precede the TWT by a hard limiter since this signal will present to the tube a more constant level input. Hard limiting degrades the performance of random access techniques only slightly. The distortion generated in the desired frequency band will resemble thermal noise as far as random access systems are concerned. In the signal-to-noise ratio analysis, this distortion has the effect of degrading the ground-based receiver slightly.

Mathematical analysis of the output of a hard limiter shows that it contains the desired signal plus a distortion component involving intermodulation products, some of which will be outside the signal band. Let the output signal have the form,

$$S_o(t) = \sum_{i=1}^{K} A_i \cos\left[\omega_o t + \theta_i(t)\right] + d(t) \tag{4-110}$$

The total power output must be a constant. Then, by computing the output power as in the case of the linear amplifier, (assuming that d(t) and the signal components are uncorrelated) we have,

$$P = \sum_{i=1}^{K} \frac{A_i^2}{2} + \overline{D^2} \tag{4-111}$$

where,

$$\overline{D^2} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} d^2(t)\,dt \tag{4-112}$$

From Equation (4-111) it is seen that the effective signal power is now reduced by $\overline{D^2}$. Using Equation (4-110) and (4-111) we have,

$$P = K\overline{A^2} + \overline{D^2} \tag{4-113}$$

Equation (4-110) then becomes,

$$S_{oi}(t) = \sqrt{\frac{P}{K}\left(1 - \frac{\overline{D^2}}{P}\right)} \left\{ \sum_{i=1}^{K} \frac{A_i}{\sqrt{\overline{A^2}}} \cdot \cos\left[\omega_o t + \theta_i(t)\right] + \sqrt{\frac{\overline{D^2}}{\frac{P}{K}\left(1 - \frac{\overline{D^2}}{P}\right)}}\, d(t) \right\} \tag{4-114}$$

When $\frac{\overline{D^2}}{P} << 1$, the power in the distortion term is negligible compared to the total power, we obtain an equivalent result to multiplexing in a linear amplifier. This condition is generally satisfied in random access systems; hence, the limiter appears to be a linear multiplexer with the indicated power division. In principle it is therefore possible to supply a constant level signal to the TWT.

In general, the limiter should be followed by a bandpass amplifier so as to filter the limited signal in the neighborhood of the major zone. This results in a fluctuating signal although the fluctuation has been reduced. The fluctuation can be further reduced by further limiting and bandpass filtering. This problem will be investigated during the next phase.

4.5.3 Signal-to-Noise Ratio at the Output of the Receiver

The signal-to-noise ratio at the output of a random access receiver will now be obtained. This measure is of fundamental importance in these systems since the important theoretical performance criteria depend on it.

4.5.3.1 General Calculations

The signals are represented here in complex notations (i.e., the analytical signal representation is used). This is a mathematical convenience which simplifies the operations.

The general mathematical expression for the n^{th} signal address is

$$z_n(t) = A_n \exp\{j[\omega_o t + n\Delta\omega t + \phi_n(t)]\} \; ; \quad 0 \leq t \leq T \qquad (4\text{-}115)$$

$$n = 1, 2, \ldots, M$$

where

$$T = \text{time duration of signal}$$

$$\Delta\omega = \frac{2\pi}{T} = \text{frequency shift}$$

$$\omega_o = \text{carrier (or IF) frequency}$$

$$\phi_n(t) = \text{pseudo random phase modulation.}$$

This angle can contain a message component as well, which varies at a much slower rate. Of particular interest is,

$$\exp\{j\phi_n(t)\} = (1, -1, \ldots, -1, -1, 1) = N \text{ bit pseudo random signal} \qquad (4\text{-}116)$$

Then,

$$\Delta T = \frac{T}{N} = \text{time duration of message bit}$$

$$W = \frac{1}{2\Delta T} = \frac{N}{2T} = \text{ideal low-pass bandwidth.}$$

The received signal has the form,

$$z_R(t) = z_n(t) + z_c(t) + n(t) \qquad (4\text{-}117)$$

$z_c(t)$ = clutter signal

$n(t)$ = complex white gaussian noise process of spectral density N_o watts per cps.

The clutter is given by,

$$z_c^{(N)}(t) = \sum_{\substack{p \\ p \neq n}} A_p \exp\{j[(\omega_o + p\Delta\omega)(t - T_p) + \phi_p(t - T_p)]\} + n(t) \qquad (4\text{-}118)$$

The output of the matched filter is,

$$Z_n(\tau) = \int_o^T z_n(t) z_n^*(t+\tau)dt + \int_o^T z_c(t) z^*_n(t+\tau)dt$$

$$+ \int_o^T n(t) z^*_n(t+\tau)dt \qquad (4\text{-}119)$$

At the instant of match the output of a matched filter (or active correlator) is given by,

$$Z_n(0) = \int_o^T |z_n(t)|^2 \, dt + \int_o^T z_c(t) z^*_n(t)dt + \int_o^T n(t) z_n^*(t)dt \qquad (4\text{-}120)$$

The predetection signal-to-noise (plus clutter) power ratio is,

$$\eta^2 = \frac{\left| \int_o^T |z_n(t)|^2 \, dt \right|^2}{\frac{1}{2} \left| \int_o^T [z_c(t) + n(t)] \, z^*_n(t) \, dt \right|^2} \qquad (4\text{-}121)$$

The peak energy obtained at the output of the matched filter is given by,

$$E_n(\text{peak}) = \int_o^T |z_n(t)|^2 \, dt = \int_o^T A_n^2 \, dt = A_n^2 T \qquad (4\text{-}122)$$

Thus,

$$\eta^2 = \frac{2(A_n^2 T)^2}{\left| \int_o^T [z_c(t) + n(t)] z_n^*(t) \, dt \right|^2} \qquad (4\text{-}123)$$

It is now necessary to calculate the denominator of Equation (4-123), for example, the clutter plus thermal noise power at the output of the matched filter. (From here on the term matched filter will mean any correlation device.)

From Equations (4-115), (4-118), and (4-119) we have,

$$Z_n = \sum_{\substack{p \\ p \neq n}} A_n A_p \exp\{j\Theta_p\} \int_0^T \exp\{j[(p-n)\Delta\omega t + \phi_p(t - \tau_p) - \phi_n(t)]\}dt + \int_0^T n(t) A_n \exp\{-j[\omega_o t + n\Delta\omega t + \phi_n(t)]\}\, dt \tag{4-124}$$

where $\Theta_p = (\omega_o + p\Delta\omega)\ \tau_p$ is an arbitrary(constant) phase angle. Let,

$$\Phi_{np} = \frac{1}{T} \int_0^T \exp\{j[(p-n)\Delta\omega t + \phi_p(t - \tau_p) - \phi_n(t)]\}\, dt \tag{4-125a}$$

and

$$\Phi_N = \int_0^T n(t) A_n \exp\{-j[\omega_o t + n\Delta\omega t + \phi_n(t)]\} dt \tag{4-125b}$$

Substituting Equations (4-125a) and (4-125b) into Equation (4-124) yields,

$$Z_n = T \sum_{\substack{p \\ p \neq n}} A_n A_p \Phi_{np} \exp j\Theta_p + \Phi_N \tag{4-126}$$

It is now necessary to compute $\overline{|Z_n|^2}$, the mean square noise plus clutter;

$$\overline{|Z_n|^2} = (A_n T)^2 \sum_{\substack{p \\ p \neq n}} \sum_{\substack{r \\ r \neq n}} A_p A_r \overline{\Phi_{np} \Phi_{nr}^*}\ \overline{\exp j(\Theta_p - \Theta_r)} + \overline{|\Phi_N|^2} + 2TA_n \sum_{\substack{p \\ p \neq n}} A_p \overline{\Phi_N} \overline{\Phi_{np}}\ \overline{\exp j\Theta_p} \tag{4-127}$$

Now,

$$\overline{\exp j\,(\Theta_p - \Theta_r)} = 1 \quad p = r \qquad (4\text{-}128a)$$
$$= 0 \quad p \neq r$$

$$\overline{\Phi_N} = 0 \qquad (4\text{-}128b)$$

Thus, the total noise power is

$$\overline{|Z_n|^2} = (A_n T)^2 \sum_{\substack{p \\ p \neq n}} A_p^2 \, \overline{|\Phi_{np}|^2} + \overline{|\Phi_N|^2} \qquad (4\text{-}129)$$

where

$$|\Phi_N|^2 = \overline{\left| \int_o^T A_n \exp\{-j[\omega_o + n\Delta\omega)t + \phi_n(t)]\} \cdot n(t)dt \right|^2} \qquad (4\text{-}130)$$

The complex noise has the form

$$n(t) = A_N(t) \exp\{j[\omega_o t + \Theta_N(t)]\} \qquad (4\text{-}131)$$

$$= [X_N(t) + j\,Y_N(t)] \exp j\omega_o t \qquad (4\text{-}132)$$

where

$$\overline{X_N^2} = \overline{Y_N^2} = \frac{1}{2}\overline{n^2} = \sigma_N^2 = \text{thermal noise power} \qquad (4\text{-}133)$$

The functions $\{X(t), Y(t)\}$ are Hilbert transforms of each other . Thus,

$$\overline{|\Phi_N|^2} = 2 \overline{\left| \int_o^T X_N(t) A_n \exp\{-j[n\Delta\omega t + \phi_n(t)]\} dt \right|^2} \qquad (4\text{-}134)$$

If we now represent Equation (4-134) by samples at points separated by $t = 1/W$, (independent sample points of the Hilbert component) and replace the integral by a sum we have,

$$\overline{|\Phi_N|^2} = 2\left(\frac{A_n}{W}\right)^2 \sum_{i=1}^{WT} \sum_{q=1}^{WT} \exp\{-j[n\Delta\omega(t_i - t_q)]\}$$

$$\cdot \exp\{-j[\phi_n(t_i) - \phi_n(t_q)]\ \overline{X_N(t_i)X_N(t_q)} \qquad (4\text{-}135)$$

where,

$$\overline{X_N(t_i)X_N(t_q)} = \sigma_N^2 = W N_o \ ; \ i = q \qquad (4\text{-}136)$$
$$= 0 \ ; \ i \neq q$$

Hence,

$$\overline{|\Phi_N|^2} = 2(A_n^2 T)N_o \ . \qquad (4\text{-}137)$$

From Equations (4-129) and (4-137) we have for the noise power,

$$\overline{|Z_n|^2} = (A_n T)^2 \sum_{\substack{p \\ p \neq n}} A_p^2 \overline{|\Phi_{np}|^2} + 2(A_n^2 T)N_o \qquad (4\text{-}138)$$

where,

$$\overline{|\Phi_{np}|^2} = \frac{1}{T^2} \overline{\left| \int_o^T \exp\{j[(p-n)\Delta\omega t + \phi_p(t - \tau_p) - \phi_n(t)]\}\, dt \right|^2}$$
(4-139)

Equation (4-139) is the famous Ambiguity Function. The signal-to-noise ratio at the output of the matched filter is,

$$\eta_n^2 = \frac{1}{\frac{1}{2} \sum_{\substack{p \\ p \neq n}} \left(\frac{A_p}{A_n}\right)^2 \overline{|\Phi_{np}|^2} + \frac{N_o}{A_n^2 T}} \qquad (4\text{-}140)$$

If the clutter is zero we obtain the standard thermal noise signal-to-noise ratio $\eta^2 = \frac{A_n^2 T}{N_o} = \frac{E_n(\text{peak})}{N_o} = \frac{2E_n}{N_o}$, where E_n is the average signal energy. The term to the left is the clutter contributed at the output of the n^{th} matched filter (or after active correlation and filtering) by the common channel signals.

4.5.3.2 Peak Signal-to-Noise Ratio for Bernoulli Signal Addresses

Let us now assume that the signal addresses are binary signals which take on positive and negative values and that the frequency shift is zero (i.e., $\Delta\omega = 0$). Then from Equation (4-139)

$$\overline{|\Phi_{np}|^2} = \frac{1}{T^2} \overline{\left| \int_o^T \exp\{j\,\phi_n(t - \tau_p)\} \cdot \exp\{-j\phi_n(t)\}\, dt \right|^2} \tag{4-141}$$

At the sample points the integrand is itself a binary signal. If each signal is a Bernoulli sequence the product is also a Bernoulli sequence. The integrand contains ($N - \tau_p$) bits; τ_p can be assumed to be an integer random variable which has a flat distribution. Changing Equation (4-141) from an integral to a sum we find that the variance of Equation (4-141) for a fixed value of τ_p is simply

$$\overline{|\Phi_{np}(\tau_p)|}^2 = \left(\frac{\Delta T}{T}\right)^2 [N - \tau_p] \tag{4-142}$$

If Equation (4-142) is now averaged over variations in τ_p we have

$$\begin{aligned} \overline{|\Phi_{np}|}^2 &= \left(\frac{\Delta T}{T}\right)^2 \sum_{\tau_p=1}^{N} \frac{2}{N}(N - \tau_p) \\ &= \left(\frac{\Delta T}{T}\right)^2 (N-1) \approx N\left(\frac{\Delta T}{T}\right)^2 \quad ; \quad N >> 1\,. \end{aligned} \tag{4-143}$$

Since, $\frac{T}{\Delta T} = N$, we have

$$\overline{|\Phi_{np}|^2} = \frac{1}{N} . \tag{4-144}$$

Thus, in this special case the peak signal-to-noise power ratio is,

$$\begin{aligned} \eta^2 &= \frac{1}{\frac{1}{2N} \sum\limits_{\substack{p \\ p \neq n}} \left(\frac{A_p}{A_n}\right)^2 + \left(\frac{N_o}{A_n^2 T}\right)} \\ &= \frac{A_n^2 T}{\frac{1}{2W} \sum\limits_{\substack{p \\ p \neq n}} \frac{A_p^2}{2} + N_o} \end{aligned} \tag{4-145}$$

The left term in the denominator shows the average clutter energy spread over the RF bandwidth 2W. It is evident that as the bandwidth is increased the mutual clutter is spread over a broadband and its effect is decreased until thermal noise becomes the limiting factor. Thus, when $W \to \infty$

$$\eta_\infty^2 = \frac{A_n^2 T}{N_o} = \frac{2E}{N_o} \tag{4-146}$$

as required.

Let P be the total satellite power output. Then

$$P = \frac{A_n^2}{2} + \sum_{\substack{p=1 \\ p \neq n}}^{K} \frac{A_p^2}{2} \tag{4-147}$$

Substituting Equation (4-147) for the clutter gives

$$\eta^2 = \frac{A_n^2 \; T}{\frac{1}{2\,W}\left(P - \frac{A_n^2}{2}\right) + N_o}$$

$$= \frac{2 \cdot \left(\frac{A_n^2}{2P}\right) T}{\frac{1}{2W}\left(1 - \frac{A_n^2}{2P}\right) + \frac{N_o}{P}} \tag{4-148}$$

The term $\frac{P}{N_o}$ is a constant of the satellite system. It is commonly referred to as the noise bandwidth.

The channel capacity of the satellite system as the bandwidth becomes very large is,

$$C = W \log \left(1 + \frac{P}{W\,N_o}\right)$$

$$= C_\infty = \frac{P}{N_o} \quad \text{as} \quad W \rightarrow \infty \tag{4-149}$$

Hence,

$$\eta^2 = \frac{2W\,C_\infty\,T\left(\frac{A_n^2}{2P}\right)}{C_\infty\left(1 - \frac{A_n^2}{2P}\right) + 2W} \tag{4-150}$$

Note that the effective bandwidth is,

$$2W_e = \frac{2\,W\,C_\infty}{C_\infty\left(1 - \frac{A_n^2}{2P}\right) + 2W} \tag{4-151}$$

and

$$\eta^2 = 2W_e T \left(\frac{A_n^2}{2P}\right) \tag{4-152}$$

If all signals are multiplexed at equal power we have, from Equation (4-147),

$$\frac{A_n^2}{2P} = \frac{1}{K}$$

and Equation (4-152) becomes

$$\eta^2 = \frac{2W_e T}{K} \tag{4-153}$$

In particular, if $2W \gg C_\infty$,

$$\eta_\infty = \frac{C_\infty T}{K} \tag{4-154}$$

and communication is thermal noise limited. On the other hand if $C_\infty \gg 2W$ and $K \gg 1$,

$$\eta_W = \frac{2WT}{K} \tag{4-155}$$

and communications is clutter limited.

Another important class of signals is the PN frequency shifted alphabet. Here an address is a frequency shift; each signal has the same pseudo-noise binary phase modulation. In this case,

$$|\Phi_{np}|^2 = \frac{1}{T^2} \overline{\left| \int_0^T \exp \{j[(p-n)\Delta\omega\, t + \phi(t-\tau_p) \cdot - \phi(t)]\} \right|^2} \tag{4-156}$$

The clutter power is given by the ambiguity function.

4.5.3.3 Bounds on Clutter Power

We will now obtain bounds on the mean square clutter. Let us express Equation (4-139) as a double sum, at the sampling points. Then,

$$\overline{|\Phi_{np}|^2} = \frac{1}{(2WT)^2} \sum_{i=1}^{2WT} \sum_{q=1}^{2WT} \overline{\exp\{j[(p-n)\Delta\omega(t_i - t_q) + \phi_p(t_i - \tau_p) - \phi_p(t_q - \tau_p) - \phi_n(t_i) + \phi_n(t_q)]\}}$$

$$= \frac{1}{(2WT)^2} \Big[2WT + \sum_{\substack{i=1 \\ i \neq q}}^{2WT} \sum_{\substack{q=1 \\ q \neq i}}^{2WT} \overline{\exp\{j[(p-n)\Delta\omega(t_i - t_q) + \phi_p(t_i - \tau_p) - \phi_p(t_q - \tau_p) - \phi_n(t_i) + \phi_n(t_q)]\}} \Big] \tag{4-157}$$

Let ρ_{np} equal the ensemble average shown in Equation (4-157) independent of the coordinates (t_i, t_q). Then,

$$\overline{|\Phi_{np}|^2} = \frac{1}{(2WT)^2} [2WT + (2WT - 1)^2 \rho_{np}] \equiv (2WT)^{\gamma - 2} \tag{4-158}$$

where γ is a constant to be determined. Equation (4-158) simply shows how the clutter power varies with signal dimensionality. Then,

$$\rho(\gamma, 2WT) = \frac{(2WT)^{\gamma} - (2WT)}{(2WT - 1)^2} \tag{4-159a}$$

$$= \frac{(2WT)^{\gamma - 2} - \frac{1}{2WT}}{(1 - \frac{1}{2WT})^2} \tag{4-159b}$$

$$\lim_{WT \to \infty} \rho(2;\, 2WT) = 1 \tag{4-160a}$$

$$\lim_{WT \to \infty} \rho(\gamma < 2;\, 2WT) = 0 \tag{4-160b}$$

$$\lim_{WT \to \infty} \rho(\gamma > 2;\, 2WT) = \infty \tag{4-160c}$$

From Equation (4-140)

$$\eta^2 = \frac{2 \cdot \left(\frac{A_n^2}{2P}\right) T}{(2WT)^{\gamma - 1} \frac{\left(1 - \frac{A_n^2}{2P}\right)}{2W} + \frac{N_o}{P}} \tag{4-161}$$

4.5.4 Capacity Theorems for Random Access Systems

4.5.4.1 Post-Detection Decision M-ary Orthogonal Alphabets

Let

$$H = \frac{\log M}{T} = \text{information rate (units of natural base)} \tag{4-162a}$$

and

$$M = \exp(m) \tag{4-162b}$$

where,

M = Number of signals in alphabet

m = Number of message nats per signal.

Hence,

$$H = \frac{m}{T} \tag{4-162c}$$

Substituting for T into Equation (4-161) gives

$$\eta_n^2 = \frac{2\left(\frac{A_n^2}{2P}\right)\left(\frac{m}{H}\right)}{\left(\frac{2Wm}{H}\right)^{\gamma-1}\frac{\left(1-\frac{A_n^2}{2P}\right)}{2W}+\frac{N_o}{P}} \tag{4-163}$$

Let

$$r(\gamma, m) = \frac{2m}{\eta_n^2} \tag{4-164}$$

Following Turin[21]

$$P_c = 1 - \int_{-\infty}^{+\infty} p(y)\left[1 - \exp\left(-\frac{1}{2}y^2\right)\right]^{M-1} dy \tag{4-165}$$

where

$$\begin{aligned} p(y) &= y \exp -\frac{1}{2}(y^2 + \eta_n^2)\, I_0(\eta_n y) \;;\quad y \geq 0 \\ &= 0 \;;\quad y < 0 \end{aligned} \tag{4-166}$$

where η_n is the peak signal to RMS noise ratio at the output of the coherent system. Using Equation (4-164)

$$P(y) = y \exp -\frac{1}{2}\left(y^2 + \frac{2m}{r(\gamma, m)}\right) I_0\left(\sqrt{\frac{2m}{r(\gamma, m)}}\; y\right) \tag{4-167}$$

Following Turin, we let,

$$z = y - \eta_n = y - \sqrt{\frac{2m}{r(\gamma, m)}} \tag{4-168}$$

Then,

$$P_c = 1 - \int_{-\infty}^{+\infty} p(z + \sqrt{\frac{2m}{r(\gamma, m)}}) \left\{1 - \exp\left[- \frac{1}{2} (z + \sqrt{\frac{2m}{r(\gamma, m)}})^2\right] \right\}^{(\exp m-1)} \tag{4-169}$$

Holding z fixed and substituting $\log M = m$ we obtain the limit of the bracketed expressions as

$$L = \lim_{M \to \infty} \left\{1 - \exp\left[-\frac{1}{2} \left(z + \sqrt{\frac{2 \log M}{r(\gamma, \log M)}}\right)^2\right] \right\}^{M-1} \tag{4-170a}$$

$$= \lim_{M \to \infty} \left\{1 - \exp\left(- \frac{\log M}{r(\gamma, \log M)}\right)\right\}^{M} \tag{4-170b}$$

$$= \lim_{M \to \infty} \exp\left\{ - M \exp\left[- \frac{\log M}{r(\gamma, M)} \right]\right\} \tag{4-170c}$$

$$= \lim_{M \to \infty} \exp\left\{ -M^{\left[1 - \frac{1}{r(\gamma, \log M)}\right]} \right\} \tag{4-170d}$$

$$\left.\begin{aligned} &= 1 \text{ when } \lim_{M \to \infty} \frac{1}{r(\gamma, m)} > 1 \\ &= 0 \text{ when } \lim_{M \to \infty} \frac{1}{r(\gamma, m)} < 1 \end{aligned}\right\} \tag{4-171}$$

In addition, we have

$$\lim_{m \to \infty} p\left(z + \sqrt{\frac{m}{r(\gamma, m)}} \right) = 0 \qquad (4\text{-}172)$$

As $m \to \infty$, Equation (4-172) tends to a normal density which is zero in the limit. Then,

$$P_c = 0 \; ; \quad \lim_{m \to \infty} r(\gamma, m) < 1 \qquad (4\text{-}173a)$$

$$= 1 \quad \lim_{m \to \infty} r(\gamma, m) > 1 \qquad (4\text{-}173b)$$

Equation (4-173) states that errorless transmission can be achieved if Equation (4-173a) is satisfied. This is accomplished at the expense of infinite delay and infinite complexity. The analysis also assumes that each message would be encoded into orthogonal signals which would require infinite bandwidth . The random access signals approach orthogonality in the limit as $T \to \infty$ for finite bandwidth. The analysis therefore holds for these as well. Similar results can be obtained for non-orthogonal higher-order alphabets as well.

From Equations (4-163) and (4-164),

$$r(\gamma, m) = \frac{\left(\frac{2\,Wm}{H}\right)^{\gamma - 1} \dfrac{\left(1 - \frac{A_n^2}{2P}\right)}{2W} \; \dfrac{N_o}{P}}{\left(\frac{A_n^2}{2P}\right) \frac{1}{H}} \qquad (4\text{-}174)$$

Case 1: $\gamma = 1$;

$$\lim_{m \to \infty} r(1,m) = \frac{\left(1 - \frac{A_n^2}{2P}\right)\frac{1}{2W} + \frac{N_o}{P}}{\left(\frac{A_n^2}{2P}\right)\frac{1}{H}} < 1 \tag{4-174}$$

Therefore,

$$H_n < \frac{\left(\frac{A_n^2}{2P}\right)}{\left(1 - \frac{A_n^2}{2P}\right)\frac{1}{2W} + \frac{N_o}{P}} \tag{4-175}$$

Thus errorless transmission can be achieved as long as the information rate of the source does not exceed the bound shown. Therefore, when $\gamma = 1$, the capacity is limited by thermal noise and by clutter. The clutter can be reduced, however, by increasing the system bandwidth. As $W \to \infty$, the thermal noise capacity is obtained.

If equal power multiplexing is achieved in the satellite then, Equation (4-175) becomes,

$$H < \frac{\left(\frac{1}{K}\right)}{\frac{\left(1 - \frac{1}{K}\right)}{2W} + \frac{1}{C_\infty}} = \frac{\left(\frac{2\ W\ C_\infty}{K}\right)}{C_\infty\left(1 - \frac{1}{K}\right) + 2W} \tag{4-176}$$

This is the capacity of a random access system using Bernoulli signal addresses.

Case 2: $\gamma < 1$

$$\lim_{m \to \infty} r(\gamma < 1;\ m) = \frac{H N_o}{\left(\frac{A_n^2}{2}\right)} < 1 \tag{4-177}$$

or

$$H < \frac{A_n^2}{2N_o} \tag{4-178}$$

In this case the clutter goes to zero even for a finite bandwidth and the system is thermal noise limited. The significance of this result is not apparent at this time. It may very well be that $\gamma < 1$ implies that the bandwidth tends to infinity with m.

Case 3: $\gamma > 1$

$$\lim_{m \to \infty} r(\gamma > 1\ ;\ m) = \infty \tag{4-179}$$

Hence capacity can only be achieved if $\gamma \leq 1$.

The results obtained are important in that they give an absolute limit on the information rate. Although this limit cannot be attained physically, it can be approached for finite size alphabets and with practical implementation.

4.5.4.2 Predetection Decision M-ary Orthogonal Alphabets

The capacity theorems for gaussian channels using predetection and post-detection decision are the same. The predetection case will only be outlined here since it will become apparent that asymptotically predetection and post-detection error probabilities approach the same limits.

For the predetection decision case we have,

$$P_c = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \exp\left\{-\frac{1}{2}\left[y - \eta\sqrt{(1-\lambda)}\right]^2\right\} \left[1 - \int_{y}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)dx\right]^{M-1} dy \qquad (4\text{-}180)$$

where λ is the maximum crosscorrelation coefficient among the elements of the M-ary alphabet. Making the same change of variable as in the post-detection case we have,

$$P_c = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)\left[1 - \int_{x}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(z+\eta\sqrt{1-\lambda})^2\right]dz\right]^{M-1} \qquad (4\text{-}181)$$

If we now substitute $\eta^2 = 2m/r$ and recognize that the integrand in the brackets can be written as,

$$\int^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(z+\eta\sqrt{1-\lambda})^2\right]dx \sim \frac{1}{\sqrt{2\pi}\,x} \exp\left[-\frac{1}{2}(x+\eta\sqrt{1-\lambda})^2\right] \qquad (4\text{-}182)$$

Then,

$$P_c = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)\left[1 - \frac{1}{\sqrt{2\pi}\,x} \exp\left[-\frac{1}{2}(x+\eta\sqrt{1-\lambda})^2\right]\right]^{M-1} dx \qquad (4\text{-}183)$$

We now have essentially the same forms as previously. For PN signals considered here, $\lambda \rightarrow 0$ as $m \rightarrow \infty$. Thus, the same capacity relationships will be obtained. Figure 4-70 is a family of curves which gives the error probability versus $\frac{1}{r} \log 2$ with alphabet order M as a parameter. (The x-axis has units 1/bits.)

4.5.5 Error Rates for Random Access Systems

4.5.5.1 Post-Detection M-ary Alphabet Using Greatest-of Decision

Turin[21] has graphed the error probability for the post-detection case. In order to use his results it is essential to obtain his value of r in terms of the constants used here. Interest lies primarily in the case $\gamma = 1$. This is also the Bernoulli alphabet case. Here,

$$\frac{1}{r} = \frac{\frac{A_n^2}{2P} \; \frac{1}{H}}{\left(1 - \frac{A_n^2}{2P}\right) \frac{1}{2W} + \frac{N_o}{P}} \tag{4-184}$$

From previous consideration $H = m/T$. Then,

$$\frac{1}{r} = \frac{\frac{A_n^2}{2P} \; \frac{T}{m}}{\left(1 - \frac{A_n^2}{2P}\right) \frac{1}{2W} + \frac{N_o}{P}}$$

$$= \frac{2\,W\,C_\infty \left(\frac{A_n^2}{2P}\right) \frac{T}{m}}{C_\infty \left(1 - \frac{A_n^2}{2P}\right) + 2W} \tag{4-185}$$

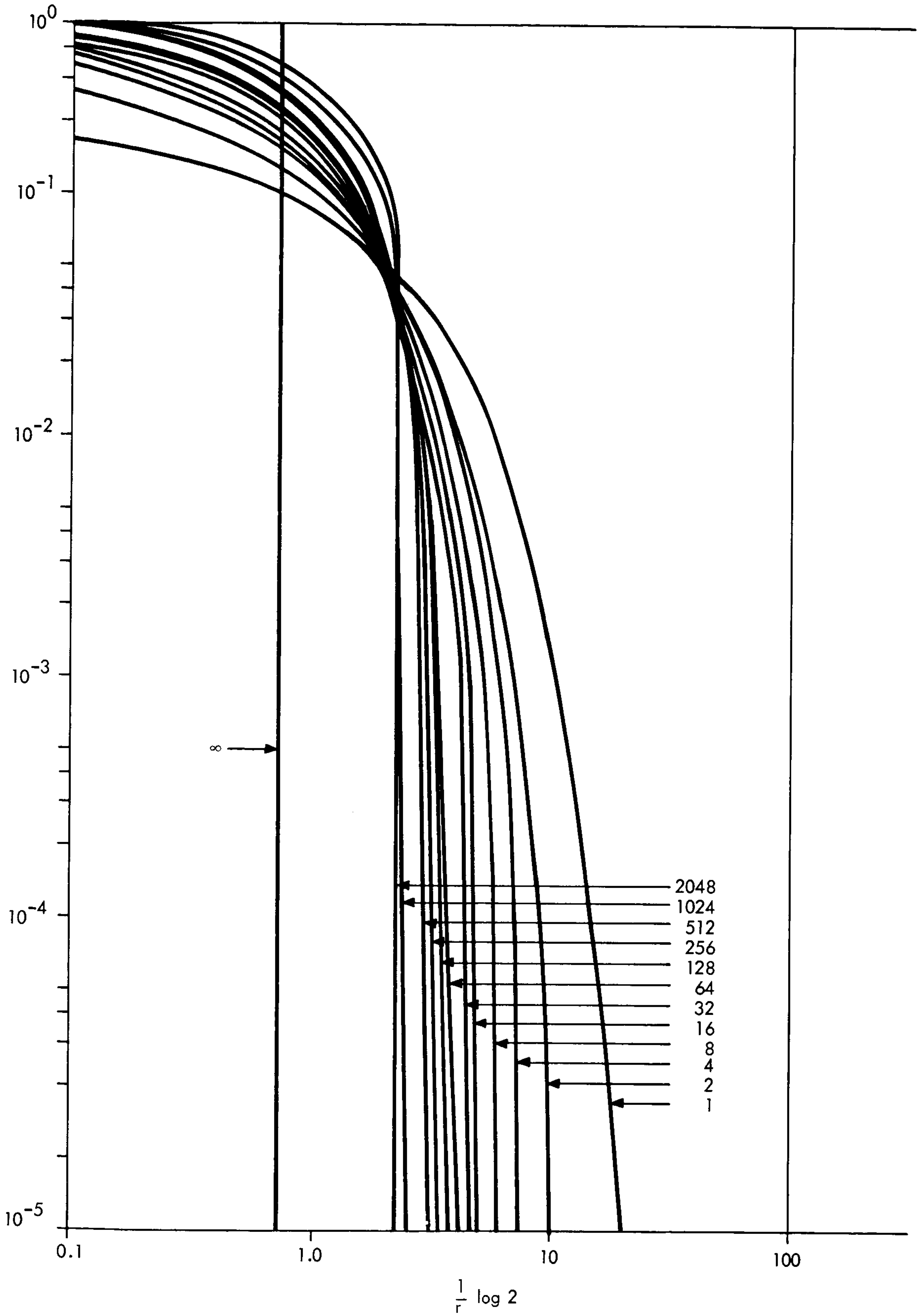

Figure 4-70. Pre-detection M-ary Word Error Probability Orthogonal Codes

If we wish to use bit instead of units to the base e, we simply substitute $m_b = m \log_2 e$.

If we now solve for $\frac{P}{\frac{A_n^2}{2}}$ we obtain

$$\frac{P}{\left(\frac{A_n^2}{2}\right)} = \frac{\frac{2}{m} W C_{\infty} T\, r+1}{C_{\infty} + 2W} \approx \frac{2W C_{\infty} T}{C_{\infty} + 2W} \left(\frac{r}{m}\right) \tag{4-186}$$

For equal power multiplexing $K = \frac{P}{\left(\frac{A_n^2}{2}\right)}$. Then Equation (4-186) gives the number of equal power users of unity duty ratio. If the signals have a duty factor d_0, then the number of active users of the satellite is simply

$$K_0 = \frac{K}{d_0} \tag{4-187}$$

For an error rate of 10^{-3}, $M=32$, $r = \frac{1}{5.5}$. For a noise bandwidth $\frac{P}{N_o} = 10^7$ and a receiver bandwidth $W = 10^8$ cps, we obtain $K = 63$. This information is obtained from Figure 4-71. Figure 4-71 is a family of curves which gives the error probability versus $\frac{1}{r}$ with order of the alphabet M as a parameter.

If we further assume that an active talker sends signals only one-third of the time and that one of the parties is usually listening, we have a duty factor $d_0 = \frac{1}{6}$. Thus, the number of full duplex

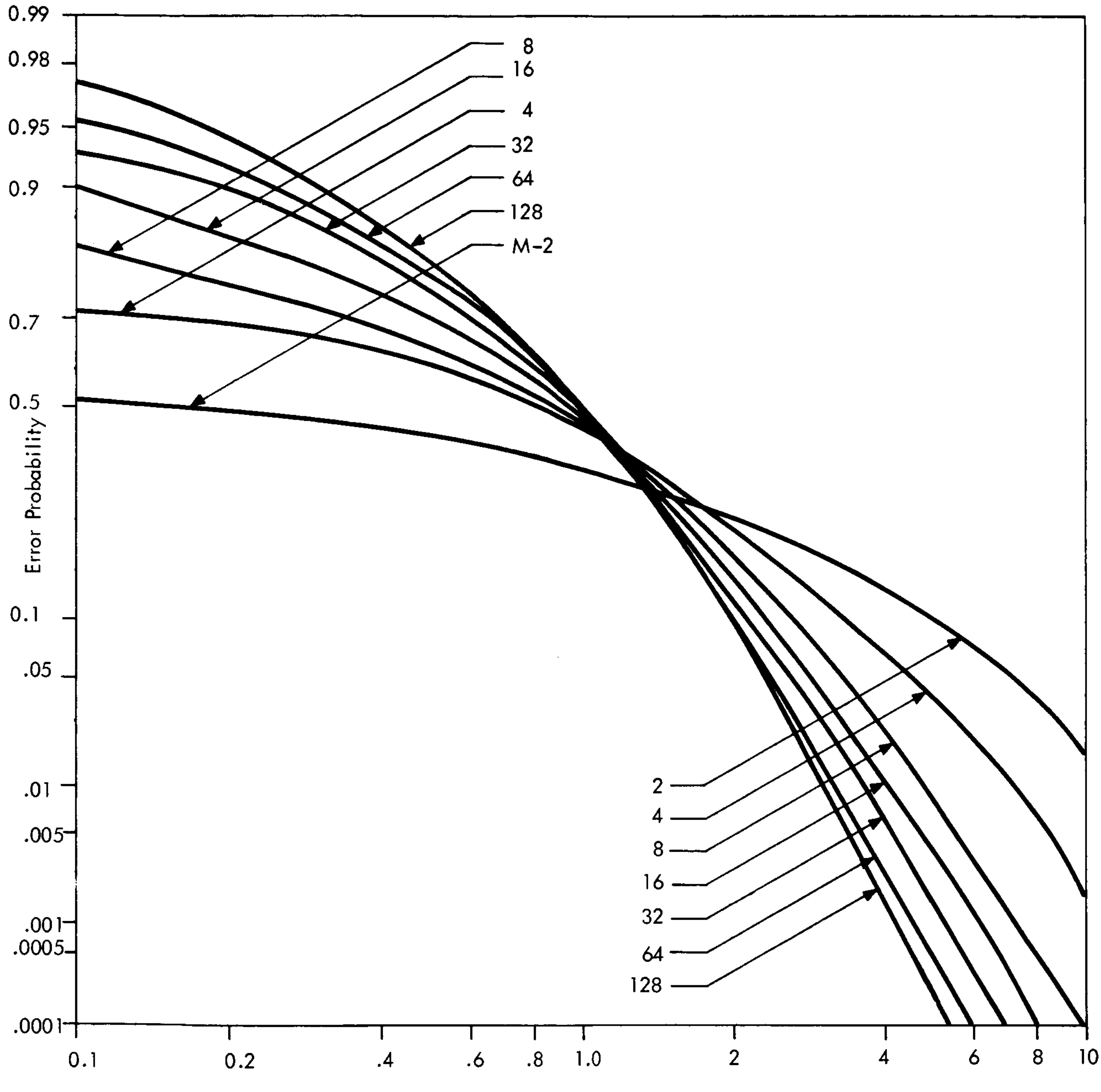

Figure 4-71. Greatest-of Post-Detection M-ary Decision

conversations which the system can accommodate is, $K_0 = 63 \times 3 = 189$. For the parameters chosen, the system is thermal noise limited and not clutter limited.

We will obtain an approximate expression for the error probability valid for large signal-to-noise ratios. This expression is an upper bound on the error rate.

$$1 - \alpha = \int_0^{\infty} y \exp[-\tfrac{1}{2}(y^2 + \eta^2)]\, I_0(\eta y)[1 - \exp(-\tfrac{1}{2}y^2)]^{M-1} dy \qquad (4\text{-}188)$$

$$= \int_0^{\infty} y \exp[-\tfrac{1}{2}(y^2 + \eta^2)]\, I_0(\eta y)\, [1 - (M-1)\exp(-\tfrac{1}{2}y^2)]\, dy \qquad (4\text{-}189)$$

If we change the variable to $x = \frac{y}{2}$ then,

$$\alpha = \frac{(M-1)}{2} \exp(-\frac{\eta^2}{4}) \int_0^{\infty} y \exp[-\tfrac{1}{2}(y^2 + \frac{\eta^2}{2})]\, I_0(\frac{\eta y}{2})\, dy \qquad (4\text{-}190)$$

The integral is unity. Then

$$\alpha = \frac{(M-1)}{2} \exp(-\frac{\eta^2}{4}) \qquad (4\text{-}191)$$

where η is the peak signal to RMS noise ratio. If we let $\eta^2 = 2m/r$, then, Equation (4-191) becomes.

$$\alpha(m_b) = \exp_2\{-m_b(\frac{1}{2r} - 1) - 1\} - \exp_2\{(\frac{-m_b}{2r}) - 1\} \qquad (4\text{-}192)$$

There are two other M-ary post-detection decision procedures of interest which will be studied during the next phase. These are null-zone detection and threshold detection. The former is a greatest-of-decision with a threshold. The largest value is chosen and a decision is made if this value exceeds a predetermined threshold.

The threshold detection procedure is completely asynchronous, and wide-open in time. Whenever a signal exceeds the threshold, a pulse is generated. The pulse train is fed directly into a shaped low-pass filter and the message is recovered.

4.5.5.2 Predetection Greatest-of M-ary Decision Procedure

The error probability for the predetection case can be approximated just as in the post-detection case. The approximation is given as

$$a_m = \frac{(M-1)}{\eta\sqrt{\pi\,(1-\lambda)}} \qquad \exp\left\{-\frac{\eta^2}{4}(1-\lambda\right\} \tag{4-193}$$

where λ is the pair-wise crosscorrelation among the M-ary signals. Letting $\eta^2 = 2m/r$, we obtain

$$a_1 = \frac{1}{\sqrt{2\pi\left(\frac{1-\lambda}{r}\right)}} \qquad \exp_2\left\{-m_b\frac{1-\lambda}{2r}\right\} \tag{4-195}$$

For $\exp_2(m_b) \gg 1$,

$$a_m = \frac{1}{\sqrt{2\pi\left(\frac{1-\lambda}{r}\right)}} \qquad \exp_2\left\{-m_b\left(\frac{1-\lambda}{2r}-1\right)\right\} \tag{4-196}$$

4.5.6 Audio-Signal-to-Noise Ratio

In this section we will discuss briefly the audio-signal-to-noise ratio of random access systems. The signal-to-noise ratio will depend on the type of modulation used.

4.5.6.1 Correlation Lock

Here we assume that the 4 kc audio-signal is converted to a form of pulse-time modulation at an 8 kc sampling rate which in turn is converted to a square-wave. The message is recovered by feeding the square-wave into a low-pass filter. The square-wave is used to phase-reversal modulate a PN subcarrier. At the receiver the PN subcarrier is removed and the square-wave output is fed into a 4 kc filter and recovered. This system has no threshold. The signal-to-noise ratio at the output of the audio filter is given by

$$\eta^2 = \frac{2C_\infty WT}{C_\infty + 2W} \left(\frac{A_n^2}{2P}\right) \tag{4-197}$$

where T = 1/4000 = 250 microseconds. For a 20 db audio signal-to-noise ratio $\eta^2 = 100$. For $2W = 10^8$, $C_\infty = 10^7$,

$$\frac{P}{\left(\frac{A_n^2}{2}\right)} = K = \frac{C_\infty T}{\eta^2} = \frac{10^7 \times 250 \times 10^{-6}}{100} = 25$$

Thus 25 equal power talkers can be handled.

4.5.6.2 Maximum Likelihood Sequence

This is essentially a digital system. We will neglect the quantization noise. If companding of the audio is used, then for all practical purposes the signal-to-noise ratio is given by,

$$\left(\frac{S}{N}\right)_{\text{audio}} = \frac{1}{2\alpha + \exp_2(-2m_b)} \tag{4-198}$$

and in db we have,

$$\left(\frac{S}{N}\right)_{\text{db}} = -10 \log_{10} \left[2\alpha + \exp_2(-2m_b)\right] \tag{4-199}$$

From Equation (4-192),

$$\left(\frac{S}{N}\right)_{\text{db}} = -10 \log_{10}\left[\exp_2\left(-m_b\left(\frac{1}{2}r - 1\right)\right) + \exp_2(-2m_b)\right] \tag{4-200}$$

$$= 6m_b - 10 \log_{10}\left[\exp_2\left(-m_b\left(\frac{1}{2}r - 3\right)\right) + 1\right] \quad (4\text{-}201$$

When $1/2r = 3$; we have,

$$\left(\frac{S}{N}\right)_{\text{db}} = 6m_b - 3 = 3[2m_b - 1] \tag{4-202}$$

Equations (4-200) and (4-202) will be examined in more detail in Phase III.

4.5.7 Call-up Error Probability-Matched Filter Case

There are two types of errors which can occur during call-up when the called party is in a standby mode. It is possible that a false call can be indicated if noise crosses a predetermined threshold. It is also possible that a call will not be indicated when, in fact, one is being placed due to signal not crossing the threshold. The false call is more serious and should not be permitted to occur too often. When a call is not indicated when it is being placed, the calling party can try again.

In the matched filter case the false call error probability is simply

$$\alpha_F = \exp\left(-\frac{1}{2} r_0^2\right) \tag{4-203}$$

where r_0 is a preassigned threshood.

The call probability is given by

$$1 - \beta = \int_{r_0}^{\infty} y \exp - \frac{1}{2}\left(y^2 + \eta^2\right) I_0(\eta y) dy \tag{4-204}$$

where η is the peak signal-to-noise ratio given by Equation (4-163).

An approximation to $1 - \beta$ valid for large signal-to-noise ratios is

$$1 - \beta \approx \int_{r_0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \frac{1}{2}(y - \eta)^2 dy \tag{4-205}$$

$$1 - \beta \approx \int_{r_0 - \eta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) dy \; ; \quad r_0 - \eta << 0$$

The error β is given by

$$\beta \approx \int_{-(\eta - r_0)}^{-\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) dy$$

$$\approx \frac{1}{\sqrt{2\pi}\;(\eta - r_0)} \exp - \frac{1}{2}(\eta - r_0)^2 \tag{4-207}$$

From Equation (4-203) we can substitute for r_0 and obtain

$$\beta \approx \frac{1}{\sqrt{2\pi}\left(\eta - \sqrt{2 \log \frac{1}{\alpha_F}}\right)} \exp - \frac{1}{2}\left(\eta - \sqrt{2 \log \frac{1}{\alpha_F}}\right)^2 \tag{4-208}$$

A special case of interest is $r_0 = \frac{\eta}{2}$. Then,

$$\alpha_F = \exp\left(-\frac{\eta^2}{4}\right) \tag{4-209a}$$

$$1 - \beta \approx \frac{1}{\sqrt{\frac{\pi}{2}}\,\eta} \exp\left(-\frac{\eta^2}{4}\right) . \tag{4-209b}$$

4.6 Detailed Logic Design of Random Access Noise Signal Address Communications (RANSAC) Systems

Two basic RANSAC techniques and configurations applicable to the synchronous repeater satellite will be discussed in this section in detail. These are,

Maximum Likelihood Sequence (MLS-RANSAC)

Correlation Locked (CL-RANSAC)

(We will refer to the systems by the abbreviated names for the purpose of simplicity.)

All the techniques use PN signal addresses. These differ primarily in the manner in which the message signal is transformed into the PN address and in the technique of reception. The calling procedure is the same for the systems although transmission and reception of the calling signals is different. The matched filter technique uses digital pulse position modulation and requires quasi-synchronous operation. On the other hand, the correlation-locked technique has a phase-reversal modulated PN subcarrier, modulated by either a binary message or an analog (square wave) message which is demodulated by using correlation lock both in PN bit and in RF phase. However, once lock is established, communication is equivalent to transmission over a wire line. It is apparent from this brief discussion that each technique has favorable and unfavorable characteristics.

4.6.1 Characteristics Common to the Chosen RANSAC Systems

The following characteristics are common to the two RANSAC systems chosen for this study:

- Call-up procedure.
- Optimum from communication theory standpoint.
- Phase-reversal PN modulation.
- Message is addressed directly.
- Major operations are performed by digital circuits.
- Multiplexing in the satellite occurs in a hard limiter prior to applying signal to TWT.
- Message rate can be varied as desired; can accommodate mixture of voice messages, teletype, data, etc.
- Inherent privacy.
- Minimum interference to conventional systems.
- Conventional systems introduce minimum interference to PN systems.
- Capacity of system limited only by number of active users.

4.6.2 Characteristics in which the Two RANSAC Systems Differ Significantly

The chosen RANSAC systems differ significantly in the following:

- Implementation of call-up procedure.
- Receiver implementation.

- Degree of synchronization required.
- Method of detection at receiver.
- Sensitivity to doppler.
- Time required for placing a call.
- Capability to turn carrier on-off with voice and still maintain communications.

The common characteristics of these systems reflect those operations that characterize the class of PN communication systems in general. The characteristics in which they differ reflect primarily practical considerations which are closely related to implementation.

4.6.3 Call-Up Procedure Common to RANSAC

The procedure for calling a party is the same in the two RANSAC systems. Each channel is assigned a unique signal address on which a call is received and via which a connection is made. Dialing the called party's telephone number selects the called party's signal address. The calling party's receiver is also set to the called party's signal address. If the called party's signal is received, a busy signal is indicated. (Provisions are made at each end to prevent "snooping.") If a busy signal is not received a call has been initiated. After a suitable delay the calling party's address ("self-code") is transmitted via the PN signal. That is, each "self-code" bit is addressed with the PN signal. This code is received reliably, and subsequently selects automatically the calling party's address on the PN sequence generator. This code is then transmitted back to the calling party. When this return signal is received the call-up procedure is terminated and a two-way connection exists for full duplex message communication.

Although the call-up procedure is the same for the two RANSAC systems, the techniques of implementing it are different.

4.6.4 Basic Functions of MLS-RANSAC

4.6.4.1 Call-Up Subunit

The call unit shown symbolically in Figure 4-72 consists of an M-bit maximal length sequence generator and an M-bit shift register. In addition (not shown here) an N-bit binary counter is used to register the number of shift pulses applied to the M-sequence generator. There are two feedback paths, one of which is selected in response to the control signal.

A signal address or telephone number consists of M-bits. A call is dialed by loading an M-bit number into the hold shift register. This number is used to preset the M-sequence generator into the initial portion of the called party's address.

Assume that M = 17. Then the sequence generator will generate $2^{17} - 1$ bits before recycling. If each subscriber is assigned a signal address $2^7 = 128$ bits long, then $\frac{2^{17}}{2^7} = 1024$ subscribers can be

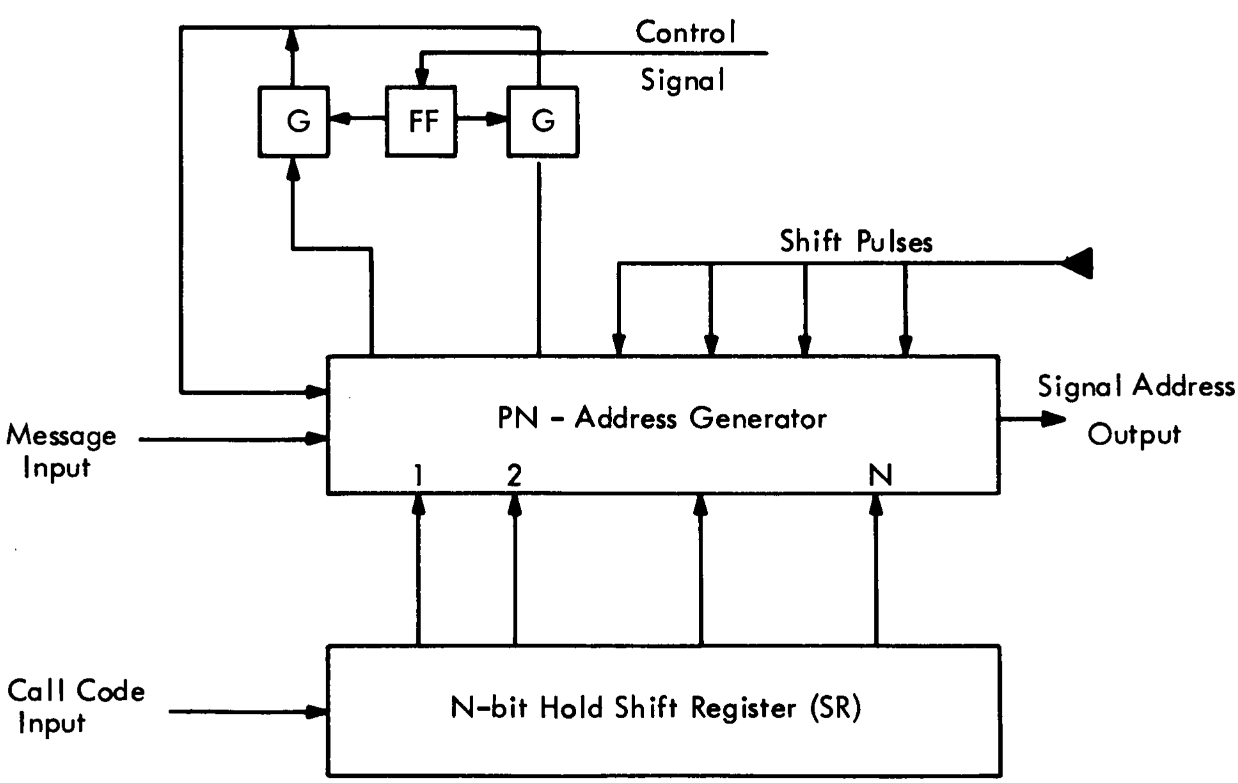

Figure 4-72. Diagram of Call Unit

accommodated. However, to select a called party's 128-bit signal address, only 17 bits are required to specify its initial starting point. That is the telephone number is 17 bits long (or shorter if, say, an octal number is used). This procedure can be visualized with the diagram in Figure 4-73.

Each channel is assigned a 128-bit segment of the sequence which has a total cycle time of $2^{17} - 1$ bits. The segments do not overlap. If the called party is, say, number 4, then a 17-bit number corresponding to the called party's phone number is preset into the sequence generator. This initializes the sequence generator uniquely to party number 4's signal address. The telephone number represents the first 17 bits of the 128-bit signal address of party 4. A message pulse starts the shift pulse generator, which in turn causes the sequence generator to produce party number 4's address. A binary counter senses the shift pulses. After 128 counts the sequence generator is reset to its initial state determined by party number 4's phone number held in the shift register.

The 128-bit codes are unique, and statistical considerations show that the 1024 addresses, each 128 bits long, will be sufficiently far apart from each other.

There are two feedback paths; only one is selected at a time. The paths cause sequences to be generated that are reversed in time with respect to each other. The purpose of having two unique addresses

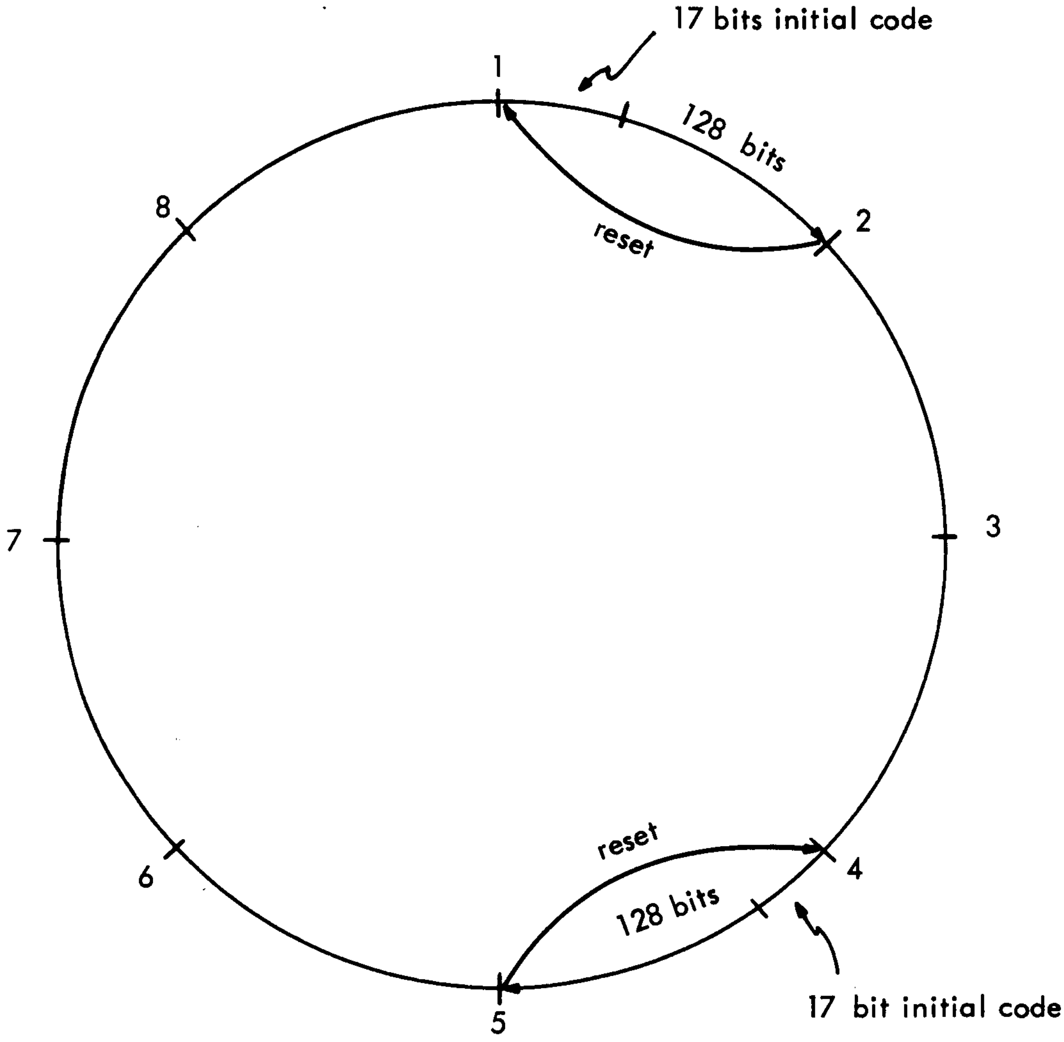

Figure 4-73. Graphical Representation of Calling Unit Operation

per subscriber will be discussed in the detailed block diagram of the MLS-RANSAC system.

An identical unit is available at the receiver which is automatically preset to the calling party's address. In this mode when the calling party's code is received it is automatically shifted into the hold register. After 17 bits the register is full and the called party is "switched" to the calling party's signal address. A pulse at the receiver causes the sequence generator to produce the calling party's address. This is transmitted back to the calling party; when received the two-way connection is completed. This connection has been obtained automatically; human intervention appears only in dialing or when initiated a call.

4.6.4.2 PPM System with Voice Switching

Figure 4-74 is a block diagram of a PPM system with voice switching.

The 4 kc voice signal is fed into a compander which attempts to convert the voice signal so that the sample values will have a flat probability density. This will maximize the audio signal-to-noise ratio at the output of the audio-amplifier at the receiver for the type of detection procedure used. The output of the compander is added to the 8 kc sawtooth wave-form where the zero crossings are changed in accordance with the audio wave. The zero crossings are then sensed by the slicer and pulse generator.

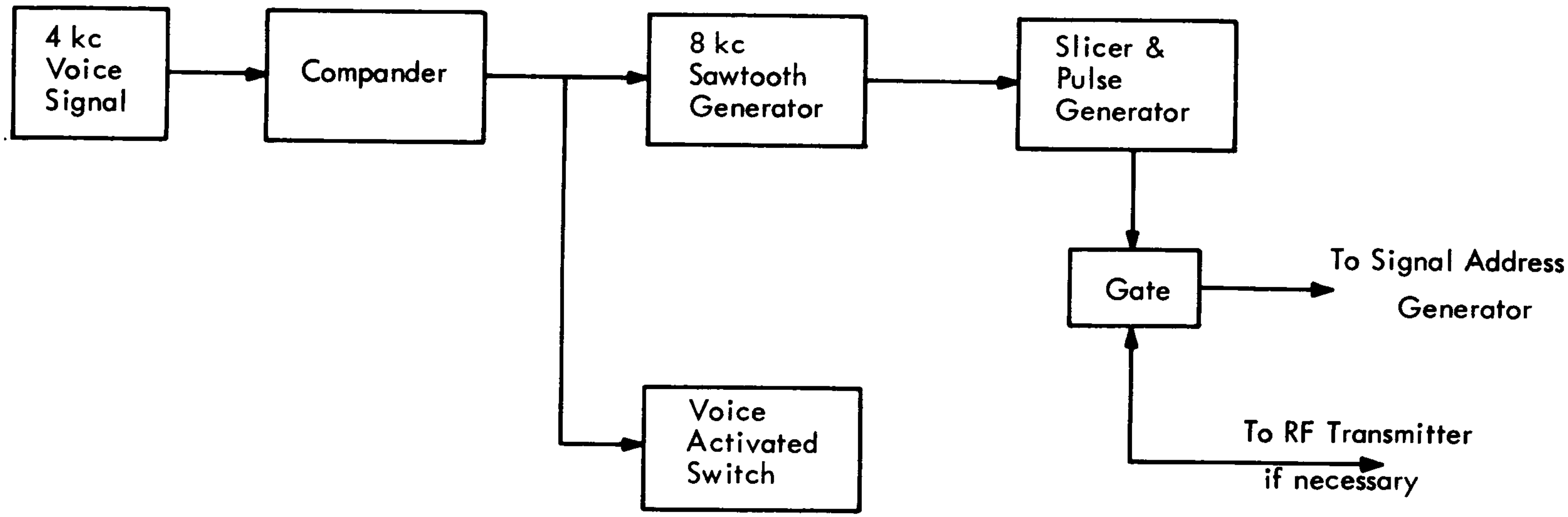

Figure 4-74. Block Diagram of PPM With Voice Switching

The output of the compander is applied to a voice switching circuit which generates a gate whenever audio power is detected. This has the very important function of keeping RF carrier off the air when not actively talking. (A talker is active on the average only 1/2 of the time.) This reduces the clutter and results in more efficient use of satellite power. The PPM (pulse-position modulation) signal output is fed to the signal address generator. If required, the voice actuated switch can gate the appropriate section of the transmitter.

4.6.4.3 Transmitter and Message Receiver

Figure 4-75 is a simplified block diagram of the transmitter.

The output of the PN-signal address generator is fed into a balanced modulator; the output signal is a double sideband suppressed carrier signal. A crystal controlled-frequency synthesizer is the local oscillator which, in response to a control signal, generates a frequency which places the particular address in a particular frequency band. Each address is hopped to a PN selected frequency band. This mode of operation requires quasi-synchronous operation between transmitter and receiver. In this manner each signal is, in effect, spread over the entire satellite band. This results in efficient mutual interference rejection.

Figure 4-76 is the receiver section of the MLS-RANSAC. The RF output feeds a bandpass limiter. The advantage of limiting at low

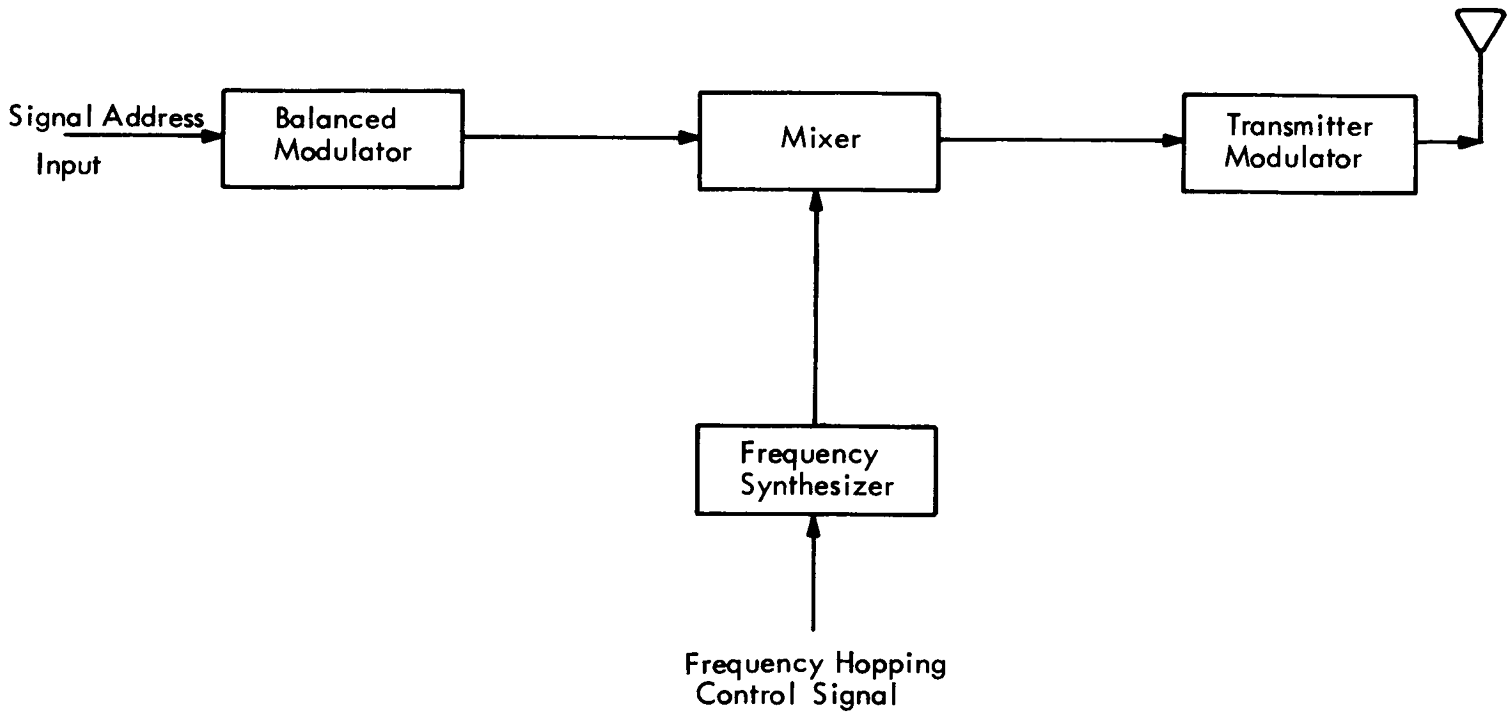

Figure 4-75. Transmitter Section

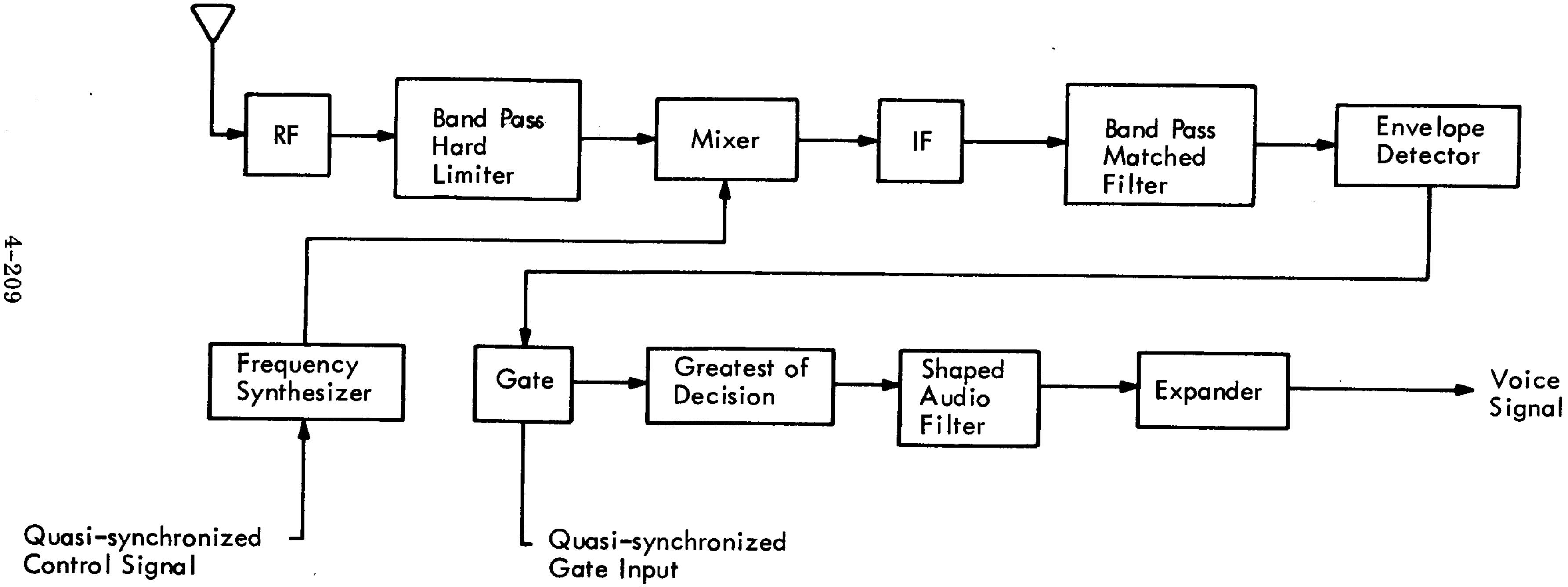

Figure 4-76. Message Receiver

signal-to-noise ratio is that only 1 db is lost by this operation. The frequency synthesizer at the receiver injects the proper local frequency so that the signal will pass through the IF filter. The output of the IF feeds the matched filter which will respond strongly only to the address to which it is matched. This output feeds a conventional envelope detector and then a gate. The quasi-synchronous gate allows the signal to pass into the greatest-of decision circuit during the interval when it is expected. The greatest-of decision is optimal; a time interval containing the desired pulse is selected if this pulse is the largest one in the interval. A low-pass filter having a 6 db per octave roll-off will demodulate the message. Receiver operation in the call mode will be described when the detailed configuration is presented.

4.6.5 Basic Functions of CL-RANSAC

The correlation locked RANSAC transmits information by phase reversal modulating a binary PN subcarrier. Active correlation is used for establishing a connection between two parties. A connection here establishes correlation lock between transmitter and receiver PN signals. Once lock is established it is equivalent to a wire connecting the terminals. Communication can then proceed in a normal way. Both analog and digital messages can be transmitted by modulating the PN subcarrier. It is essential to maintain correlation lock during transmission.

4.6.5.1 Call-Up Logic

Figure 4-77 shows the sequence of operations during the ground station's endeavor to determine the presence of the calling party. The incoming transmission from the calling party is a phase-reversal modulated PN signal. The PN sequence is unique to this ground station. It is desirable, from the point of view of minimizing the call-up acquisition time, to use a maximal length sequence whose length is relatively short. With a sequence whose repetition period is short, the search process over the ambiguous bits can be made easier. For example, the length of a maximal length sequence of 13 stages is $2^{13} - 1 = 8191$. If the PN sequence is generated at a 10 mega bit rate, it takes .82 milliseconds to receive the entire call-up sequence. Suppose the decision as to whether or not the incoming sequence is in step with the locally generated sequence is made by integrating over 1,000 pseudo random bits. Then, every 0.1 milli-second a comparison can be made. In a maximum of .82 seconds, all the necessary comparisons can be made and the locally generated sequence can be brought into synchronization with the pseudo random sequence modulating the incoming signal.

Since each ground station's call-up system is assigned a unique PN sequence, it becomes necessary to choose a class of sequences

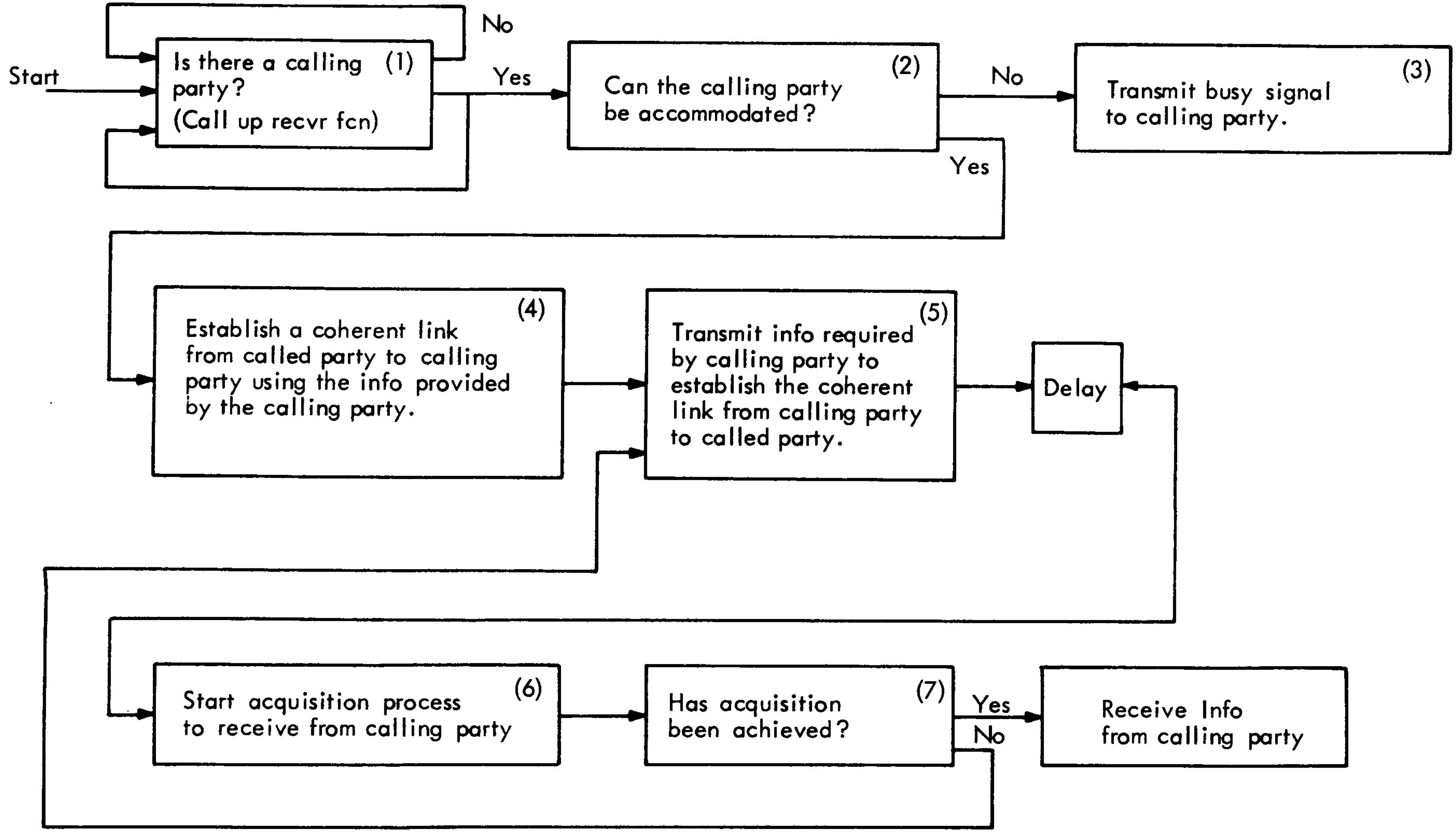

Figure 4-77. Callup System—Called Party Mode of Mode of Operation at a Ground Station

from which it would be possible to choose all the required sequences. Consulting a table of Irreducible Polynomials, one finds that for degree 13 there are 630 sequences. For the problem at hand this provides a sufficiently large number from which to make the choice for the call-up sequence.

Figure 4-78 shows the information m (t) transmitted during the call-up process. The function m (t) takes on the values ± 1. The acquisition block contains a series of ones so the call-up receiver can perform the PN sequence acquisition. It is necessary for the calling party to know the pseudo-random sequence to use in transmitting to the called party.

4.6.5.2 PN Subcarrier Modulator

Figure 4-79 is a block diagram of a PN modulator. The message can either be in binary or simply in a form of square-wave time modulation.

The PN subcarrier is at a much higher bit rate than the message. Hence, many PN bits are transmitted per message bit. Message and PN subcarrier are combined a a modulo-2 adder (exclusive-OR circuit). The modulated PN subcarrier is then fed into a balanced modulator.

Acquisition Block ($m(t) = 1$)	Called Party's Address	Calling Party's Address	Information Concerning the Sequence Required to Contact the Calling Party

Figure 4-78. Information Modulating the Callup Sequence from Calling Party to Called Party

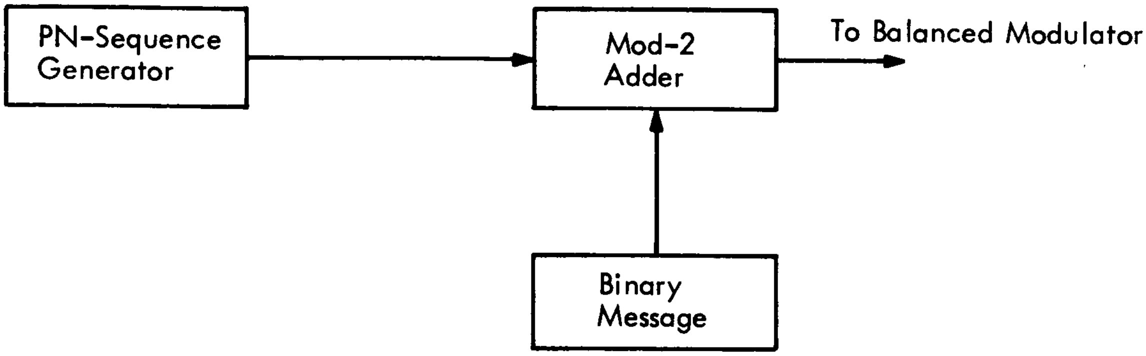

Figure 4-79. PN Subcarrier Modulator

4.6.5.3 Transmitter and Message Receiver

The modulated PN subcarrier is fed into a balanced modulator and then to a mixer as shown in Figure 4-80. In order to obtain complete frequency coverage some addresses are assigned to different frequency bands. Thus, an address is a frequency shift with a PN sequence. Frequency-hopping is not used in the CL-RANSAC, but frequency assignment is used to reduce the mutual clutter.

The local oscillator is fixed by the frequency band assigned to the receiver as shown in Figure 4-81. The output of the bandpass limiter feeds the phase-locked circuits (shown in detail in the next section). The phase-locked circuits must extract carrier from the signal to obtain carrier phase-lock and must also obtain PN bit lock. The resultant signal injected into the multiplier is RF-phase and PN-bit synchronous with the received signal. The output of the multiplier is the message immersed in broad-band noise. This output is then fed into a low-pass filter matched to the message signal. If the analog message is in a form of square wave modulation, it is recovered at the filter output. On the other hand if the message is in digital form, the output of the filter must be fed into conventional digital demodulating circuits.

4.6.6 Detailed Logical Design of MLS-RANSAC

The MLS-RANSAC (Random Access Noise Signal Address

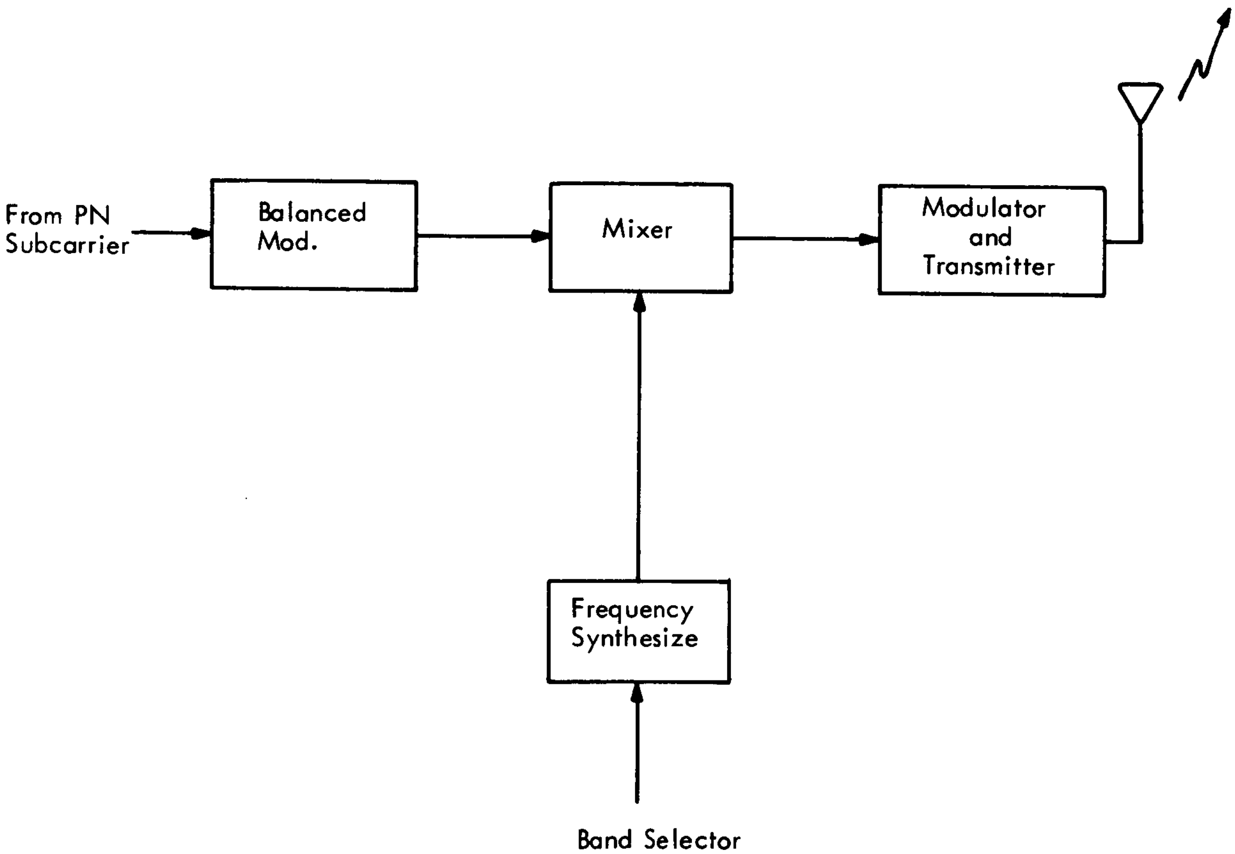

Figure 4-80. Simplified Block Diagram of CL-Transmitter

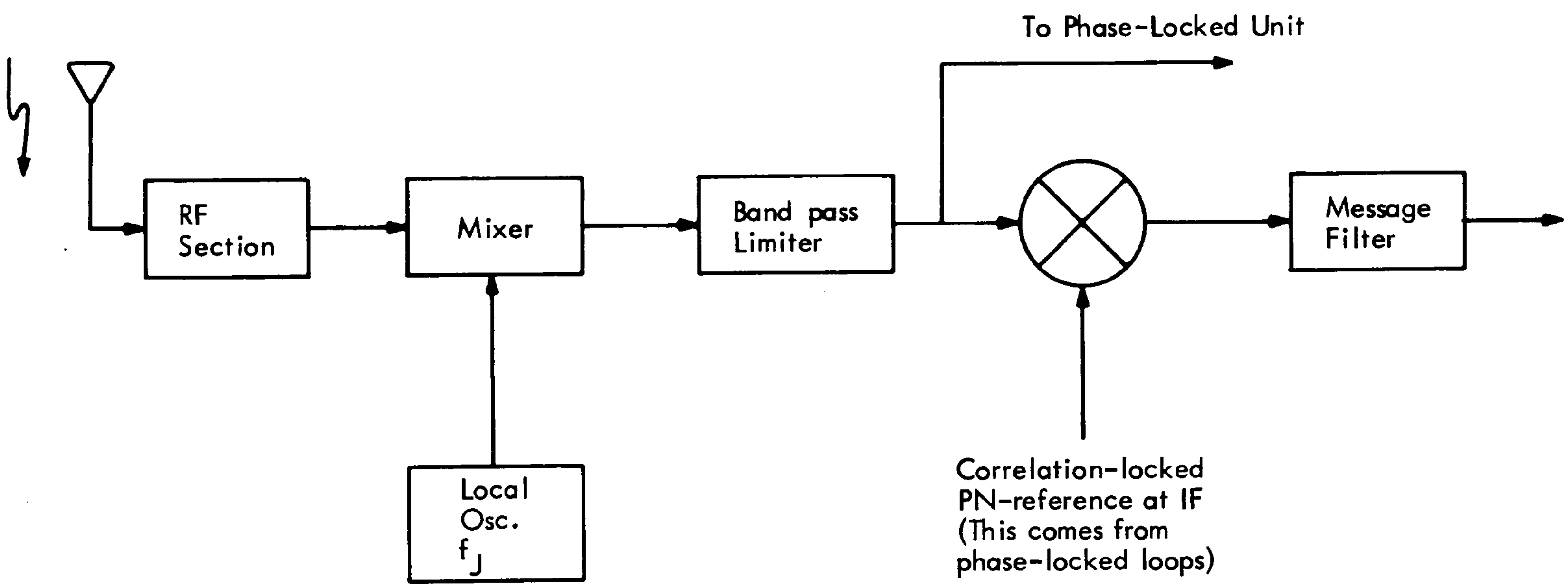

Figure 4-81. Simplified Block Diagram of Receiver

Communication) system is a communication system which permits direct user-to-user communications via a common frequency band. The number of active users of the channel at any instant is substantially smaller (perhaps 10%) than the number of subscribers. The number of active users of the satellite is a random variable which is governed by the signal, power, noise, and message statistics. The fidelity with which each message is received depends on the number of active users and on the channel parameters. The MLS-RANSAC is an optimum communications concept which makes use of the most fundamental theorem of statistical communications theory: The Channel Capacity Theorem. The RANSAC technique is equally effective against thermal noise and mutual interference of equal power. (It has a degree of immunity against strong signal mutual interference as well.)

In the MLS-RANSAC each subscriber is assigned a PN binary signal address having a large bandwidth-time (WT) product. In this application a binary signal address of 128 bits is used. Each binary message sequency is mapped into a time shift proportional to the value of the binary number. The resulting message is therefore converted to (digital or analog) pulse-position modulation. Each message pulse is then encoded directly into the called party's PN address signal. Each receiver has a matched filter which responds strongly to its own address. The unintended messages contribute mutual signal clutter which degrades the message. (For the parameters of the design point model communication is thermal-noise limited rather than clutter limited for small stations. The larger stations are both clutter and thermal-noise limited.)

Since the signal addresses have sharp autocorrelation functions, and hence the delay resolution capability, the output of the receiver (i.e., the autocorrelation function) which is matched to its intended signal will generate a sharp pulse at the instant of match, thus reproducing the PPM signal. The message is then demodulated in a conventional manner.

In the MLS-RANSAC frequency-hopping is also introduced in order to reduce the effect of strong signal mutual interference. This is also a practical way of increasing the effective WT product of the signal alphabet and hence reducing mutual clutter.

4.6.6.1 Signal Addresses

There are two addresses per subscriber, one for placing a call and subsequently for maintaining partial synchronization (if required) and the other for message transmission.

There are 128 bits per address. Each address is a binary sequence derived from a 17-bit M-sequence generator. An address is selected by specifying a 17-bit binary code which identifies the starting point in the M-sequence. The end of the sequence is derived by counting 128 and then resetting the sequence generator to its initial starting position. The synchronization address signal is selected by switching-in a different feedback path. The feedback path will result in a sequence that is the mirror image of the message sequence.

4.6.6.2 System Parameters and Characteristics

For voice applications we assume a message rate of 40,000 bits per second. (There are 5 bits per analog sample.) The low-pass signal address bandwidth is approximately 2.5 MC for an address bit rate of 5×10^6. (The RF bandwidth is 5 mcps ideally.)

Every 5-bit sequence that corresponds to an analog sample value is mapped into one of 32 delay increments. The delay resolution is $1/5 \times 10^6 = .2\ \mu$ sec. The maximum pulse position deviation is $= .20 \times 32 = 6.4\ \mu$ sec. Since each address is 25μ sec. long, mapping each 5-bit message sequence into one of 32 delays reduces the duty factor by a factor of five. This reduces the average power by a factor of five as well as the mutual clutter, and, in addition, is particularly effective against strong signal mutual interference. A strong signal falling in a time gap contributes the same amount of clutter as a weak signal since the hard limiter makes the clutter power at the output of the matched filter in the absence of signal, and is independent of the input. Thus, higher-order alphabets based on delay resolution are effective in reducing strong signal mutual interference. The strong signal mutual interference is not a problem in the system considered.

Pseudo-random frequency-hopping techniques are introduced as an additional mode of operation for the purpose of increasing the effective WT product. This mode of operation permits the accommodation of more active users and, in addition, reduces further the probability of strong signal mutual interference. This type of additional spectrum spreading has the effect of making the satellite system thermal noise limited. Nineteen disjoint frequency bands are designed into the system, each of which is selected at random for each transmitted signal address. As long as random frequency-hopping is used, the hopping need not be in discrete steps; it may be in continuous steps, although the former is preferable.

When frequency-hopping is used, the transmitter and receiver operate in a quasi-synchronous mode. If the synchronization is made more precise performance is improved.

4.6.6.3 Automatic Switchboard-Band (ASB)

This is a 5 MC (or perhaps 1 mcps) frequency band which is used only for establishing a connection. Thus, a call will be accepted only if this band is addressed. When clutter is a problem this will be a low clutter band since establishing a connection requires a code burst of short duration. Since frequency-hopping is used for message transmission, a fixed band must be assigned for calling. In addition the automatic switchboard (ASB) can accept special calls if desired. A single matched filter receiver is used for monitoring the switchboard band and for the message band. When in the standby mode the receiver is set to the calling band. It is switched to the message band after a connection is established.

4.6.6.4 Procedure for Establishing a Connection

Let A be the calling party and B the called party. A calls B by dialing B's calling address, which automatically switches A's transmitter to the switchboard band. This start or synchronization address signal is followed by a 17-bit code which identifies the calling party's address. B's message address is used for this purpose. Each binary one is transmitted as a signal address burst using the message address of B. Hence, each bit is protected against interference, noise, and ambiguity. When frequency-hopping is used, the calling party's hopping code is always used. Here A simply switches the PN generator to his "self-code," initializing it to the proper hopping sequence.

When B receives the first synchronization signal (this is an asynchronous mode) timing information is derived for sampling and detecting the 17-bit switching information which identifies the caller. This number sets up the B's message PN generator to the starting position of A's address by selecting the proper switches on the sequence. In addition, the frequency-hopping code is also preset to the received sequence. The end of the address is detected by a 7-bit binary counter generator which resets the PN generator to its initial position after 128 shift pulses.

When the calling party's address is received by B, a confirmation synch signal is generated and transmitted via the switchboard band. At the same instant B starts the frequency-hopping PN generator. When this signal is received by A in the message band the circuit is completed. This signal starts A's PN generator for frequency-hopping.

4.6.6.5 Message Reception

There is a matched filter receiver in the message band. The modulation used is PPM and the demodulation procedure ranges from asynchronous operation to post-detection synchronous operation. In the case of voice or analog messages, the demodulation process is essentially PPM, while for data sources it is a digital PPM multiple decision process.

The various modes of operation can be adjusted by a manual threshold control. When the threshold is at zero, there will be strong background noise in the absence of signal, while in the presence of signal, quieting results much like in FM. As the threshold is raised the background noise level in the absence of signal is reduced. The noise level users. The system is designed for a noise level which is acceptable when a predetermined number of users are active. System performance degrades gradually as the number of users is increased. Beyond a certain point the degradation is rapid, particularly after channel capacity is exceeded.

4.6.6.6 Detailed Description of Logical Design of RANSAC

Figure 4-82 is a block diagram of RANSAC demonstrating the logical functions of calling, being called, and sending back message.

A party is called by selecting manually the called party's address. This operation presets the called party's address into the shift register called the address selector. In addition, it inhibits those functions when the system is in a standby condition. The address selector presets PN (Generator I), the Address Generator, to the called party's code. This operation also presets flip-flop FF_4 so that the called party's address is prevented from presetting PN Generator II, the Frequency-Hopping Generator. PN II is normally preset to the calling party's code. Hence, frequency-hopping is always achieved by using the calling party's address.

In addition, the manual dial (1) presets FF_2 so that the output (4) which drives G_2 is active while G_3 is inactive. The gate, G_1, is held off by the signal at (22) while the signal at terminal (21) maintains G_2

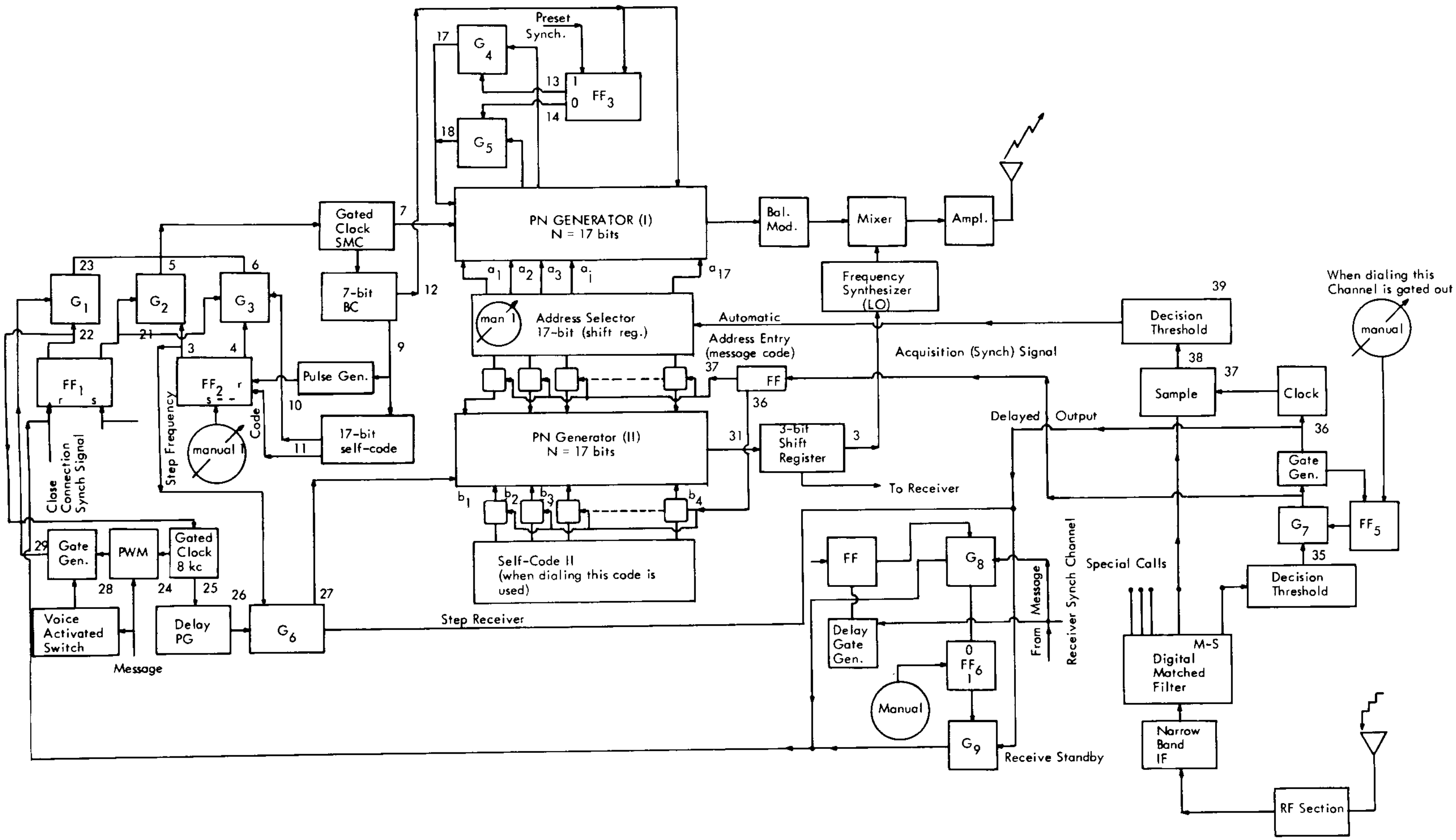

Figure 4-82. MLS-RANSAC Logical Design

and G_3 in the standby condition. The act of gating on G_2 turns on the clock which drives PN I at a 5 MC rate.

PN I is initially in the synch signal state determined by the initial state of FF_3. This flip-flop selects one of two feedback paths of PN I, which determines whether the transmitted sequence is the synch or message waveform. The initial quiescent state of PN I is the synch state. The clock output at (7) drives the PN generator such that the binary output is the called party's address. The binary signal is fed into the balanced modulator, then to a mixer and amplifier for transmission. The initial state of the VCO's is set to the switchboard band.

The clock output at (7) is also fed into a 7-bit binary counter. At the count of 128, a pulse is generated which resets PN I to the initial state determined by the 17-bit sequence stored in the Address Selector. The binary counter output specifies the end of address. In addition, the pulse at (12) resets FF_3 so that G_5 is turned on. The gate G_5 selects the second feedback path at terminal (16) while the path at (15) is inhibited. PN I will now generate the called party's message signal address. This address is the mirror image (in time) of the calling address.

The output of the 7-bit binary counter which specifies the end of address is also used at (9) to drive a 17-bit "self-code" generator. The 17-bit "self-code" represents the Address Selector switch position which identifies the starting position of PN I for generating the calling party's address. The 17-bit "self-address" is stored in a 17-bit shift register which is driven by the output at (9). The output at (9) also triggers a pulse generator which resets FF_2, selecting G_3 and inhibiting G_2. The 17-bit "self-code" then gates the clock, which in turn drives the Address Generator PN I. In this manner the calling party's PN Generator connections are transmitted to the called party. Each binary one symbol is transmitted as a complex signal having 128 PN bits while binary zero is represented by the absence of signal. Thus, the calling party's PN I connections are transmitted by means of signal which has noise and interference immunity. In addition, the triggering of FF_2 turns on G_6 momentarily, which starts PN II. This action switches the LO into the message band.

Figure 4-82 will also be used to describe the called party's receiver operation. The digital matched filter receiver in the switchboard band is normally in the standby condition. When the synch address signal is received, the decision circuit generates a pulse which passes through G_7 which is conditioned to the on-state by FF_5. The pulse turns on the gate generator which in turn gates the clock. The clock samples the received signal at precise instants of time. At each sampling instant a decision is made if a one or a zero is present. The sequence resulting from the 17 decisions is the received version of the "self-code." The result of each decision is shifted into the address selector shift register. After the 17th decision FF_5 is reset, inhibiting the synch channel of the switchboard receiver.

The Address Selector presets PN I of the called party to the calling party's address. The received synch address sets FF_4 so that the received 17-bit code is also applied to the hopping generator, PN II (the "self-code" is inhibited by this action). At the end of the received "self-code" a pulse is applied to G_6 stepping the transmitter to a different frequency. (It should be recalled that the same operation is performed by the called party after the "self-code" is transmitted.)

The called party now must transmit a confirmation signal. For this purpose the synch address will be used over the message band. Since the called party is always in a standby mode, FF_6 selects G_9. The delayed pulse passing through G_9 triggers FF_1 which transmits a confirmation signal.

The calling party has FF_6 in the state which selects G_8. The confirmation message address is now received on a different frequency. The confirmation signal passes through G_8 and sets FF_1 in the communication mode; that is the circuit is now closed. The confirmation signal comes via the message receiver band. The purpose of receiving the confirmation on the message band is to prevent another caller from introducing a signal in the calling party's receiver which is awaiting a confirmation. A third address can be used for this purpose.

With FF_1 in the on state, G_1 is gated which, of course, activates PN I for sending message. In addition, the sampling 8 kc clock which is now free running, is turned on. This pulse subcarrier triggers a pulse generator at (24) which is pulse-width modulated (PWM) by the message. The trailing edge of the pulse triggers a gate generator of duration equal to 128 clock pulses (5 MC). The clock pulses at (7) then activates PN I.

In order to inhibit the pulse subcarrier when information is not transmitted, a voice activated switch is used. This turns the pulse signal off in the absence of the voice message.

4.6.6.7 Message Receiver Block Diagram

For the purpose of this discussion it will suffice to discuss the message receiver beginning with the matched filter output shown in Figure 4-83. The matched filter is pre-wired to receive one of two addresses, the synch signal or the message signal. The synch signal at (13) passes through a gate which is opened for a time interval when the synch pulse is expected. The synch pulse drives the clock at (14) so that it synchronizes with it. At (15) the clock in turn triggers a delay gate generator which inhibits the gate until the next synch signal is expected, when it opens. In addition, the clock triggers a gate

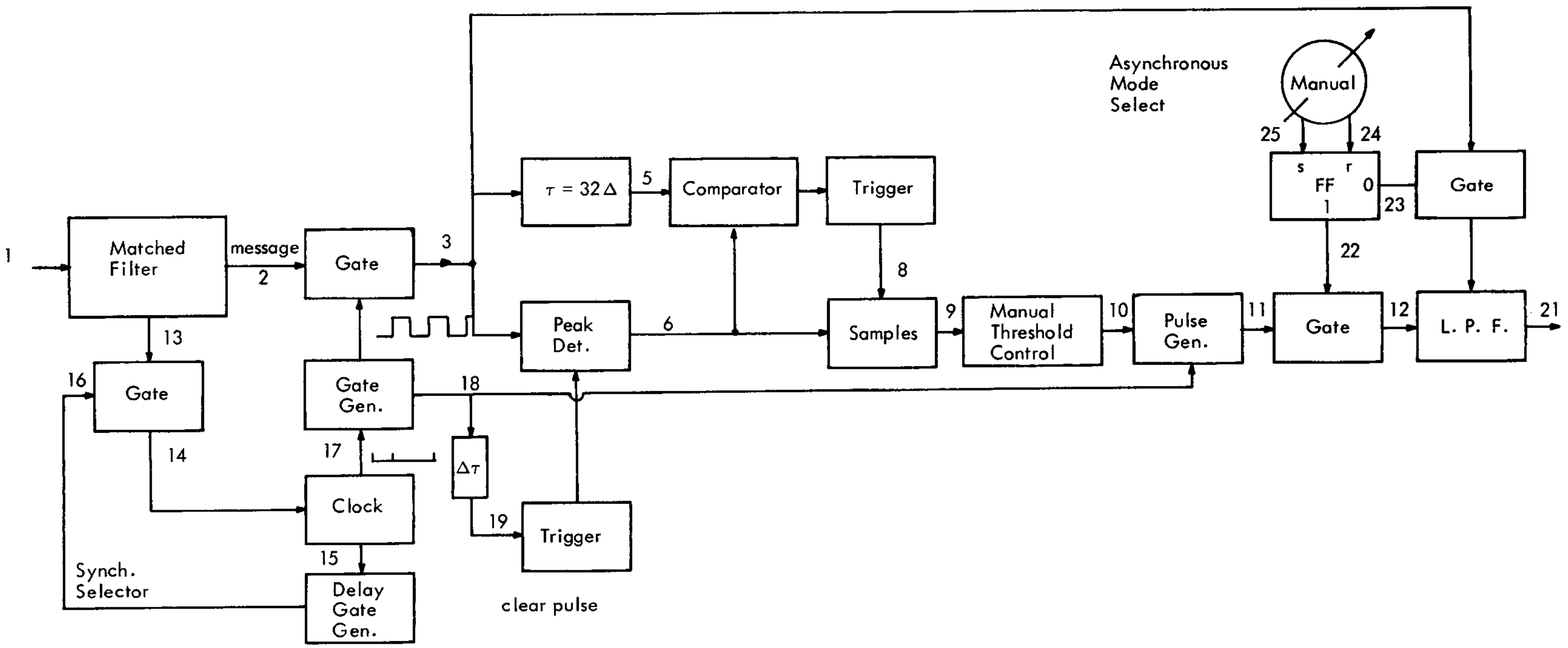

Figure 4-83. Message Demodulator

generator at (7) which is in this state for a length of time equal to the maximum pulse time duration. That is, signal is gated through at (3) only when the message signal is expected, and inhibited for the time interval when the message signal is not expected. This operation is equivalent to a 32 level higher-order PPM alphabet. Widening the gate increases the error probability. (Precise timing is, however, not required.) The signal at (3) enters a peak detector which holds the maximum value of the signal which entered the gate. This is essential since the optimum decision procedure decides that a pulse is present in the time interval which has the largest value. In order to effect this procedure, the received signal at (3) is also delayed in an analog line for 32 increments. The delayed output at (5) is compared with the stored peak in the comparator and a pulse is generated at (7). The stored values are sampled and fed into the manual threshold control for a final decision. If the threshold is exceeded, the pulse generator which has been preset by a pulse at (8) is reset by the output of the threshold detector. The signal output at (11) is therefore a pulse whose width is proportional to the transmitted analog message sample. The analog message is recovered by the low-pass filter when the gate is open.

The manual threshold control, (9), is extremely important for optimizing reception. It permits several modes of operation depending on the clutter statistics of the environment.

Mode 1: Greatest of Decision; Zero Threshold

In this mode the threshold is set at zero. When a party is talking the receiver will not receive message although it will be open. Since the threshold is zero, a decision will always be made as to the maximum based noise. Clearly, this condition will generate strong background noise during pauses and in the talking party's receiver. This condition is equivalent to FM in the absence of carrier. In this system too, if carrier is transmitted in the absence of modulation, quieting will occur during gaps of zero modulation. Mode 1 is an optimum procedure when carrier is transmitted. When carrier is turned off it may be possible to switch off the audio section when noise only is present.

Mode 2: Greatest of Decision; Finite Threshold

The decision procedure here is identical to Mode 1 except that to obtain an indication of the presence of signal a threshold must be exceeded. In the absence of carrier this threshold has the effect of reducing false indications, and hence the noise background is reduced. This mode of operation also reduces the probability of a signal indication when the signal is present. Mode 2 is therefore a compromise between background noise in the absence of carrier and noise generated during the message.

Mode 3: Threshold Detection

In this mode the threshold is set at some level, and every crossing is indicated as a signal. This operation can be completely asynchronous, if desired. A true indication requires that a pulse cross the threshold at the instant when signal is present and that there be an absence of crossing at all other instants. This decision procedure is the most inefficient, but also the simplest.

In Figure 4-84 the Manual Select Switch chooses either asynchronous threshold detection or "greatest-of" detection. In low density environments the former is a good operating mode. In face, in low density signal application the synchronous mode can be eliminated, completely reducing the circuit complexity. In addition, the frequency-hopping mode can be eliminated, reducing complexity even further. This will be an area of study in Phase III.

4.6.6.8 The Busy Signal

We do not show the logic of transmitting or receiving the busin signal. There are two types of operations possible. In mode 1, when dialing, the matched filter taps are adjusted to the called party's address by the use of auxiliary switching logic. The calling receiver can then listen to the environment for a busy signal. Provisions would have to be made to avoid "snooping," for example, by disconnecting the audio in the calling mode. This mode requires the addition of complex logic.

The second mode must have an auxiliary "call-code" generator at each terminal. The receiver must also have a third address for receiving switching information. The calling party dials in the usual manner and transmits the called party's address followed by his "self-code." This switches the auxiliary sequence generator to the calling party's address. The calling party can then generate the busy signal for a predetermined time. This mode has the advantage of simplicity, although only one busy signal can be transmitted at a time. A potentially serious disadvantage in this mode is discussed in the next section.

4.6.6.9 Detailed Block Diagram of the Receiver IF Section

Figure 4-84 is a more detailed block diagram of the MSL-RANSAC receiver through the IF. There are two channels; the calling band and the message band. The two IF's are both 5 mcps wide. The call is placed via the IF_1, while message arrives via IF_2. That is, the oscillator at the receiver is stepped through 19 frequency bands in synchronism with the transmitter. Thus, a 5 mcps portion of the spectrum is translated into the message IF. The output of the IF is then fed into a matched filter followed by an envelope detector. In this mode the matched filter must be switched to the called party's address in order to receive the busy signal.

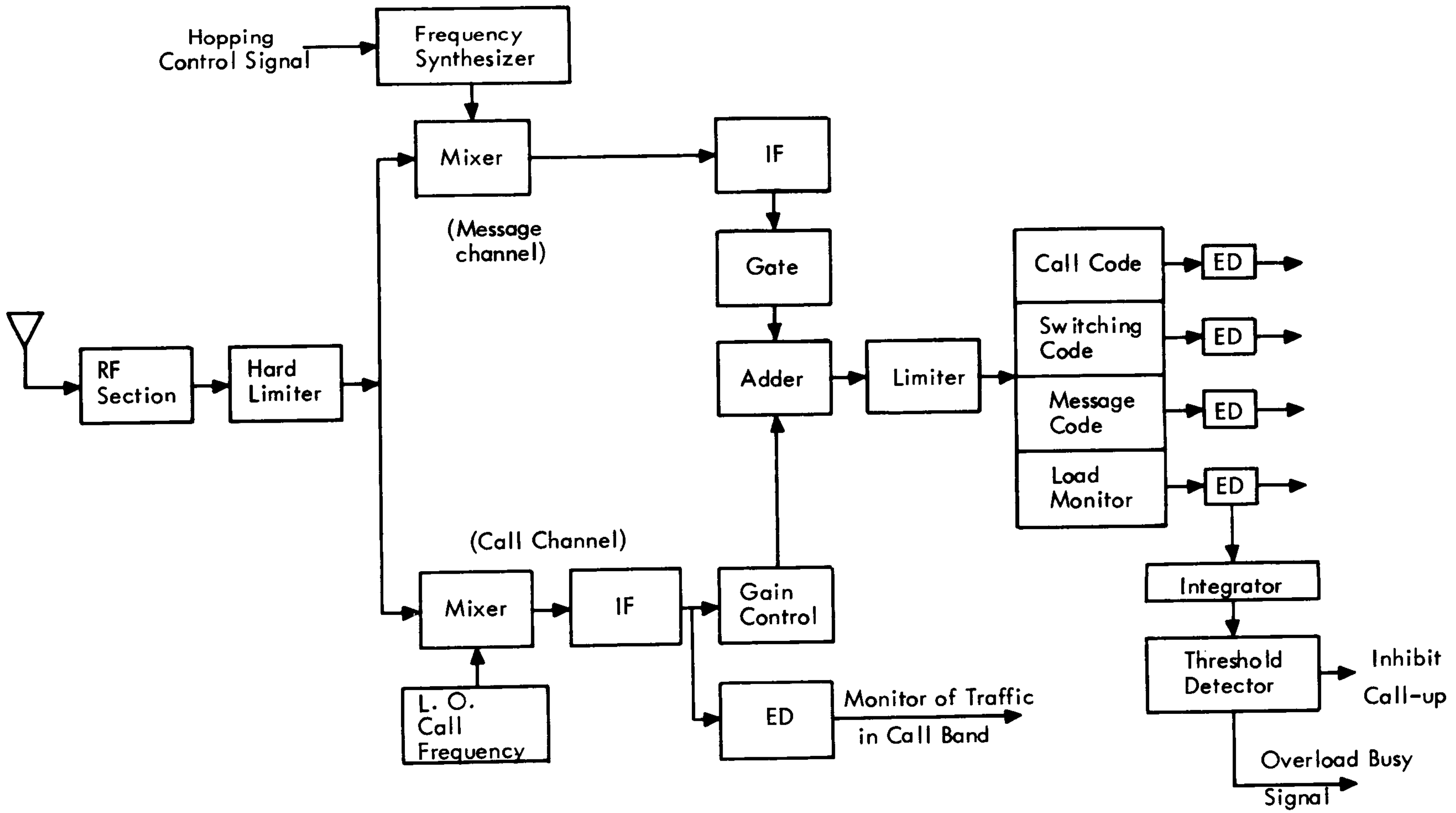

Figure 4-84. RANSAC Front-End of Receiver

If the called party is to send back a busy signal the operation is still the same in the standby mode. However, when the called party is busy it is essential to combine the output of the message channel and the call band. This, unfortunately, doubles the noise at the input to the matched filter and results in a 3 db loss in signal-to-noise ratio. This penalty may be more serious than adding complexity to the equipment.

Another possible approach to busy signal reception in the second mode is to have a single 5 mcps frequency band and to increase the dwell time for each hop to, say, 34 message signal durations. Since 25 μ sec message signal is used this will result in a dwell time of 850 μ sec. or approximately seven message sample periods. That is, the frequency-hopping occurs every eight message samples. Furthermore, if it is ensured that, say, every ninth hop the oscillator returns to the call band, then a busy signal can be obtained in less than 7650 microseconds.

The problems associated with busy signal reception in the MLS-RANSAC will receive more detailed attention during Phase III.

4.6.6.10 Automatic Traffic Monitoring and Regulation

In the MLS-RANSAC traffic density can be monitored at each station quite simply. We assume that multiplexing at the satellite occurs at a hard limiter. Thus the average power in the down link is not a good measure of traffic. Assume that a given station transmits a periodic traffic test signal at the message duty factor. This can be a burst of RF without PN coding, transmitted at the message rate outside the message band. The output of the IF is then square-law detected and integrated. In dense traffic, the condition of interest, the average power will be proportional to the number of signals being multiplexed since power division will occur at the limiter. Even during low activity, the condition not of particular interest, this will probably be a good indication of traffic conditions. This measurement may be used to regulate traffic automatically and prevent the system from overloading. The overload channel is shown in the receiver, Figure 4-84.

This type of automatic traffic regulation will receive further attention in the next phase.

4.6.7 Detailed Logical Block Diagram of CL-RANSAC

Here the detailed logic of the CL-RANSAC will be discussed.

4.6.7.1 Signal Addresses and System Parameters.

Each station is assigned a unique maximal length sequence. A 13-stage M-sequence generator is used. This generator must be capable of generating a sequence of period, 8191 bits at a 10 megabit rate. Thus, 0.82 milliseconds are required to receive the entire call-up sequence. The maximum time for obtaining a connection is 0.82 seconds. A 13-bit M-sequence has 630 distinct sequences which can be obtained by changing the feedback paths.

We will assume that the voice message is encoded into 5 bits per sample giving a message rate of 40,000 bits/sec.

4.6.7.2 Call-Up Logic

The operation of the call-up receiver is shown in Figure 4-85. The Synch Programmer has a stored program that performs a systematic search in resolving the PN bit ambiguity and the small uncertainty in the carrier frequency due to doppler. The program specifies the order in which the ambiguous bit and frequency search will be performed, and will terminate the process when acquisition has been achieved. Since a short sequence is being used, the initial call-up receiver continuously searches over the bits of the calling sequence. In order to accomplish the PN bit search, the Synch Programmer calls for a local replica to be generated and mixed with a possible incoming signal. The VCO sweep should be performed first over a small number of PN bits and if the phase lock loop receiver indicates a tentative synch condition by exceeding a threshold then the integrate and dump process is performed to obtain the required WT process gain. If acquisition is not achieved, the following possibilities exist:

(1) There is an input and the local replica is OK, but an error in recognizing this condition had been made.

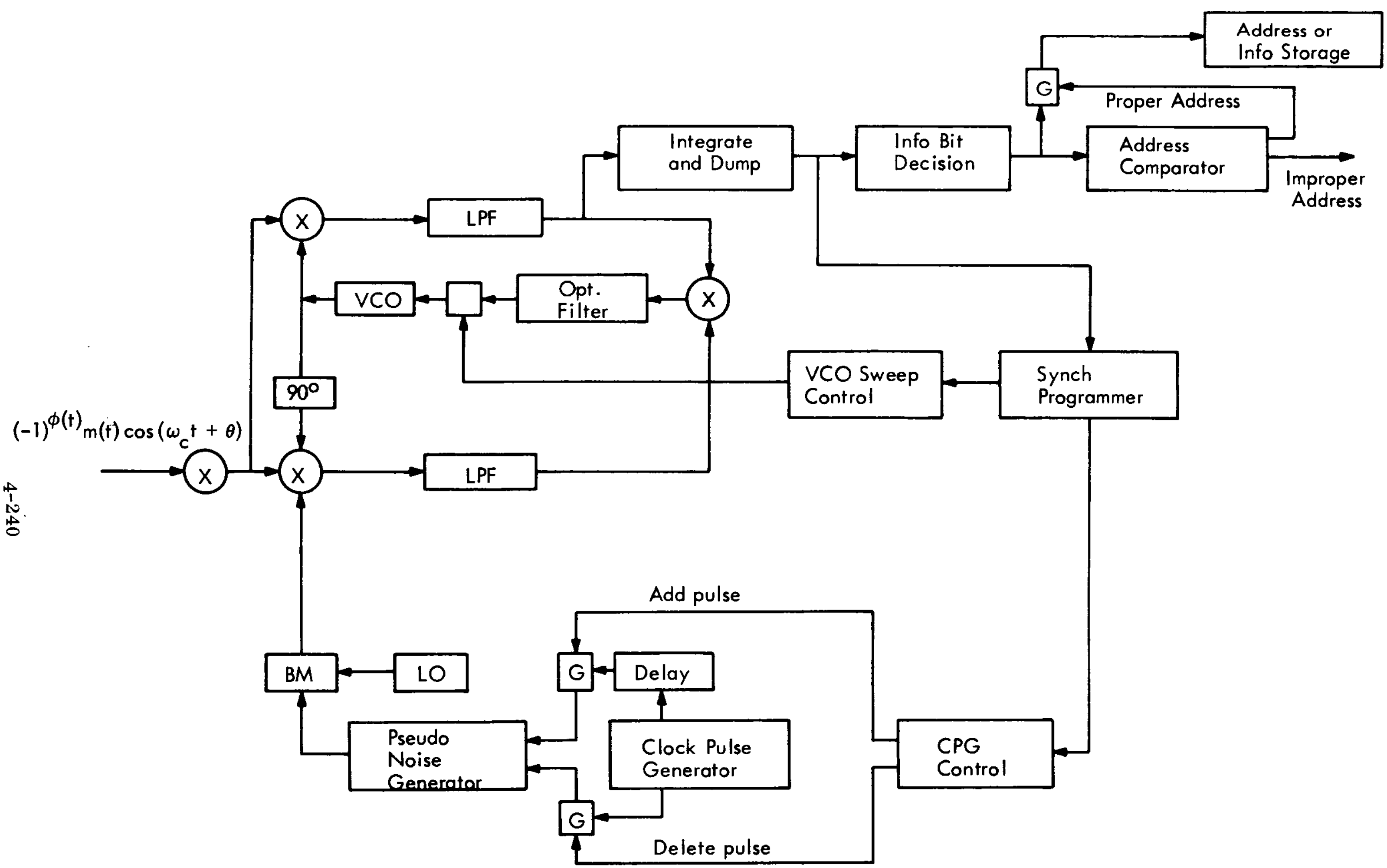

Figure 4-85. Functional Diagram of Callup Receiver

(2) There is an input, but the local pseudo noise generator is not in synchronization with the incoming pseudo noise modulation.

(3) There is no input.

The probability of the event described as (1) above occurring will be minimized, but there is nothing that can be done about (3). The Synch Programmer next generates a portion of the PN sequence which is displaced by one pseudo-random bit with regard to the sequence used for the previous comparison. This is accomplished by the Clock Pulse Generator Control by either adding an extra pulse or deleting a clock pulse into the pseudo sequence generator. Having obtained the desired portion of the sequence over which a comparison with the possible incoming sequence is to be made, the process described previously is repeated. The Synch Programmer controls this operation as described in Section 4.6.5.1.

In Figure 4-86, Block (2), the called party has gone into an acquisition search anticipating the calling party's response.

4.6.7.3 Message Reception

The Information Receiver shown in Figure 4-87 resembles the Call Up Receiver of Figure 4-85. The PN sequence used in this is longer than the one used for the call-up process. The Coarse Synch Programmer performs the same type of function as the Synch Programmer considered in the call-up receiver. In addition there is an early and

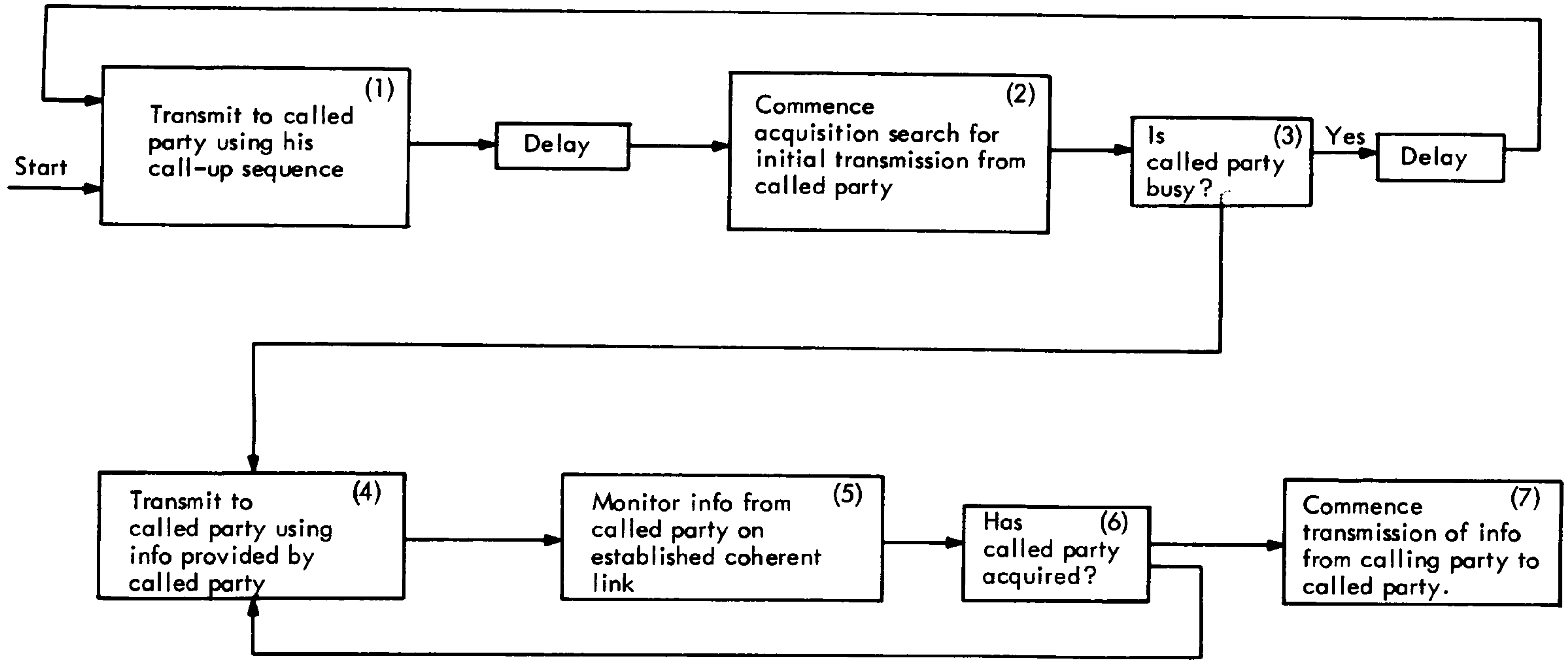

Figure 4-86. Callup System—Calling Party Mode of Operation at Ground Station

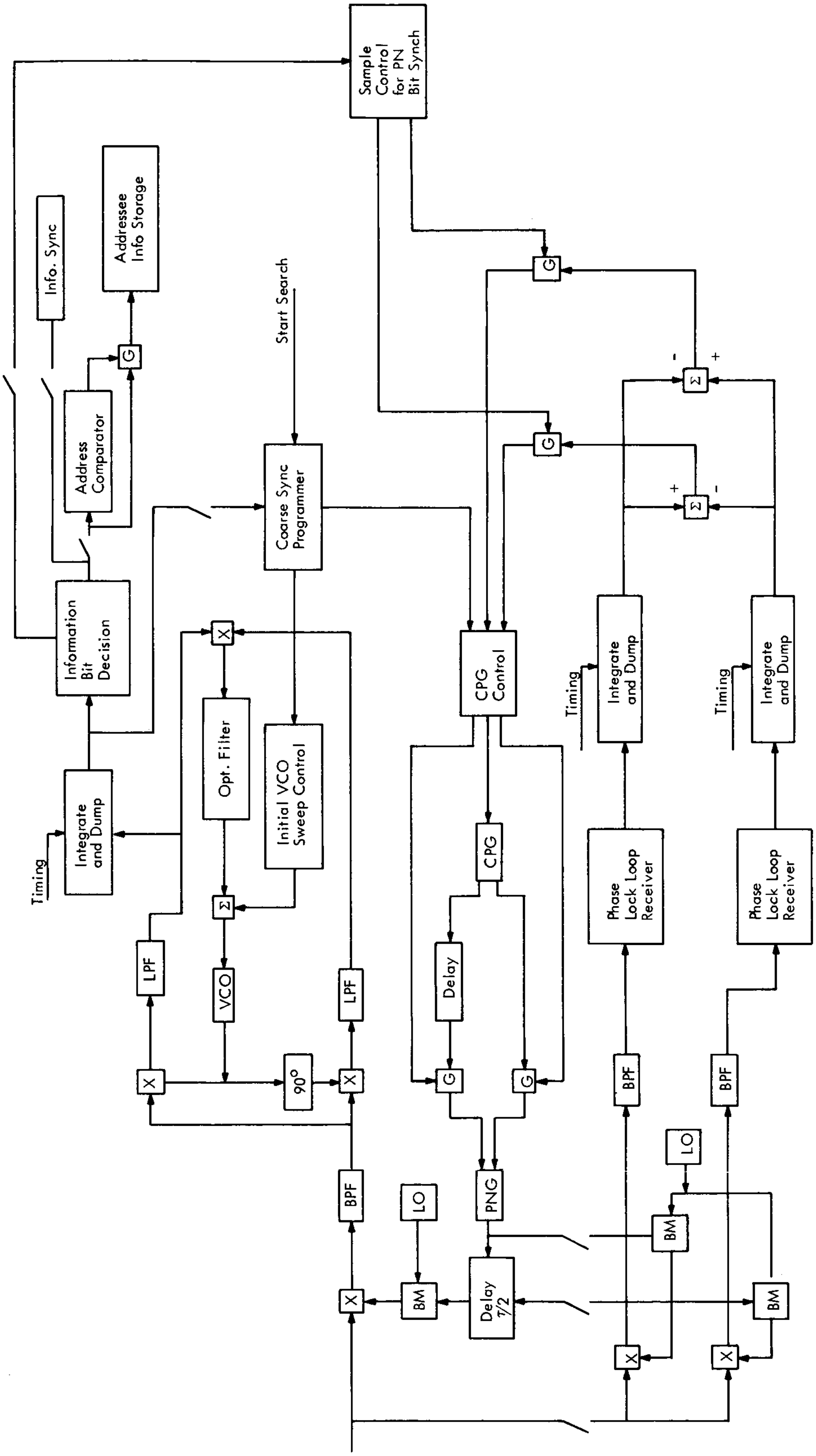

Figure 4-87. Information Receiver

late correlator to maintain bit synchronization once the PN bit ambiguity has been reduced to a single bit.

The CPG instability both at the transmitter and receiver gives rise to the need to track the incoming pseudo noise bits once synchronization has been established to within a bit.

This goal is accomplished by generating an early and late version of the incoming pseudo noise sequence. These sequences are used to generate the early and late local replicas to feed into the early and late correlators.

We first assume $M(t) \equiv 1$ and later will show how this condition is removed. Taking the low frequency component of the output of the two multipliers, we have going into the PLL in the late correlator,

$$(-1)^{\phi(t) + \phi\left(t - \frac{T}{2}\right)} \cos\left[\left(\omega_c - \tilde{\omega}_c\right) t + \theta\right] \tag{4-210a}$$

and into the PLL in the early correlator

$$(-1)^{\phi(t) + \phi\left(t + \frac{T}{2}\right)} \cos\left[\left(\omega_c - \tilde{\omega}_c\right) t + \theta\right] \tag{4-210b}$$

In the expression above, $\phi(t)$ denotes the pseudo noise modulation on the incoming sequence. In actuality, the incoming sequence will be perturbed in time and this is represented as $\phi(t + \Delta)$, where Δ is a random variable. The perturbation results from the instability associated with the oscillator that feeds into the clock pulse generator at the transmitter. The instability of the oscillator at the receiver also contributes to the time perturbation Δ. Thermal noise effects will not be considered to affect Δ, for in the presence of thermal noise the incoming signal is,

$$\cos(\omega_c t + \alpha(t) + \theta(t)) + x(t)\cos\omega_c t + y(t)\sin\omega_c t$$

$$= \cos\left(\omega_c t + \alpha(t) + \tan^{-1}\frac{\sin\theta(t) + y(t)}{\cos\theta(t) + x(t)}\right) \qquad (4\text{-}211)$$

where $x(t)$ and $y(t)$ are independent gaussian random variables and (t) is the pseudo-randomly generated phase angles 0 and π. In other words the incoming phase noise, $\tan^{-1}\frac{\sin\theta(t) + y(t)}{\cos\theta(t) + x(t)}$, will enter the phase lock loop receiver, but the slippage between the incoming pseudo-random bit and the locally generated PN bit will be minimized by tracking the incoming slow jitter on the PN bit.

If the incoming PN sequence $\phi(t)$ is delayed by half a PN bit, $\frac{T}{2}$, we have going into the PLL receiver in the late correlator,

$$(-1)^{\phi(t - \frac{T}{2}) + \phi(t - \frac{T}{2})}\cos[(\omega_c - \omega_c)t + \theta]$$

$$= \cos[(\omega_c - \omega_c)t + \theta] \qquad (4\text{-}212a)$$

and into the PLL receiver in the early correlator

$$(-1)^{\phi(t - \frac{T}{2}) + \phi(t + \frac{T}{2})}\cos[(\omega_c - \omega_c)t + \theta] \qquad (4\text{-}212b)$$

In the latter case, $\phi\,(t - \frac{T}{2})$ and $\phi\,(t + \frac{T}{2})$ are displaced by a single PN bit so $\phi\,(t - \frac{T}{2}) + \phi\,(t + \frac{T}{2})$ gives rise to a shifted version of the original sequence due to the cycle and add property of maximal length sequence. In this case the late correlator provides the indication that the incoming sequence is in step with the sequence $\phi\,(t - \frac{T}{2})$. This will be obtained at the output of the integrate and dump circuit of the late correlator.

When the incoming sequence approaches the sequence $\phi\,(t + \frac{T}{2})$ the situation will be reversed in comparison to the previous case.

If the outputs of the early and late correlators are combined at the sampling instant by subtracting the output of the late correlator from that of the early correlator the characteristic in Figure 4-88 will be obtained.

When the PN modulation, $\phi\,(t)$, on the incoming signal is in synch with the locally generated sequence there will be no output signal exceeding the threshold from either of the two correlators. The phase lock loop response will be designed so that there will be no response to phase changes that are constant over a time interval shorter than half a PN bit length, $\frac{T}{2}$. As the incoming PN sequence, $\phi\,(t)$, approaches $\phi\,(t + \frac{T}{2})$, for example, the phase excursions due to radians

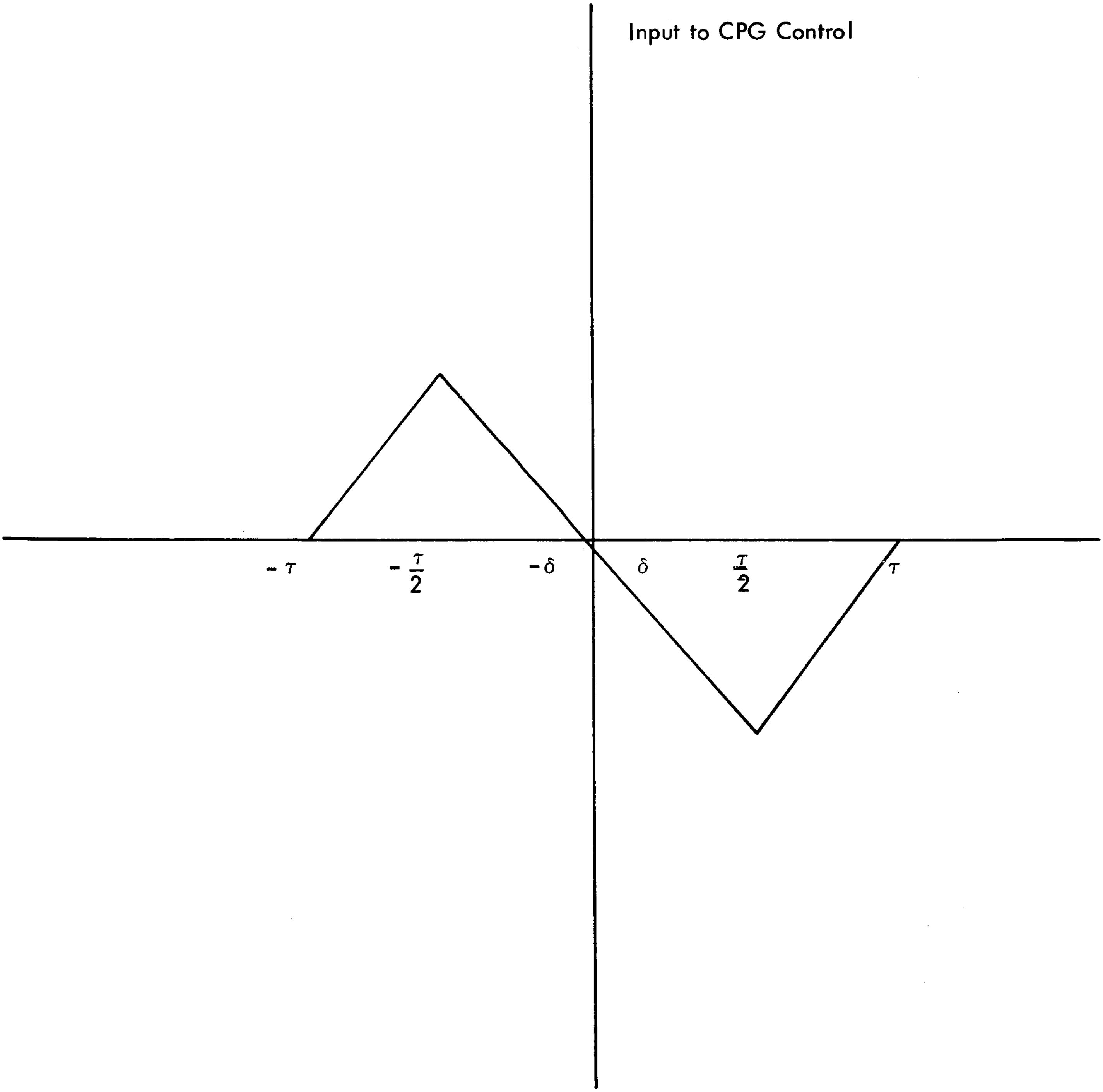

Figure 4-88. Error Function for PN Bit Sync System

$$(-1)^{\phi(t + \Delta) + \quad (t + \frac{T}{2})}$$

will occur over a period $\left|\frac{T}{2} - \Delta\right|$, where $\Delta < \frac{T}{2}$, in the early correlator will be designed to lock up to phase 0 radians when Δ reaches some predetermined value. In this case, at the late correlator, the phase excursions to π due to $(-1)^{\phi(t + \Delta) + \phi(t - \frac{T}{2})}$ will occur over a time interval of $\Delta + \frac{T}{2}$. The phase lock loop will not lock, for it will attempt to respond to the phase changes.

So far, it has been assumed, that there is no information modulating the incoming signal. That is, we have assumed $m(t) = 1$. When information is transmitted, the combining of the outputs of the early and late correlators requires consideration. It is proposed that the output of the information bit decision circuitry be used to read out the properly combined output of the early and late correlators.
(See Figure 4-89)

Returning to the error fcn of Figure 4-88, when the slippage between the incoming and locally generated PN bits is between $-\Delta$ and $+\Delta$, there will be no attempt to compensate for the slippage. The effect of this slippage between the PN bits by the amount $\pm\Delta$ is to deteriorate the signal to noise ratio at the output of the integrate and dump circuitry, which precedes the information bit decision system. This presumes that the main phase lock loop receiver has not lost lock

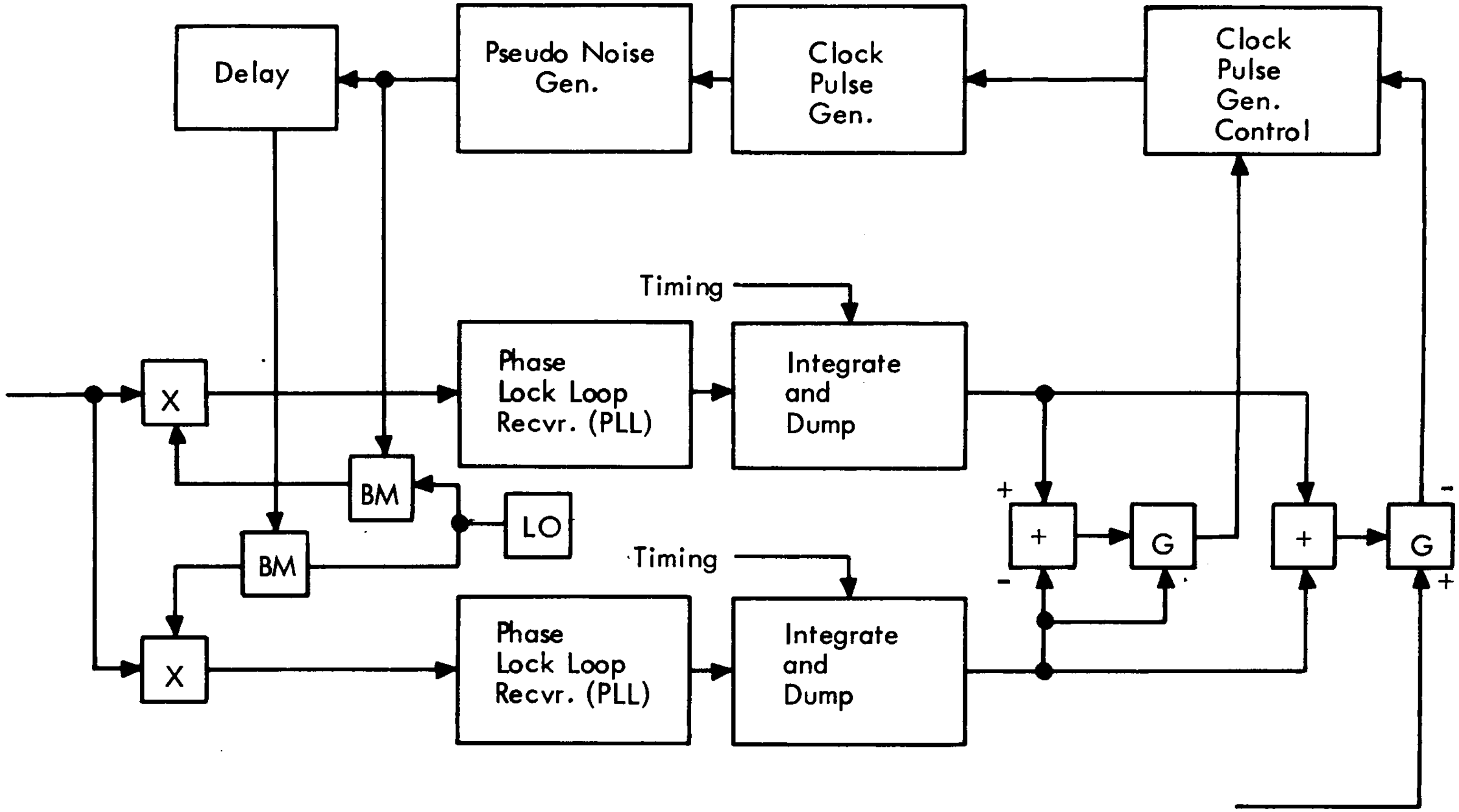

Figure 4-89. PN Bit Sync System

during the time that this maximum slippage, Δ , has occurred. The loop filter will require proper design to achieve this.

The CL-RANSAC will receive further study in Phase III with the aim of optimizing this technique.

GLOSSARY

PN = Pseudo noise.

MODEM = Modulation-demodulation equipment.

$Z_o(t)$ = Complex modulated carrier.

$Z(t)$ = Complex envelope function.

$a(t)$ = Envelope of modulated carrier.

$\phi(t)$ = Phase of modulated carrier.

M = The order of a alphabet (i.e., the number of possible results of a measurement).

T = Duration of a message symbol.

ΔT = Duration of a PN bit.

W = Bandwidth of a message symbol (wideband).

W_o = Bandwidth of information (narrow band).

N = Number of PN bits per message symbol.

$\Delta \omega$ = Frequency shift.

FSK = Frequency shift keying.

$D(W_o)$ = Energy density of narrow band signal.

$D(W)$ = Energy density of wideband signal.

E = Signal energy

N_o = Noise density (watts/csp).

T_o = Multipath duration.

$x(t)$ = In-phase component of a bandpass signal.

$y(t)$ = Quadriture component of a bandpass signal.

DELTIC = Matched filter incorporating active correlator.

T_o = Duration of a message bit.

$D(\varepsilon)$ = Discriminator characteristic.

BPL = Band pass limiter.

VCO = Voltage controlled oscillator.

ED = Envelope detector.

MF = Matched filter.

CW = Carrier wave.

σ_N = Noise power.

s^2 = Signal power.

A-D = Analog to digital.

PPM = Pulse position modulation.

ρ = Correlation function.

$F(\omega)$ = Signal spectrum.

d_{jk} = Distance between signals in the mean square sense.

h = Entropy.

$E\{\cdot\}$ = Expected valus of $\{\cdot\}$.

RANSAC = Random Access Noise Signal Address Communications.

CL = Correlation-locked.

MLS = Maximum likelihood sequence.

mod-2 = Modulo 2 (exclusive OR).

ASB = Automatic switchboard band.

FF = Flip-flop.

G = Gate.

CPG = Clock pulse generator.

PNRM = Pseudo noise subcarrier rate modulation.

PLL = Phase locked loop.

SECTION 5

SPECIAL DEVICE STUDIES

5.1 Introduction

A satellite communication system involves many physical operations which affect system performance. Of particular interest in this program are those operations that are intimately related to the modulation techniques. The TWT in the satellite is an extremely important device which affects system performance depending on the modulation used. The hard limiter is another important device which is of particular importance to PN modulation techniques and therefore deserves special attention. Frequency compressive feedback is an important operation in the ground receiver if FM is used in the down link. These are examples of special devices whose operation affects performance and which are related to a particular class of modulation techniques.

An effort along these lines was initiated recently with emphasis on the TWT. A preliminary survey of the TWT characteristics and the manner in which communications is affected by this device is discussed here.

5.2 TWT Characteristics

During this reporting period an investigation of the TWT operating characteristics as applied to a multiple-access problem was initiated. The effects of TWT fading, compression, and AM to PM conversion when a single signal is applied as the input to a TWT have been investigated and reported in the literature. However, the output properties of a TWT in response to many input signals are less known, and this is the area that requires work.

A summary of the Bell Telephone M-1789 tube characteristics when operated under nominal input conditions serves to describe the TWT phenomena mentioned above. This tube was designed to operate as an amplifier which provides a gain of 30 db at 5 watts of output in the 5925 mc to 6425 mc band. The measurements were made under a nominal input signal with the following characteristics:

a. Frequency: 6175 mc (band center)

b. Beam current: 40 ma

c. Magnetic flux density: 600 Gauss

d. Collector voltage: 1200 volts

Figure 5-1 shows the power output as a function of the power input when the nominal input signal is applied. In this case the helix voltage was varied to examine the effects of different voltage levels.

From the curves shown in Figure 5-1, it is clear that at higher helix voltages a larger output power is obtained at saturation. One further point of interest is that at higher helix voltages linear operation is maintained to higher output levels. From the standpoint of gain,

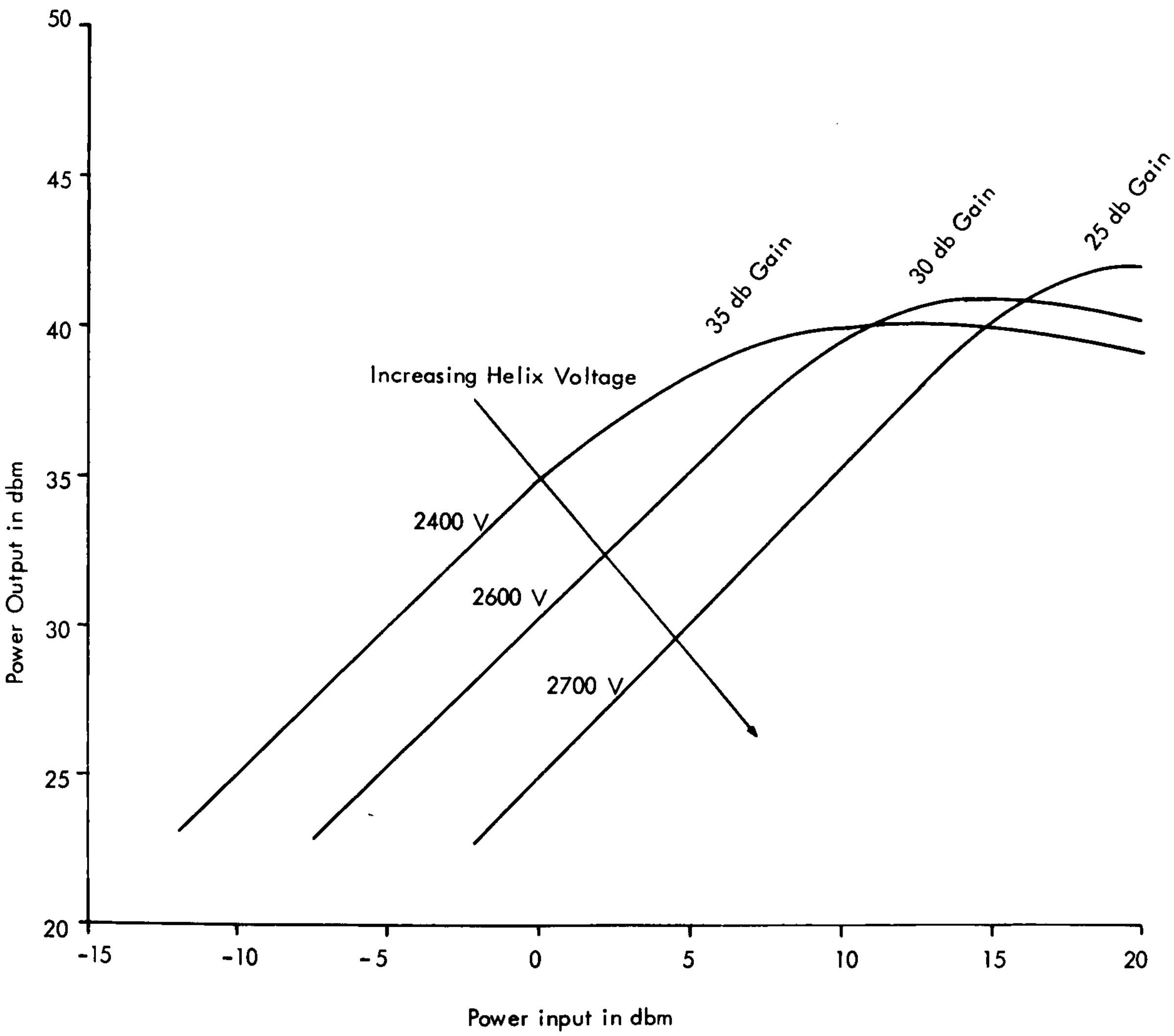

Figure 5-1. Power Output vs Power Input with Helix Voltage as Parameter

a larger gain is achieved at lower helix voltages.

In making the power measurements of Figure 5-1, a self-balancing bridge is used in conjunction with a bolometer. The heat dissipated by the signal in a temperature sensitive resistor determines the RF signal power. A phenomena in TWT's, referred to as fading, makes it mandatory that one performs the power measurement properly. Fading occurs when the TWT is driven to a high output level after having been operated for several minutes without an input signal. In this case the initial output will be greater than what is shown in Figure 5-1 and will decrease to a stable level in several minutes. This fading phenomena is caused by an increase in the intrinsic attenuation of the helix near the output end as a result of the applied RF signal dissipating its power as heat. In making the measurement for Figure 5-1, sufficient time must be allowed to elapse between an input power level change.

Figure 5-2 shows the maximum output power after fading as a function of frequency for the cases in which the helix voltage is adjusted for maximum gain and for the case in which the helix voltage gives rise to maximum output power. It is of interest to note that the fading effect is less in the case of tube operation with maximum gain.

Returning to the power out versus power in relationship under nominal operating conditions, the phenomenon of compression and

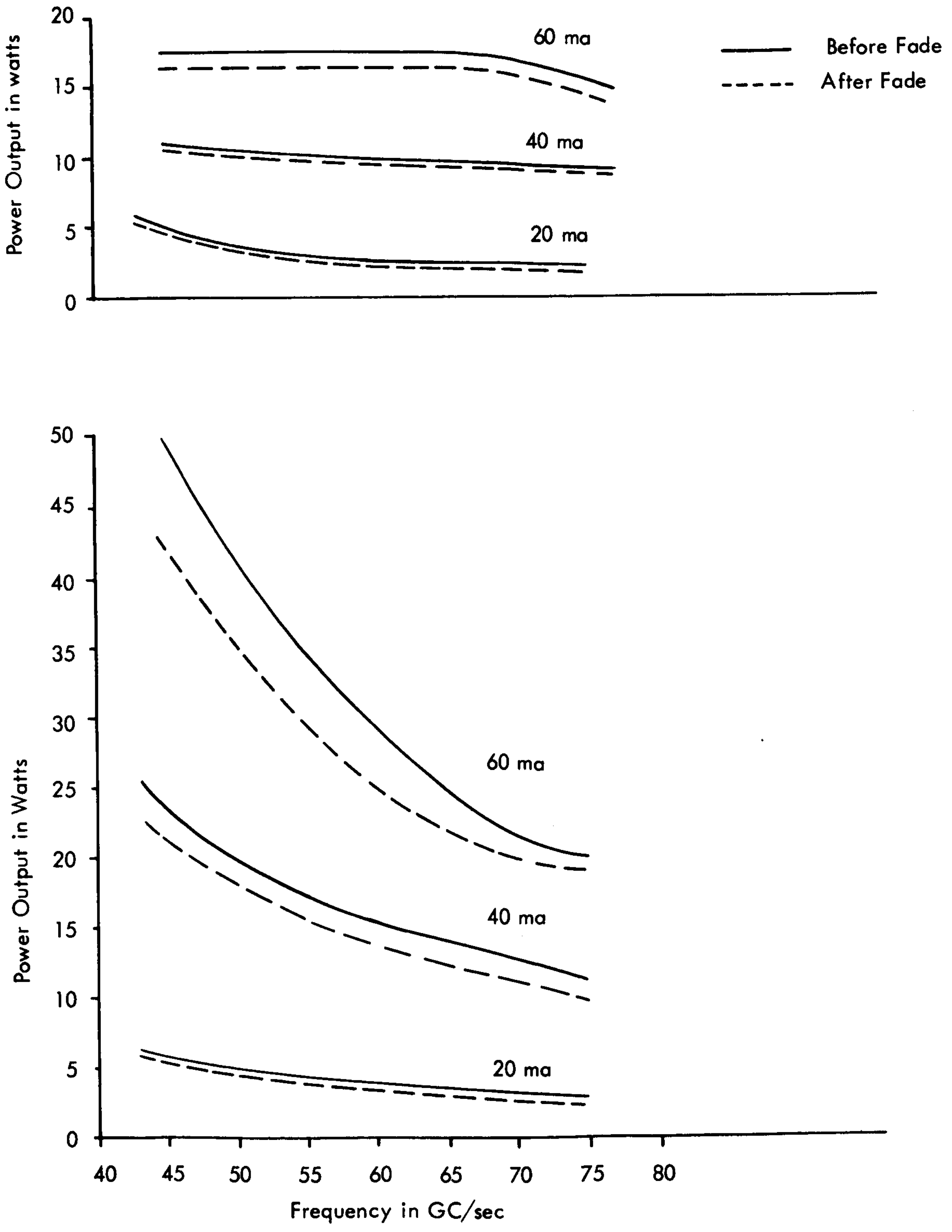

Figure 5-2. Max Power Output After Fading vs Frequency

AM to PM conversion by the TWT warrants discussion. With regard to the first, the percentage of compression is defined as follows:

$$\left[1 - \frac{\dfrac{\Delta V_o}{V_o}}{\dfrac{\Delta V_i}{V_i}} \right] 100 = c \times 100 \qquad (5\text{-}1)$$

where V_o is the voltage at the TWT output, V_i is the input voltage, and ΔV_o is the change in the output voltage when there is a small change in the input voltage by an amount of ΔV_i. In the case of a linear amplifier, the percent compression is zero, and in the saturation region of an ideal limiter the percent compression is 100.

Figure 24[22] discusses a method for measuring the percentage amplitude modulation at the input and output of a TWT with crystal monitors. A carrier at the nominal frequency of $\frac{\omega_c}{2\pi} = f_c = 6175$ mc is modulated with a $\frac{\omega_m}{2\pi} = f_m = 60$ cps sinusoid with an amplitude modulation of 1 db. The input signal at the TWT input is

$$V_1(1 + a \sin \omega_m t) \sin \omega_c t \qquad (5\text{-}2a)$$

The amplitude modulation on the output signal is

$$1 + a(1-c) \sin \omega_m t \qquad (5\text{-}2b)$$

since

$$\frac{\Delta V_o}{V_o} = (1 - c) \frac{\Delta V_i}{V_i} \qquad (5\text{-}2c)$$

Figure 5-3 shows the percentage compression plotted as a function of the input power with the helix voltage as a parameter. The curves corresponding to the helix voltage of 2700 v show that the linear amplification range is wider than in the case of the 2400 v helix voltage.

The next phenomena of interest is the AM to PM conversion which occurs due to the fact that the electrical length of a TWT operated in the non-linear region is dependent upon the input power level. As a result of this, any AM on the input signal results in the PM of the output signal. The same test procedure referred to for the compression measurement is used to measure the conversion. Figure 5-4 shows the AM to PM conversion in terms of degrees per decibel change in amplitude as a function of input power. The helix voltage is also a parameter in this case.

As a result of compression and conversion, the output of the TWT in response to the input given in Equation(5-2) will be

$$KV_1 \{1 + a\,(1-c)\,\sin\omega_m t\}\ \sin(\omega_c t + k_p\, a \sin\omega_m t) \tag{5-3}$$

where K is the amplification and k_p is the conversion factor. For an input amplitude modulation change of a , a phase change in $\Delta\,\theta$ radians results.

$$k_p = \frac{\Delta\theta}{a} \tag{5-4}$$

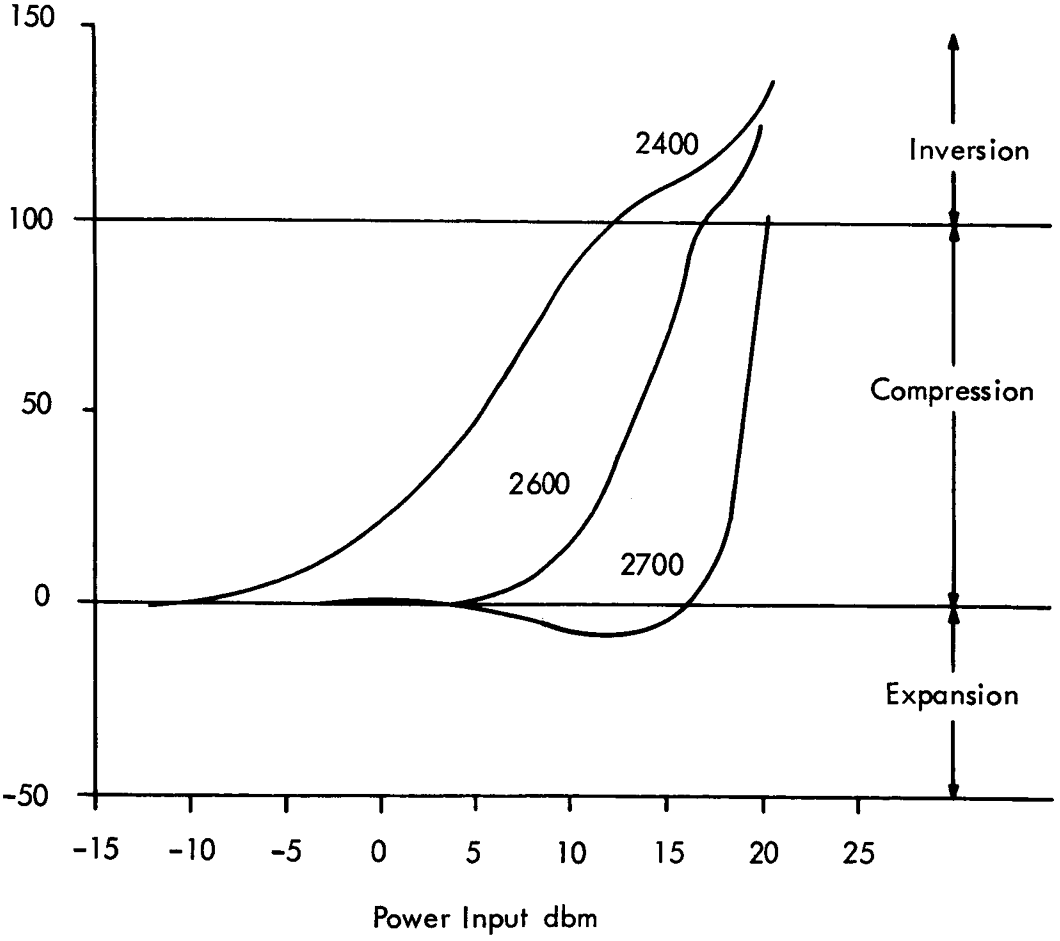

Figure 5-3. Compression vs Input Power

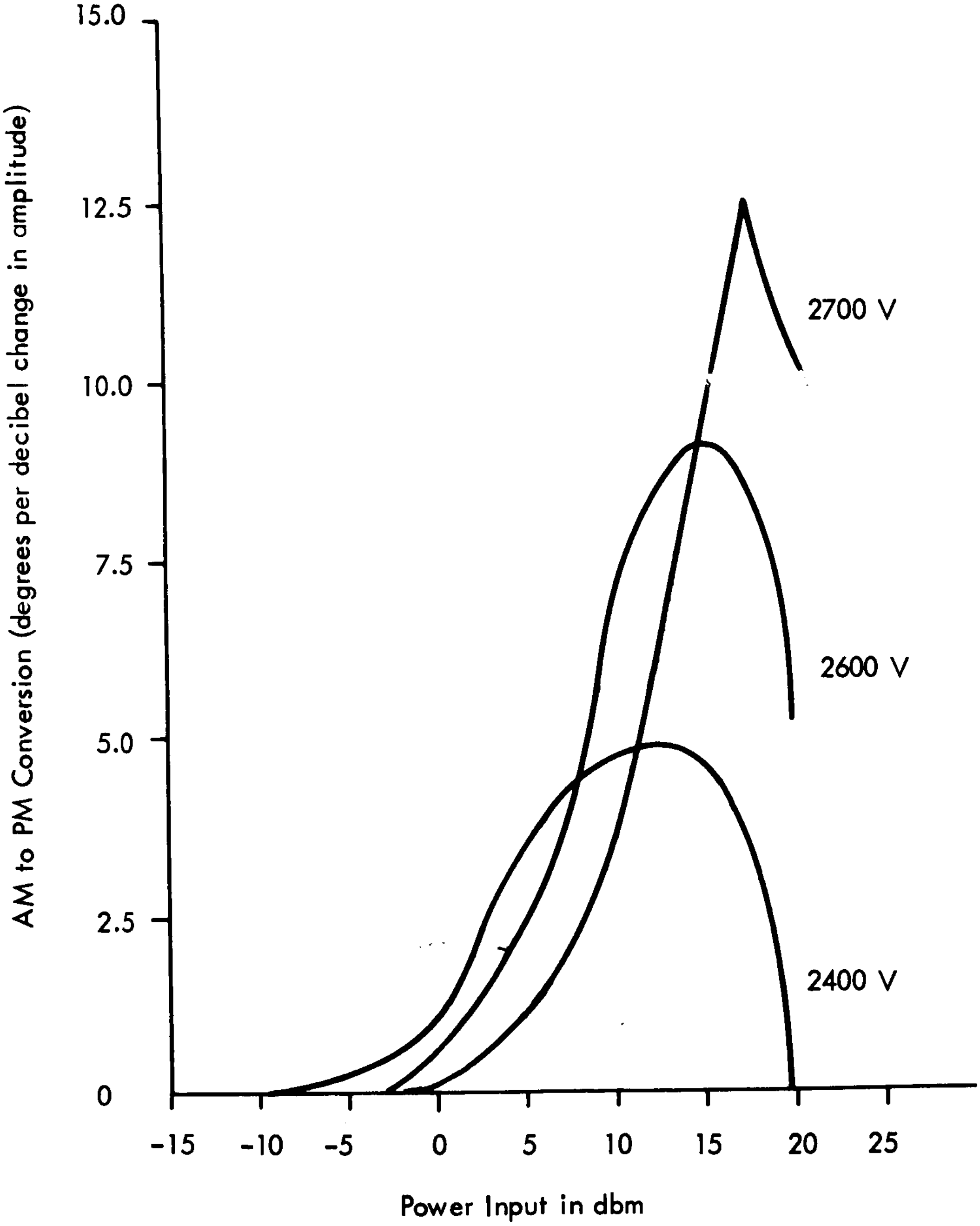

Figure 5-4. AM to PM Conversion in Degrees Per db

In order to conform with the measure of conversion in terms of degree per db of amplitude change of Figure 5-4, it can be readily shown that

$$k_p = 0.151 \ \frac{\text{phase angle change in degree}}{\text{amplitude change in db}}$$

In addition to the phase distortions discussed above, the supply voltage stability and output matching conditions are additional factors that give rise to phase distortion. For the large gain, small output power region, Reference 23 develops numerical relationships between the phase distortion and the independent variables that give rise to the phase distortion. The functional relationship is based on Pierce's small signal theory[24] which does not apply outside of the large gain region.

In satellite applications, as in the Telstar experiment, in order to improve the efficiency and to obtain maximum output power, the tube will be operated under over voltage conditions. In the case of the KP 24 tube used in the Telstar experiment, the maximum low level gain is 47 db and is obtained with a helix voltage of 1380 volts. However, the operating point was chosen at a helix voltage of 1520 volts in order to obtain the desired output power and to achieve higher efficiencies. The output power at saturation in this case is approximately 37 dbm. It appears that an improvement has been achieved in the Bell Laboratories

KP 24 tube[25] with regards to AM to PM conversion than that shown in Figure 5-4 for the Bell Laboratories M1789 tube.

5.3 TWT Effects on System Performance — Preliminary Discussion

The preceding discussion points out the fact that the satellite electronics design must be based on choosing an operating point for the tube to obtain the required output power and higher efficiencies, but a thorough examination of the effects of fading, compression, and conversion at the operating point must be performed. The effects of fading are difficult to assess in light of the lag in time of the output change after the input. This behavior of the TWT can be considered as that ascribed to a non-linear device with memory. If the tube operating point is chosen at a place where the fading effects are severe, as shown in Figure 5-2, where the helix voltage is adjusted for maximum output power, the modulation technique used in the multiple-access system requires careful selection. Off-on signal modulation techniques such as f-t hopping techniques may be susceptible to the effects of fading more so than in the case of a CW modulation method.

The distortion effects of compression due to multiple input signals can be analyzed through the techniques used to handle the no memory non-linear devices. The work in References 26, 27, 28, and 29 are pertinent.

Reference 26 deals with the conversion problem to some extent. There is some question as to whether the effects of conversion are considered at all in the tube operating point selection discussed in Reference 27, where the CCIR standard concerning the noise power ratio is used to determine the level of intermodulation distortion. Further investigation of the distortion problem will be continued during the next reporting period.

GLOSSARY

TWT	=	Traveling Wave Tube
PN	=	Pseudo Noise
V_o	=	TWT output voltage
V_i	=	TWT input voltage
ΔV_o	=	Change in TWT output voltage
ΔV_i	=	Change in TWT input voltage
K_p	=	Measure of AM to PM conversion
c	=	Compression
f-t	=	frequency-time

SECTION 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 PN Modulation Techniques

Pseudo-noise communications techniques show significant promise as possible solutions to the multiple access satellite communications problem. The number of modulation techniques within the PN class is extremely large. The techniques which are recommended for more detailed study and optimization will meet the preliminary requirements of the Design Point System Model. These techniques are representative members of two broad sub-classes of pseudo-noise techniques: those that use passive correlation (matched filter) reception and those that use active correlation (locked) reception. Both techniques can be implemented with state of the art components.

Preliminary results indicate that the small station system capacity is thermal noise limited while the larger stations are both thermal noise and clutter limited. Efficient communications through clutter and thermal noise require the use of higher-order signal alphabets. This requirement is compatible with PN modulation techniques. Beyond a point (say, a 128-order alphabet) however, the case of higher order signal alphabets leads to extremely complex apparatus.

The hard limiter is an important signal amplitude normalization device which should precede the TWT in the satellite. The degradation caused by the hard limiter is in the neighborhood of 1 db. This loss can be more than overcome by applying a constant level signal to TWT at the optimum operating point.

The effect of the TWT on communications efficiency and fidelity is significant. This is true for pseudo-noise and conventional modulation techniques. This problem must be studied and understood thoroughly.

Finally, the systems configurations which have been developed in Section 4.6 are preliminary. These will be optimized in Phase III.

The two PN multiplexing techniques which have been selected as valid candidates for further study and optimization in Phase III are:

a. PN modulation with Matched Filter Reception

b. PN modulation with Correlation Locked Reception

The preliminary study indicates that matched filter reception will use the channel capacity more efficiently than correlation-locked reception, although the latter appears to require less hardware. The channel utilization efficiency of the correlation locked-techniques can approach matched filter reception at the expense of increased complexity.

These two PN configurations will be the starting point for detailed investigations and optimization in Phase III. They will then be compared with the selected conventional techniques.

6.2 Conventional Modulation Techniques

It has been shown in preliminary analyses that conventional modulation techniques can function effectively with a communication satellite in a multiple access mode of operation, both with and without a central coordinating ground station. Three principal types of satellite signal processors have been identified as follows:

a. 'Transparent" —attractive because of its simplicity, but requires wideband up-link transmission.

b. Compound Modulation — additional stage of modulation applied to the composite signal arriving at satellite to make up for low power available for down-link transmission. (Increase degree of modulation —with multiple access, apparently feasible only as a special case using SSBFM).

c. Detect-and-remodulate—principally of advantage in an uncontrolled electromagnetic environment.

All satellites need frequency translation to isolate transmitters from receivers.

Simultaneous, uncoordinated multiple carriers lead to a signal with peaks much larger than their average values. If this type of signal is used on the down-link, the satellite power output must be significantly decreased. The most efficient use of the TWT requires a constant level input. Therefore, either angle modulation of the composite signal or TDM is recommended.

In the case of TDM, it is important to employ signals having both high average power and a high duty factor. To reduce TDM synchronization problems, burst transmission can be used. The technique is applicable to both pulse and analog systems. Burst transmission allows high utilization of the satellite TWT power with (say) PCM or FM. Also, ancillaries on the ground (such as compandors, or encoders) can be shared. Added delays (using a stationary satellite)of 10 to 100 milliseconds will not appreciably affect the system's performance. Burst transmission requires storage devices at the transmitter (capable of fast read out) and at the receiver (capable of fast read in). Similarly, shared ancillaries must be capable of fast (wide band) response.

Preliminary calculations and block diagramming of representative combinations of modulation and multiplexing (using parameters and objectives of the Design Point System Model) indicate that the equipment penalty required to implement the callup portion of a conventional system is sufficiently small as to be an unimportant factor in the overall system selection. Further, the analysis also indicates that the most promising conventional modulation techniques do not require components either overly expensive or beyond the state-of-the-art.

The system which has shown the best signal-to-thermal noise performance (viz., SSB/FDM - Composite FM) is recommended for study in depth, optimization, and comparison with pseudo-noise systems.

In addition, it is recommended that a second conventional technique, PCM/TDM - PCM/TDM, be included in the Phase III evaluation. Its good performance in the signal-to-thermal-noise ratio analysis, the simplicity of the satellite repeater, and the possible future requirement for secure communications make this technique a valid candidate for further study.

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